Zadatak 101 (Viky, maturantica)

Riješite jednadžbu: $\log_{0.5} x \cdot \log_2 x \cdot \log_{\sqrt{16}} x = 4.$

Rješenje 101

Ponovimo!

$$\log_b a = \frac{1}{\log_a b} \quad , \quad \log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \log_b f(x) = \log_b g(x) \Rightarrow f(x) = g(x).$$

$$\log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log_b a = c \Leftrightarrow b^c = a.$$

1.inačica

$$\log_{0.5} x \cdot \log_2 x \cdot \log_{\sqrt{16}} x = 4 \Rightarrow \begin{bmatrix} \text{Diskusija!} \\ x > 0 \end{bmatrix} \Rightarrow \log_{\frac{5}{10}} x \cdot \log_2 x \cdot \log_4 x = 4 \Rightarrow \log_{\frac{1}{2}} x \cdot \log_2 x \cdot \log_{2^2} x = 4 \Rightarrow$$

$$\Rightarrow \log_{2^{-1}} x \cdot \log_2 x \cdot \log_{2^2} x = 4 \Rightarrow -\log_2 x \cdot \log_2 x \cdot \frac{1}{2} \cdot \log_2 x = 4 \Rightarrow -\frac{1}{2} \cdot \left(\log_2 x\right)^3 = 4 \,/\cdot(-2) \Rightarrow$$

$$\Rightarrow \left(\log_2 x\right)^3 = -8 \,/\sqrt[3]{} \Rightarrow \log_2 x = \sqrt[3]{-8} \Rightarrow \log_2 x = -2 \Rightarrow \log_2 x = \log_2 2^{-2} \quad x = 2^{-2} \Rightarrow x = \frac{1}{2^2} \Rightarrow x = \frac{1}{4}.$$

2.inačica

$$\log_{0.5} x \cdot \log_2 x \cdot \log_{\sqrt{16}} x = 4 \Rightarrow \begin{bmatrix} \text{Diskusija!} \\ x > 0 \end{bmatrix} \Rightarrow \frac{\log x}{\log 0.5} \cdot \frac{\log x}{\log 2} \cdot \frac{\log x}{\log \sqrt{16}} = 4 \Rightarrow$$

$$\Rightarrow \frac{\log x}{\log \frac{5}{10}} \cdot \frac{\log x}{\log 2} \cdot \frac{\log x}{\log 4} = 4 \Rightarrow \frac{\log x}{\log \frac{1}{2}} \cdot \frac{\log x}{\log 2} \cdot \frac{\log x}{\log 2^2} = 4 \Rightarrow \frac{\log x}{\log 1 - \log 2} \cdot \frac{\log x}{\log 2} \cdot \frac{\log x}{2 \cdot \log 2} = 4 \Rightarrow$$

$$\Rightarrow \frac{\log x}{0 - \log 2} \cdot \frac{\log x}{\log 2} \cdot \frac{\log x}{2 \cdot \log 2} = 4 \Rightarrow \frac{\log x}{-\log 2} \cdot \frac{\log x}{\log 2} \cdot \frac{\log x}{2 \cdot \log 2} = 4 \Rightarrow -\frac{1}{2} \cdot \frac{\log x}{\log 2} \cdot \frac{\log x}{\log 2} \cdot \frac{\log x}{\log 2} = 4 \Rightarrow$$

$$\Rightarrow -\frac{1}{2} \cdot \left(\frac{\log x}{\log 2}\right)^3 = 4 \,/\cdot(-2) \Rightarrow \left(\frac{\log x}{\log 2}\right)^3 = -8 \,/\sqrt[3]{} \Rightarrow \frac{\log x}{\log 2} = \sqrt[3]{-8} \Rightarrow \frac{\log x}{\log 2} = -2 \Rightarrow \log x = -2 \cdot \log 2 \Rightarrow$$

$$\Rightarrow \log x = \log 2^{-2} \Rightarrow x = 2^{-2} \Rightarrow x = \frac{1}{2^2} \Rightarrow x = \frac{1}{4}.$$

3.inačica

$$\log_{0.5} x \cdot \log_2 x \cdot \log_{\sqrt{16}} x = 4 \Rightarrow \begin{bmatrix} \text{Diskusija!} \\ x > 0 \end{bmatrix} \Rightarrow \frac{1}{\log_x 0.5} \cdot \frac{1}{\log_x 2} \cdot \frac{1}{\log_x \sqrt{16}} = 4 \Rightarrow$$

$$\Rightarrow \frac{1}{\log_x \frac{5}{10}} \cdot \frac{1}{\log_x 2} \cdot \frac{1}{\log_x 4} = 4 \Rightarrow \frac{1}{\log_x \frac{1}{2}} \cdot \frac{1}{\log_x 2} \cdot \frac{1}{\log_x 2^2} = 4 \Rightarrow \frac{1}{\log_x 1 - \log_x 2} \cdot \frac{1}{\log_x 2} \cdot \frac{1}{2 \cdot \log_x 2} = 4 \Rightarrow$$

$$\Rightarrow \frac{1}{0 - \log_x 2} \cdot \frac{1}{\log_x 2} \cdot \frac{1}{2 \cdot \log_x 2} = 4 \Rightarrow \frac{1}{-\log_x 2} \cdot \frac{1}{\log_x 2} \cdot \frac{1}{2 \cdot \log_x 2} = 4 \Rightarrow -\frac{1}{2} \cdot \left(\frac{1}{\log_x 2}\right)^3 = 4 \,/\cdot(-2) \Rightarrow$$

$$\Rightarrow \left(\frac{1}{\log_x 2}\right)^3 = -8 \,/\sqrt[3]{} \Rightarrow \frac{1}{\log_x 2} = \sqrt[3]{-8} \Rightarrow \frac{1}{\log_x 2} = -2 \Rightarrow \log_x 2 = -\frac{1}{2} \Rightarrow x^{-\frac{1}{2}} = 2 \Rightarrow$$

$$\Rightarrow \frac{1}{x^{\frac{1}{2}}} = 2 \Rightarrow \frac{1}{\sqrt{x}} = 2 \Rightarrow \sqrt{x} = \frac{1}{2} \,/^2 \Rightarrow x = \frac{1}{4}.$$

Vježba 101

Riješite jednadžbu: $\log_2 x \cdot \log_{\sqrt{4}} x = 4.$

Rezultat: x = 4.

