**Zadatak 181 (Goran, gimnazija)**

Riješi jednadžbu: $8^x = 7^{x-1} + 7^x$.

**Rješenje 181**

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a \quad , \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

$$8^x = 7^{x-1} + 7^x \Rightarrow 8^x = 7^x \cdot 7^{-1} + 7^x \Rightarrow 8^x = 7^x \cdot \left(7^{-1} + 1\right) \Rightarrow 8^x = 7^x \cdot \left(\frac{1}{7} + 1\right) \Rightarrow$$

$$\Rightarrow 8^x = 7^x \cdot \frac{8}{7} \Rightarrow 8^x = 7^x \cdot \frac{8}{7} \Big/ \cdot \frac{1}{7^x} \Rightarrow \frac{8^x}{7^x} = \frac{8}{7} \Rightarrow \left(\frac{8}{7}\right)^x = \left(\frac{8}{7}\right)^1 \Rightarrow x = 1.$$

**Vježba 181**

Riješi jednadžbu: $7^x = 6^{x-1} + 6^x$.

**Rezultat:** x = 1.

**Zadatak 182 (Ljilja, srednja škola)**

Riješi jednadžbu : $7^{x+1} - 2^{x-1} = 5 \cdot 7^x + 3 \cdot 2^{x+3}$.

**Rješenje 182**

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a \quad , \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

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$$7^{x+1} - 2^{x-1} = 5 \cdot 7^x + 3 \cdot 2^{x+3} \Rightarrow 7^{x+1} - 5 \cdot 7^x = 3 \cdot 2^{x+3} + 2^{x-1} \Rightarrow$$

$$\Rightarrow 7^x \cdot 7^1 - 5 \cdot 7^x = 3 \cdot 2^x \cdot 2^3 + 2^x \cdot 2^{-1} \Rightarrow 7^x \cdot \left(7^1 - 5\right) = 2^x \cdot \left(3 \cdot 2^3 + 2^{-1}\right) \Rightarrow$$

$$\Rightarrow 7^x \cdot (7 - 5) = 2^x \cdot \left(3 \cdot 8 + \frac{1}{2}\right) \Rightarrow 7^x \cdot 2 = 2^x \cdot \left(24 + \frac{1}{2}\right) \Rightarrow 7^x \cdot 2 = 2^x \cdot \frac{49}{2} \Big/ \cdot \frac{1}{2 \cdot 2^x} \Rightarrow$$

$$\Rightarrow \frac{7^x}{2^x} = \frac{49}{4} \Rightarrow \left(\frac{7}{2}\right)^x = \left(\frac{7}{2}\right)^2 \Rightarrow x = 2.$$

**Vježba 182**

Riješi jednadžbu : $7^{x+1} - 2^{x-1} = 5 \cdot 7^x + 6 \cdot 2^{x+2}$.

**Rezultat:** x = 2.

**Zadatak 183 (Amela, studentica)**

Riješi jednadžbu : $2 + 3 \cdot \log x = -1$.

**Rješenje 183**

Ponovimo!

Definicija:

$$\log_b a = c \Leftrightarrow b^c = a.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \qquad a = b^c$$
$$\rightarrow$$

$$\log 0.1 = -1 \quad , \quad \log f(x) = \log g(x) \Rightarrow f(x) = g(x) \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a.$$

Najprije moramo napraviti diskusiju rješenja zadatka. Budući da logaritmandi (brojevi ili izrazi pod znakom logaritma) ne smiju biti negativni (logaritamska funkcija definirana je samo za pozitivne realne brojeve!), postavit ćemo sljedeću nejednadžbu koju traženo rješenje mora zadovoljiti:

$$x > 0.$$

1.inačica

$$2+3\cdot\log x=-1 \Rightarrow 3\cdot\log x=-1-2 \Rightarrow 3\cdot\log x=-3 \Rightarrow 3\cdot\log x=-3\ /:3 \Rightarrow$$

$$\Rightarrow \log x=-1 \Rightarrow \log x=\log 0.1 \Rightarrow x=0.1.$$

2.inačica

$$2+3\cdot\log x=-1 \Rightarrow 3\cdot\log x=-1-2 \Rightarrow 3\cdot\log x=-3 \Rightarrow 3\cdot\log x=-3\ /:3 \Rightarrow$$

$$\Rightarrow \log x=-1 \Rightarrow 10^{-1}=x \Rightarrow x=10^{-1} \Rightarrow x=\frac{1}{10} \Rightarrow x=0.1.$$