Zadatak 102 (Viky, maturantica)

Riješite nejednadžbu: $\log_3 \frac{x-3}{x+3} > 0.$

Rješenje 102

Ponovimo!

$$\log_b 1 = 0 \quad , \quad \log_b f(x) > \log_b g(x) \ , \ b > 1 \Rightarrow f(x) > g(x).$$

Najprije napravimo diskusiju, tj. odredimo područje definicije zadane funkcije. Budući da je domena logaritamske funkcije interval $\langle 0, +\infty \rangle$, tj. f(x) = log x => x > 0, vrijedi:

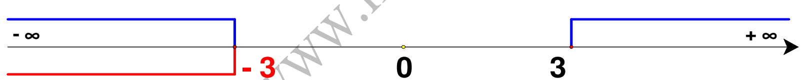
$$\frac{x-3}{x+3} > 0 \Rightarrow \left.\begin{array}{l} \left.\begin{array}{l} x-3>0 \\ x+3>0 \end{array}\right\} \\ \\ \left.\begin{array}{l} x-3<0 \\ x+3<0 \end{array}\right\} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \left.\begin{array}{l} x>3 \\ x>-3 \end{array}\right\} \\ \\ \left.\begin{array}{l} x<3 \\ x<-3 \end{array}\right\} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x>3 \\ \\ x<-3 \end{array}\right\} \Rightarrow x \in \langle -\infty, -3 \rangle \cup \langle 3, +\infty \rangle. \quad (1)$$

Rješavamo nejednadžbu:

$$\log_3 \frac{x-3}{x+3} > 0 \Rightarrow \log_3 \frac{x-3}{x+3} > \log_3 1 \Rightarrow \frac{x-3}{x+3} > 1 \Rightarrow \frac{x-3}{x+3} - 1 > 0 \Rightarrow \frac{x-3-x-3}{x+3} > 0 \Rightarrow$$

$$\Rightarrow \frac{-6}{x+3} > 0 \Rightarrow \left[\begin{array}{l} \text{brojnik je uvijek} \\ \text{negativan} \end{array}\right] \Rightarrow x+3 < 0 \Rightarrow x < -3. \quad (2)$$

Konačno rješenje je presjek rješenja (1) i (2):



$$x \in \langle -\infty, -3 \rangle \text{ ili } x < -3.$$

Vježba 102

Riješite nejednadžbu: $\log_3 \frac{x-2}{x+2} > 0.$

Rezultat: $x \in \langle -\infty, -2 \rangle$ ili $x < -2.$

Zadatak 103 (Anamarija, gimnazija)

Dokažite: $\frac{1}{\log_{\frac{1}{2}} \pi} + \frac{1}{\log_{20} \pi} > 2.$ (Uputa: koristite $\pi^2 < 10.$)

Rješenje 103

Ponovimo!

$$\log_b a = \frac{1}{\log_a b} \quad , \quad \log_b (x \cdot y) = \log_b x + \log_b y \quad , \quad \log_b x > \log_b y \ , \ b > 1 \Rightarrow x > y \quad , \quad \log_b b = 1.$$

$$\frac{1}{\log_{\frac{1}{2}} \pi} + \frac{1}{\log_{20} \pi} = \log_\pi \frac{1}{2} + \log_\pi 20 = \log_\pi \left(\frac{1}{2} \cdot 20\right) = \log_\pi 10.$$

Koristimo uvjet $10 > \pi^2$:

$$\log_\pi 10 > \log_\pi \pi^2 \Rightarrow \log_\pi 10 > 2 \cdot \log_\pi \pi \Rightarrow \log_\pi 10 > 2 \cdot 1 \Rightarrow \log_\pi 10 > 2.$$

Vježba 103

Dokažite: $\frac{1}{\log_2 \pi}+\frac{1}{\log_5 \pi}>1.$ (Uputa: koristite $10>\pi$.)

Rezultat: Dokaz analogan dokazu u zadatku.

Zadatak 104 (Mario, strojarska škola)

Ako je $\log_5 2=a$, $\log_5 3=b$ izračunajte $\log_{45} 100$.

Rješenje 104

Ponovimo!

$$\log_b a=\frac{\log_c a}{\log_c b} \quad , \quad \log_b a^n=n\cdot\log_b a \quad , \quad \log_b(x\cdot y)=\log_b x+\log_b y \quad , \quad \log_b b=1.$$

$$\log_{45}100=\frac{\log_5 100}{\log_5 45}=\frac{\log_5 10^2}{\log_5(9\cdot 5)}=\frac{2\cdot\log_5 10}{\log_5\left(3^2\cdot 5\right)}=\frac{2\cdot\log_5(2\cdot 5)}{\log_5 3^2+\log_5 5}=\frac{2\cdot\left(\log_5 2+\log_5 5\right)}{2\cdot\log_5 3+\log_5 5}=\frac{2\cdot(a+1)}{2\cdot b+1}=\frac{2\cdot a+2}{2\cdot b+1}.$$

Vježba 104

Ako je $\log_5 2=a$, $\log_5 3=b$ izračunajte $\log_5 18$.

Rezultat: a + 2 · b.

Zadatak 105 (Nina, gimnazija)

Dokažite da za svaki $x\in R^+$, $x\neq 1$ vrijedi $\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}=\frac{1}{\log_{24} x}$.

Rješenje 105

Ponovimo!

$$\log_b a=\frac{1}{\log_a b} \quad , \quad \log_b(x\cdot y)=\log_b x+\log_b y.$$

1.inačica

$$\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}=\log_x 2+\log_x 3+\log_x 4=\log_x(2\cdot 3\cdot 4)=\log_x 24=\frac{1}{\log_{24} x}.$$

2.inačica

$$\frac{1}{\log_{24} x}=\log_x 24=\log_x(2\cdot 3\cdot 4)=\log_x 2+\log_x 3+\log_x 4=\frac{1}{\log_2 x}+\frac{1}{\log_3 x}+\frac{1}{\log_4 x}.$$

Vježba 105

Dokažite da za svaki $x\in R^+$, $x\neq 1$ vrijedi $\frac{1}{\log_2 x}+\frac{1}{\log_3 x}=\frac{1}{\log_6 x}$.

Rezultat: Dokaz analogan dokazu u zadatku.

Zadatak 106 (Vedran, srednja škola)

Riješite jednadžbu: $7^{x+1}-2^{x-1}=5\cdot 7^x+3\cdot 2^{x+3}$.

Rješenje 106

Ponovimo!

$$a^{x+y}=a^x\cdot a^y \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$7^{x+1}-2^{x-1}=5\cdot 7^x+3\cdot 2^{x+3} \Rightarrow 7^{x+1}-5\cdot 7^x=3\cdot 2^{x+3}+2^{x-1} \Rightarrow$$

$$\Rightarrow 7^x\cdot 7-5\cdot 7^x=3\cdot 2^x\cdot 2^3+2^x\cdot 2^{-1} \Rightarrow 7^x\cdot(7-5)=2^x\cdot\left(3\cdot 2^3+2^{-1}\right) \Rightarrow$$

$$\Rightarrow 7^x \cdot 2 = 2^x \cdot \left(3 \cdot 8 + \frac{1}{2}\right) \Rightarrow 7^x \cdot 2 = 2^x \cdot \left(24 + \frac{1}{2}\right) \Rightarrow 7^x \cdot 2 = 2^x \cdot \frac{49}{2} \ / \cdot \frac{1}{2} \Rightarrow$$

$$\Rightarrow 7^x = 2^x \cdot \frac{49}{4} \Rightarrow 7^x = 2^x \cdot \frac{49}{4} \ / \cdot \frac{1}{2^x} \Rightarrow \frac{7^x}{2^x} = \frac{49}{4} \Rightarrow \left(\frac{7}{2}\right)^x = \left(\frac{7}{2}\right)^2 \Rightarrow x = 2.$$

Vježba 106

Riješite jednadžbu: $8^x = 7^{x-1} + 7^x$.

Rezultat: x = 1.

Zadatak 107 (2A, TUPŠ)

Riješite eksponencijalnu jednadžbu koristeći se (džepnim) računalom: $6^x = 3^{x+1}$.

Rješenje 107

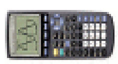
Ponovimo!

$$\log a^n = n \cdot \log a \quad , \quad \log x - \log y = \log \frac{x}{y}.$$

$$6^x = 3^{x+1} \Rightarrow \left[\begin{array}{l}\text{Budući da baze nisu jednake,} \\ \text{jednadžbu moramo logaritmirati.}\end{array}\right] \Rightarrow 6^x = 3^{x+1} \ / \log \Rightarrow \log 6^x = \log 3^{x+1} \Rightarrow$$

$$\Rightarrow x \cdot \log 6 = (x+1) \cdot \log 3 \Rightarrow x \cdot \log 6 = x \cdot \log 3 + \log 3 \Rightarrow x \cdot \log 6 - x \cdot \log 3 = \log 3 \Rightarrow$$

$$\Rightarrow x \cdot (\log 6 - \log 3) = \log 3 \Rightarrow x \cdot \log \frac{6}{3} = \log 3 \Rightarrow x \cdot \log 2 = \log 3 \ / : \log 2 \Rightarrow x = \frac{\log 3}{\log 2} \Rightarrow$$



$$\Rightarrow x = \frac{0.47712}{0.30103} \Rightarrow x = 1.58496.$$

Vježba 107

Riješite eksponencijalnu jednadžbu koristeći se (džepnim) računalom: $10^x = 5^{x+1}$.

Rezultat: x = 2.32193.

Zadatak 108 (Los-Habalos, gimnazija)

Ako je $a = \log 3$, tada je $\log_2 12$ jednako:

A) $2 + a \cdot \log_2 10$ B) $4 \cdot a \cdot \log_2 10$ C) $2 \cdot a$ D) 1 E) $4 \cdot a$

Rješenje 108

Ponovimo!

$$\log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log(x+y) = \log x + \log y \quad , \quad \log a^n = n \cdot \log a \quad , \quad \log_b a = \frac{1}{\log_a b} \quad , \quad \log_b b = 1.$$

1.inačica

$$\log_2 12 = \frac{\log 12}{\log 2} = \frac{\log(4 \cdot 3)}{\log 2} = \frac{\log 4 + \log 3}{\log 2} = \frac{\log 2^2 + \log 3}{\log 2} = \frac{2 \cdot \log 2 + \log 3}{\log 2} = \frac{2 \cdot \log 2}{\log 2} + \frac{\log 3}{\log 2} =$$

$$= 2 + \frac{\log 3}{\log 2} = [a = \log 3] = 2 + \frac{a}{\log 2} = 2 + a \cdot \log_2 10.$$

Odgovor je pod A.

2.inačica

$$\log_2 12 = \log_2(4 \cdot 3) = \log_2 4 + \log_2 3 = \log_2 2^2 + \log_2 3 = 2 \cdot \log_2 2 + \frac{\log 3}{\log 2} = 2 \cdot 1 + \frac{\log 3}{\log 2} =$$

$$= [a = \log 3] = 2 + \frac{a}{\log 2} = 2 + a \cdot \log_2 10.$$

Odgovor je pod A.

Vježba 108

Ako je $a=\log 3$, tada je $\log_2 24$ jednako:

A) $3+a\cdot\log_2 10$ B) $3\cdot a\cdot\log_2 10$ C) $3\cdot a$ D) 3 E) $6\cdot a$

Rezultat: Odgovor je pod A.

Zadatak 109 (2A, TUPŠ)

Riješite jednadžbu: $\log_2 x-\log_2(3\cdot x-1)=0.$

Rješenje 109

Ponovimo!

$$\log_b x-\log_b y=\log_b\frac{x}{y} \quad , \quad \log_b 1=0 \quad , \quad \log_b f(x)=\log_b g(x) \Rightarrow f(x)=g(x).$$

1.inačica

$$\log_2 x-\log_2(3\cdot x-1)=0 \Rightarrow \left[\begin{array}{c}\text{diskusija}\\ \left.\begin{array}{l}x>0\\ 3\cdot x-1>0\end{array}\right\} \Rightarrow \left.\begin{array}{l}x>0\\ x>\frac{1}{3}\end{array}\right\} \Rightarrow x>\frac{1}{3}\end{array}\right] \Rightarrow \log_2\frac{x}{3\cdot x-1}=\log_2 1 \Rightarrow$$

$$\Rightarrow \frac{x}{3\cdot x-1}=1 \Rightarrow x=3\cdot x-1 \Rightarrow x-3\cdot x=-1 \Rightarrow -2\cdot x=-1 \;/:(-2) \Rightarrow x=\frac{1}{2}.$$

2.inačica (Natino elegantnije rješenje)

$$\log_2 x-\log_2(3\cdot x-1)=0 \Rightarrow \left[\begin{array}{c}\text{diskusija}\\ \left.\begin{array}{l}x>0\\ 3\cdot x-1>0\end{array}\right\} \Rightarrow \left.\begin{array}{l}x>0\\ x>\frac{1}{3}\end{array}\right\} \Rightarrow x>\frac{1}{3}\end{array}\right] \Rightarrow \log_2 x=\log_2(3\cdot x-1) \Rightarrow$$

$$\Rightarrow x=3\cdot x-1 \Rightarrow x-3\cdot x=-1 \Rightarrow -2\cdot x=-1 \;/:(-2) \Rightarrow x=\frac{1}{2}.$$

Vježba 109

Riješite jednadžbu: $\log_3 x-\log_3(2\cdot x-1)=0.$

Rezultat: x = 1.

Zadatak 110 (2A, TUPŠ)

Riješite jednadžbu: $4\cdot 2^{\log_4 x}=1.$

Rješenje 110

Ponovimo!

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \log_b a=c \Leftrightarrow b^c=a.$$

$$\log_{b^n} a=\frac{1}{n}\cdot\log_b a \quad , \quad a^0=1 \quad , \quad \log_b b^n=n.$$

1.inačica

$$4\cdot 2^{\log_4 x}=1 \Rightarrow \left[\begin{array}{c}\text{diskusija}\\ x>0\end{array}\right] \Rightarrow 2^2\cdot 2^{\log_4 x}=1 \Rightarrow 2^{2+\log_4 x}=2^0 \Rightarrow 2+\log_4 x=0 \Rightarrow \log_4 x=-2 \Rightarrow$$

$$\Rightarrow x=4^{-2} \Rightarrow x=\frac{1}{4^2} \Rightarrow x=\frac{1}{16}.$$

2.inačica

$$4\cdot 2^{\log_4 x}=1 \Rightarrow \left[\begin{array}{c}\text{diskusija}\\ x>0\end{array}\right] \Rightarrow 4\cdot 2^{\log_4 x}=1 \;/:4 \Rightarrow 2^{\log_4 x}=\frac{1}{4} \Rightarrow 2^{\log_4 x}=2^{-2} \Rightarrow$$

$$\Rightarrow \log_4 x = -2 \Rightarrow x = 4^{-2} \Rightarrow x = \frac{1}{4^2} \Rightarrow x = \frac{1}{16}.$$

3.inačica

$$4 \cdot 2^{\log_4 x} = 1 \Rightarrow \begin{bmatrix} \text{diskusija} \\ x > 0 \end{bmatrix} \Rightarrow 4 \cdot 2^{\log_{2^2} x} = 1 \Rightarrow 4 \cdot 2^{\frac{1}{2} \cdot \log_2 x} = 1 \Rightarrow 4 \cdot 2^{\log_2 \sqrt{x}} = 1 \Rightarrow$$

$$\Rightarrow 4 \cdot \sqrt{x} = 1 \;/:4 \Rightarrow \sqrt{x} = \frac{1}{4} \;/^2 \Rightarrow x = \frac{1}{16}.$$

Vježba 110

Riješite jednadžbu: $2 \cdot 2^{\log_4 x} = 1.$

Rezultat: $x = \frac{1}{2}.$

Zadatak 111 (2A, TUPŠ)

Riješite jednadžbu: $3^{5^x} = 81.$

Rješenje 111

Ponovimo!

$$\log a^n = n \cdot \log a \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

$$3^{5^x} = 81 \Rightarrow 3^{5^x} = 3^4 \Rightarrow 5^x = 4 \Rightarrow \begin{bmatrix} \text{logaritmiramo} \\ \text{jednadžbu} \end{bmatrix} \Rightarrow 5^x = 4 \;/\log \Rightarrow \log 5^x = \log 4 \Rightarrow$$

$$\Rightarrow x \cdot \log 5 = \log 4 \;/:\log 5 \Rightarrow x = \frac{\log 4}{\log 5} \Rightarrow x = 0.86135.$$

Vježba 111

Riješite jednadžbu: $3^{5^x} = 27.$

Rezultat: x = 0.68261.

Zadatak 112 (2A, TUPŠ)

Riješite jednadžbu: $3^{5^x} = 7.$

Rješenje 112

Ponovimo!

$$\log a^n = n \cdot \log a \quad , \quad \log \frac{a}{b} = \log a - \log b.$$

Jednadžbu moramo dva puta logaritmirati:

$$3^{5^x} = 7 \Rightarrow \begin{bmatrix} \text{logaritmiramo} \\ \text{jednadžbu} \end{bmatrix} \Rightarrow 3^{5^x} = 7 \;/\log \Rightarrow \log 3^{5^x} = \log 7 \Rightarrow 5^x \cdot \log 3 = \log 7 \Rightarrow$$

$$\Rightarrow 5^x \cdot \log 3 = \log 7 \;/:\log 3 \Rightarrow 5^x = \frac{\log 7}{\log 3} \Rightarrow \begin{bmatrix} \text{ponovno logaritmiramo} \\ \text{jednadžbu} \end{bmatrix} \Rightarrow 5^x = \frac{\log 7}{\log 3} \;/\log \Rightarrow \log 5^x = \log \frac{\log 7}{\log 3} \Rightarrow$$

$$\Rightarrow x \cdot \log 5 = \log\log 7 - \log\log 3 \;/:\log 5 \Rightarrow x = \frac{\log\log 7 - \log\log 3}{\log 5} \Rightarrow x = 0.35521.$$

Vježba 112

Riješite jednadžbu: $3^{5^x} = 5.$

Rezultat: x = 0.23725.

Zadatak 113 (Maja, medicinska škola)

Riješite jednadžbu: $\sqrt[3]{2^x} \cdot \sqrt[3]{3^x} = 1296.$

Rješenje 113

Ponovimo!

$$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}\quad,\quad a^{x}\cdot b^{x}=(a\cdot b)^{x}\quad,\quad \sqrt[n]{a^{m}}=a^{\frac{m}{n}}\quad,\quad a^{f(x)}=a^{g(x)}\Rightarrow f(x)=g(x).$$

1.inačica

$$\sqrt[3]{2^{x}}\cdot\sqrt[3]{3^{x}}=1296\Rightarrow\sqrt[3]{2^{x}\cdot 3^{x}}=1296\Rightarrow\sqrt[3]{(2\cdot 3)^{x}}=6^{4}\Rightarrow\sqrt[3]{6^{x}}=6^{4}\Rightarrow$$

$$\Rightarrow 6^{\frac{x}{3}}=6^{4}\Rightarrow\frac{x}{3}=4\ /\cdot 3\Rightarrow x=12.$$

2.inačica

$$\sqrt[3]{2^{x}}\cdot\sqrt[3]{3^{x}}=1296\Rightarrow 2^{\frac{x}{3}}\cdot 3^{\frac{x}{3}}=1296\Rightarrow(2\cdot 3)^{\frac{x}{3}}=6^{4}\Rightarrow 6^{\frac{x}{3}}=6^{4}\Rightarrow\frac{x}{3}=4\ /\cdot 3\Rightarrow x=12.$$

Vježba 113

Riješite jednadžbu: $\sqrt[3]{2^{x}}\cdot\sqrt[3]{3^{x}}=216.$

Rezultat: x = 9.

Zadatak 114 (Carmen, ekonomska škola)

Nađite rješenje sustava eksponencijalnih jednadžbi:

$$\begin{cases}8^{2\cdot x+1}=32\cdot 2^{4\cdot y-1}\\ 5\cdot 5^{x-y}=\sqrt{25^{2\cdot y+1}}\end{cases}.$$

Rješenje 114

Ponovimo!

$$a^{n}\cdot a^{m}=a^{n+m}\quad,\quad\left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad\sqrt[n]{a^{m}}=a^{\frac{m}{n}}\quad,\quad a^{f(x)}=a^{g(x)}\Rightarrow f(x)=g(x).$$

$$\left.\begin{array}{l}8^{2\cdot x+1}=32\cdot 2^{4\cdot y-1}\\ 5\cdot 5^{x-y}=\sqrt{25^{2\cdot y+1}}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\left(2^{3}\right)^{2\cdot x+1}=2^{5}\cdot 2^{4\cdot y-1}\\ 5^{1+x-y}=\sqrt{\left(5^{2}\right)^{2\cdot y+1}}\end{array}\right\}\Rightarrow\left.\begin{array}{l}2^{6\cdot x+3}=2^{5+4\cdot y-1}\\ 5^{x-y+1}=\sqrt{5^{4\cdot y+2}}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}2^{6\cdot x+3}=2^{4\cdot y+4}\\ 5^{x-y+1}=5^{\frac{4\cdot y+2}{2}}\end{array}\right\}\Rightarrow\left.\begin{array}{l}2^{6\cdot x+3}=2^{4\cdot y+4}\\ 5^{x-y+1}=5^{2\cdot y+1}\end{array}\right\}\Rightarrow\left.\begin{array}{l}6\cdot x+3=4\cdot y+4\\ x-y+1=2\cdot y+1\end{array}\right\}\Rightarrow\left.\begin{array}{l}6\cdot x-4\cdot y=4-3\\ x-y-2\cdot y=1-1\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}6\cdot x-4\cdot y=1\\ x-3\cdot y=0\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda suprotnih}\\ \text{koeficijenata}\end{array}\right]\Rightarrow\left.\begin{array}{l}6\cdot x-4\cdot y=1\ /\cdot 3\\ x-3\cdot y=0\ /\cdot(-4)\end{array}\right\}\Rightarrow\left.\begin{array}{l}18\cdot x-12\cdot y=3\\ -4\cdot x+12\cdot y=0\end{array}\right\}\Rightarrow$$

$$\Rightarrow 14\cdot x=3\Rightarrow x=\frac{3}{14}\Rightarrow\left.\begin{array}{l}x=\frac{3}{14}\\ x-3\cdot y=0\end{array}\right\}\Rightarrow\frac{3}{14}-3\cdot y=0\Rightarrow\frac{3}{14}=3\cdot y\ /\cdot\frac{1}{3}\Rightarrow y=\frac{1}{14}\Rightarrow(x,\ y)=\left(\frac{3}{14},\frac{1}{14}\right).$$

Vježba 114

Nađite rješenje sustava eksponencijalnih jednadžbi:

$$\begin{cases}2^{6\cdot x+3}=2^{4\cdot y-4}\\ 5^{x-y+1}=\sqrt{25^{2\cdot y+1}}\end{cases}.$$

Rezultat: $(x,\ y)=\left(\frac{3}{14},\frac{1}{14}\right).$

Zadatak 115 (Carmen, ekonomska škola)

Riješite logaritamsku jednadžbu: $\frac{\log x-1}{\log x+3}+\frac{\log x-3}{\log x+1}=2.$

Rješenje 115

Ponovimo!

$$(a-b)\cdot(a+b)=a^2-b^2.$$

Najprije provedimo diskusiju:

$$\left.\begin{matrix}\log x+3\neq 0\\ \log x+1\neq 0\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\log x\neq -3\\ \log x\neq -1\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x\neq 0.001\\ x\neq 0.1\end{matrix}\right\}.$$

Sada rješavamo jednadžbu:

$$\frac{\log x-1}{\log x+3}+\frac{\log x-3}{\log x+1}=2 \Rightarrow \frac{\log x-1}{\log x+3}+\frac{\log x-3}{\log x+1}=2 \ /\cdot(\log x+3)\cdot(\log x+1) \Rightarrow$$

$$\Rightarrow (\log x-1)\cdot(\log x+1)+(\log x-3)\cdot(\log x+3)=2\cdot(\log x+3)\cdot(\log x+1) \Rightarrow$$

$$\Rightarrow \log^2 x-1+\log^2 x-9=2\cdot\left(\log^2 x+\log x+3\cdot\log x+3\right) \Rightarrow$$

$$\Rightarrow \log^2 x-1+\log^2 x-9=2\cdot\log^2 x+2\cdot\log x+6\cdot\log x+6 \Rightarrow -2\cdot\log x-6\cdot\log x=6+1+9 \Rightarrow$$

$$\Rightarrow -18\cdot\log x=16 \ /:(-18) \Rightarrow \log x=-2 \Rightarrow x=0.01.$$

Vježba 115

Riješite logaritamsku jednadžbu: $\frac{2\cdot\log x-1}{\log x+3}=1.$

Rezultat: $x = 10^4$.

Zadatak 116 (Carmen, ekonomska škola)

Riješite logaritamsku jednadžbu: $\log_2\left(2^x-3\right)=2-x.$

Rješenje 116

Ponovimo!

$$\log_b a=c \Leftrightarrow b^c=a \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Najprije provedimo diskusiju:

$$2^x-3>0 \Rightarrow 2^x>3 \Rightarrow 2^x>3 \ /\log \Rightarrow \log 2^x>\log 3 \Rightarrow x\cdot\log 2>\log 3 \Rightarrow x>\frac{\log 3}{\log 2}.$$

Sada rješavamo jednadžbu:

$$\log_2\left(2^x-3\right)=2-x \Rightarrow 2^x-3=2^{2-x} \Rightarrow 2^x-3=2^2\cdot 2^{-x} \Rightarrow 2^x-3=4\cdot\frac{1}{2^x} \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{supstitucija}\\ t=2^x\end{bmatrix} \Rightarrow t-3=\frac{4}{t} \ /\cdot t \Rightarrow t^2-3\cdot t-4=0 \Rightarrow \left.\begin{matrix}a=1 \ , \ b=-3 \ , \ c=-4\\ t_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{3\pm\sqrt{9-4\cdot 1\cdot(-4)}}{2\cdot 1} \Rightarrow t_{1,2}=\frac{3\pm\sqrt{9+16}}{2} \Rightarrow t_{1,2}=\frac{3\pm\sqrt{25}}{2} \Rightarrow t_{1,2}=\frac{3\pm 5}{2} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix}t_1=\frac{3+5}{2}\\ t_2=\frac{3-5}{2}\end{matrix}\right\} \Rightarrow \left.\begin{matrix}t_1=\frac{8}{2}\\ t_2=\frac{-2}{2}\end{matrix}\right\} \Rightarrow \left.\begin{matrix}t_1=4\\ t_2=-1\end{matrix}\right\}.$$

Vraćamo se na supstituciju:

- $\left.\begin{matrix} t=2^x \\ t=4 \end{matrix}\right\} \Rightarrow 2^x=4 \Rightarrow 2^x=2^2 \Rightarrow x=2.$
- $\left.\begin{matrix} t=2^x \\ t=-1 \end{matrix}\right\} \Rightarrow 2^x=-1$ nema smisla.

Vježba 116

Riješite logaritamsku jednadžbu: $\log_3\left(2^x-3\right)=0.$

Rezultat: x = 2.

Zadatak 117 (Carmen, ekonomska škola)

Riješite sustav jednadžbi:

$$\begin{cases} 5^x \cdot 5^y = 3125 \\ 5^x + 5^y = 150. \end{cases}$$

Rješenje 117

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

1.inačica

$$\left.\begin{matrix} 5^x \cdot 5^y = 3125 \\ 5^x + 5^y = 150 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 5^{x+y} = 5^5 \\ 5^x + 5^y = 150 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x+y=5 \\ 5^x + 5^y = 150 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y=5-x \\ 5^x + 5^y = 150 \end{matrix}\right\} \Rightarrow 5^x + 5^{5-x} = 150 \Rightarrow$$

$$\Rightarrow 5^x + 5^5 \cdot 5^{-x} = 150 \Rightarrow 5^x + \frac{5^5}{5^x} = 150 \Rightarrow \begin{bmatrix} \text{supstitucija} \\ t=5^x \end{bmatrix} \Rightarrow t + \frac{3125}{t} = 150 \; / \cdot t \Rightarrow t^2 - 150 \cdot t + 3125 = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=1 \;,\; b=-150 \;,\; c=3125 \\ t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{matrix}\right\} \Rightarrow t_{1,2} = \frac{150 \pm \sqrt{22500 - 4 \cdot 1 \cdot 3125}}{2 \cdot 1} \Rightarrow t_{1,2} = \frac{150 \pm \sqrt{22500 - 12500}}{2} \Rightarrow$$

$$t_{1,2} = \frac{150 \pm \sqrt{10000}}{2} \Rightarrow t_{1,2} = \frac{150 \pm 100}{2} \Rightarrow \left.\begin{matrix} t_1 = \frac{150+100}{2} \\ t_2 = \frac{150-100}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = \frac{250}{2} \\ t_2 = \frac{50}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = 125 \\ t_2 = 25 \end{matrix}\right\}.$$

Vraćamo se na supstituciju:

$$\left.\begin{matrix} t=5^x \\ t=125 \end{matrix}\right\} \Rightarrow 5^x = 125 \Rightarrow 5^x = 5^3 \Rightarrow x_1 = 3 \Rightarrow \left.\begin{matrix} y_1 = 5 - x_1 \\ x_1 = 3 \end{matrix}\right\} \Rightarrow y_1 = 5-3 \Rightarrow y_1 = 2 \Rightarrow \left(x_1, y_1\right) = (3, 2).$$

$$\left.\begin{matrix} t=5^x \\ t=25 \end{matrix}\right\} \Rightarrow 5^x = 25 \Rightarrow 5^x = 5^2 \Rightarrow x_2 = 2 \Rightarrow \left.\begin{matrix} y_2 = 5 - x_2 \\ x_2 = 2 \end{matrix}\right\} \Rightarrow y_2 = 5-2 \Rightarrow y_2 = 3 \Rightarrow \left(x_2, y_2\right) = (2, 3).$$

2.inačica

$$\left.\begin{matrix} 5^x \cdot 5^y = 3125 \\ 5^x + 5^y = 150 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 5^{x+y} = 5^5 \\ 5^x + 5^y = 150 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{supstitucije} \\ u=5^x \;,\; v=5^y \end{bmatrix} \Rightarrow \left.\begin{matrix} u \cdot v = 3125 \\ u+v=150 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} u \cdot v = 3125 \\ v = 150-u \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow u \cdot (150-u) = 3125 \Rightarrow 150 \cdot u - u^2 = 3125 \Rightarrow -u^2 + 150 \cdot u - 3125 = 0 \; / \cdot (-1) \Rightarrow u^2 - 150 \cdot u + 3125 = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=1 \;,\; b=-150 \;,\; c=3125 \\ u_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{matrix}\right\} \Rightarrow u_{1,2} = \frac{150 \pm \sqrt{22500 - 4 \cdot 1 \cdot 3125}}{2 \cdot 1} \Rightarrow u_{1,2} = \frac{150 \pm \sqrt{22500 - 12500}}{2} \Rightarrow$$

$$u_{1,2}=\frac{150\pm\sqrt{10000}}{2}\Rightarrow u_{1,2}=\frac{150\pm100}{2}\Rightarrow\left.\begin{array}{l}u_1=\frac{150+100}{2}\\u_2=\frac{150-100}{2}\end{array}\right\}\Rightarrow\left.\begin{array}{l}u_1=\frac{250}{2}\\u_2=\frac{50}{2}\end{array}\right\}\Rightarrow\left.\begin{array}{l}u_1=125\\u_2=25\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}v_1=150-125\\v_2=150-25\end{array}\right\}\Rightarrow\left.\begin{array}{l}v_1=25\\v_2=125\end{array}\right\}.$$

Vraćamo se na supstitucije:

$$\left.\begin{array}{l}u_1=5^x\ ,\ u_1=125\\v_1=5^y\ ,\ v_1=25\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^x=125\\5^y=25\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^x=5^3\\5^y=5^2\end{array}\right\}\Rightarrow\left.\begin{array}{l}x_1=3\\y_1=2\end{array}\right\}\Rightarrow(x_1,y_1)=(3,2).$$

$$\left.\begin{array}{l}u_2=5^x\ ,\ u_2=25\\v_2=5^y\ ,\ v_2=125\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^x=25\\5^y=125\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^x=5^2\\5^y=5^3\end{array}\right\}\Rightarrow\left.\begin{array}{l}x_2=2\\y_2=3\end{array}\right\}\Rightarrow(x_2,y_2)=(2,3).$$

Vježba 117

Riješite sustav jednadžbi:

$$\begin{cases}2^x\cdot2^y=32\\2^x+2^y=12.\end{cases}$$

Rezultat: $(x_1,y_1)=(3,2)\quad,\quad(x_2,y_2)=(2,3).$

Zadatak 118 (Carmen, ekonomska škola)

Riješite sustav jednadžbi:

$$\begin{cases}2^{x+1}\cdot y^2=32\\3^{x-1}\cdot y=6.\end{cases}$$

Rješenje 118

Ponovimo!

$$a^n\cdot a^m=a^{n+m}\quad,\quad(a^n)^m=(a^m)^n=a^{n\cdot m}\quad,\quad a^{-n}=\frac{1}{a^n}\quad,\quad a^{f(x)}=a^{g(x)}\Rightarrow f(x)=g(x)$$

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n.$$

1.inačica

$$\left.\begin{array}{l}2^{x+1}\cdot y^2=32\\3^{x-1}\cdot y=6\end{array}\right\}\Rightarrow\left.\begin{array}{l}2^x\cdot2\cdot y^2=32\ /:2\\3^x\cdot\frac{1}{3}\cdot y=6\ /\cdot3\end{array}\right\}\Rightarrow\left.\begin{array}{l}2^x\cdot y^2=16\\3^x\cdot y=18\end{array}\right\}\Rightarrow\left.\begin{array}{l}2^x\cdot y^2=16\\y=\frac{18}{3^x}\end{array}\right\}\Rightarrow2^x\cdot\left(\frac{18}{3^x}\right)^2=16\Rightarrow$$

$$\Rightarrow2^x\cdot\frac{324}{(3^x)^2}=16\Rightarrow2^x\cdot\frac{324}{(3^2)^x}=16\Rightarrow2^x\cdot\frac{324}{9^x}=16\Rightarrow\frac{2^x}{9^x}\cdot324=16\ /\cdot\frac{1}{324}\Rightarrow\frac{2^x}{9^x}=\frac{16}{324}\Rightarrow$$

$$\Rightarrow\frac{2^x}{9^x}=\frac{4}{81}\Rightarrow\left(\frac{2}{9}\right)^x=\left(\frac{2}{9}\right)^2\Rightarrow x=2\Rightarrow\left.\begin{array}{l}x=2\\y=\frac{18}{3^x}\end{array}\right\}\Rightarrow y=\frac{18}{3^2}\Rightarrow y=\frac{18}{9}\Rightarrow y=2\Rightarrow(x,y)=(2,2).$$

2.inačica

$$\left.\begin{array}{l}2^{x+1}\cdot y^2=32\\3^{x-1}\cdot y=6\end{array}\right\}\Rightarrow\left.\begin{array}{l}2^x\cdot2\cdot y^2=32\ /:2\\3^x\cdot\frac{1}{3}\cdot y=6\ /\cdot3\end{array}\right\}\Rightarrow\left.\begin{array}{l}2^x\cdot y^2=16\\3^x\cdot y=18\ /^2\end{array}\right\}\Rightarrow\left.\begin{array}{l}2^x\cdot y^2=16\\(3^x)^2\cdot y^2=324\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} 2^x \cdot y^2 = 16 \\ \left(3^2\right)^x \cdot y^2 = 324 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2^x \cdot y^2 = 16 \\ 9^x \cdot y^2 = 324 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{podijelimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow \frac{2^x \cdot y^2}{9^x \cdot y^2} = \frac{16}{324} \Rightarrow \frac{2^x}{9^x} = \frac{4}{81} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{9}\right)^x = \left(\frac{2}{9}\right)^2 \Rightarrow x = 2 \Rightarrow \left.\begin{matrix} x = 2 \\ 3^x \cdot y = 18 \end{matrix}\right\} \Rightarrow 3^2 \cdot y = 18 \Rightarrow 9 \cdot y = 18 \ /:9 \Rightarrow y = 2 \Rightarrow (x, y) = (2, 2).$$

Vježba 118

Riješite sustav jednadžbi:

$$\begin{cases} 2^{x+1} \cdot y^2 = 4 \\ 3^{x-1} \cdot y = 1. \end{cases}$$

Rezultat: $(x, y) = (1, 1).$

Zadatak 119 (Željka, gimnazija)

Ako je $\log_2 3 = a$, $\log_3 7 = b$, koliko je $\log_4 49$?

Rješenje 119

Ponovimo!

$$\log_b a^n = n \cdot \log_b a \quad , \quad \log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log_b a = \frac{1}{\log_a b} \quad , \quad \log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \log_{b^n} a^n = \log_b a.$$

1.inačica

$$\log_4 49 = \log_4 7^2 = 2 \cdot \log_4 7 = 2 \cdot \frac{\log_3 7}{\log_3 4} = 2 \cdot \log_3 7 \cdot \frac{1}{\log_3 4} = 2 \cdot \log_3 7 \cdot \log_4 3 = 2 \cdot \log_3 7 \cdot \log_{2^2} 3 =$$

$$= 2 \cdot \log_3 7 \cdot \frac{1}{2} \cdot \log_2 3 = \log_3 7 \cdot \log_2 3 = b \cdot a = a \cdot b.$$

2.inačica

$$\log_4 49 = \log_{2^2} 7^2 = \log_2 7 = \frac{\log_3 7}{\log_3 2} = \log_3 7 \cdot \frac{1}{\log_3 2} = \log_3 7 \cdot \log_2 3 = b \cdot a = a \cdot b.$$

Vježba 119

Ako je $\log_2 3 = a$, $\log_3 7 = b$, koliko je $\log_2 49$?

Rezultat: $2 \cdot a \cdot b.$

Zadatak 120 (Ivan, pomorska škola)

Logaritmirajte sljedeći izraz: $\frac{x^2 - y^2}{x \cdot y}$.

Rješenje 120

Ponovimo!

$$\log \frac{a}{b} = \log a - \log b \quad , \quad a^2 - b^2 = (a - b) \cdot (a + b) \quad , \quad \log(a \cdot b) = \log a + \log b.$$

Računamo:

$$\log \frac{x^2 - y^2}{x \cdot y} = \log\left(x^2 - y^2\right) - \log(x \cdot y) = \log[(x - y) \cdot (x + y)] - \log(x \cdot y) = \log(x - y) + \log(x + y) - (\log x + \log y) =$$

$$= \log(x - y) + \log(x + y) - \log x - \log y.$$

Vježba 120

Logaritmirajte sljedeći izraz: $\frac{x \cdot y}{x^2 - y^2}$.

Rezultat: $\log x + \log y - \log(x - y) - \log(x + y).$