**Vježba 183**

Riješi jednadžbu: $1+3\cdot\log x=-2.$

**Rezultat:** 0.1.

**Zadatak 184 (Ekipa, TUPŠ)**

U jednadžbi $100\cdot 10^x = 0.01$, nepoznanica x jednaka je:

A. – 4 B. – 3 C. – 2 D. – 1

**Rješenje 184**

Ponovimo!

Definicija:

$$a^n\cdot a^m=a^{n+m} \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad a^{-n}=\frac{1}{a^n}.$$

1.inačica

$$100\cdot10^x=0.01 \Rightarrow 10^2\cdot10^x=10^{-2} \Rightarrow 10^{2+x}=10^{-2} \Rightarrow 2+x=-2 \Rightarrow x=-2-2 \Rightarrow x=-4.$$

Odgovor je pod A.

2.inačica

Do rješenja se može doći i na jednostavniji način. Provjeravamo redom odgovore pod A, B, C i D.
Računamo pod A.

$$\left.\begin{matrix}100\cdot10^x=0.01\\ x=-4\end{matrix}\right\} \Rightarrow 100\cdot10^{-4}=0.01 \Rightarrow 10^2\cdot10^{-4}=10^{-2} \Rightarrow 10^{2-4}=10^{-2} \Rightarrow 10^{-2}=10^{-2}.$$

Dobili smo točnu jednakost (identitet).
Dakle, odgovor je pod A. Ostala tri odgovora (B, C i D) ne moramo ni računati jer je samo jedan ponuđeni odgovor točan.

**Vježba 184**

U jednadžbi $1000\cdot 10^x = 0.001$, nepoznanica x jednaka je:

A. – 6 B. – 5 C. – 4 D. – 3

**Rezultat:** Odgovor je pod A.

**Zadatak 184 (Tina, gimnazija)**

Riješi nejednadžbu: $0.75^{x-1}>\frac{\sqrt{3}}{2}.$

**Rješenje 184**

Ponovimo!

$$\sqrt{a^2}=a\ ,\ a\ge 0 \quad , \quad \sqrt[n]{a^m}=a^{\frac{m}{n}} \quad , \quad \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}.$$

$a^{f(x)} > a^{g(x)}$ , $0 < a < 1 \Rightarrow f(x) < g(x)$ , $a^n \cdot a^m = a^{n+m}$ , $a \cdot \sqrt{b} = \sqrt{a^2 \cdot b}$.

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n.$$

1.inačica

$$0.75^{x-1} > \frac{\sqrt{3}}{2} \Rightarrow \left(\frac{75}{100}\right)^{x-1} > \frac{\sqrt{3}}{2} \Rightarrow \left(\frac{3}{4}\right)^{x-1} > \frac{\sqrt{3}}{2} \Rightarrow \left(\frac{3}{4}\right)^{x-1} > \frac{\sqrt{3}}{\sqrt{4}} \Rightarrow$$

$$\Rightarrow \left(\frac{3}{4}\right)^{x-1} > \sqrt{\frac{3}{4}} \Rightarrow \left(\frac{3}{4}\right)^{x-1} > \left(\frac{3}{4}\right)^{\frac{1}{2}} \Rightarrow x-1 < \frac{1}{2} \Rightarrow x < \frac{1}{2}+1 \Rightarrow x < \frac{3}{2} \Rightarrow$$

$$\Rightarrow x \in \left\langle -\infty, \frac{3}{2} \right\rangle.$$

2.inačica

$$0.75^{x-1} > \frac{\sqrt{3}}{2} \Rightarrow \left(\frac{75}{100}\right)^{x-1} > \frac{\sqrt{3}}{2} \Rightarrow \left(\frac{3}{4}\right)^{x-1} > \frac{\sqrt{3}}{2} \Rightarrow \left(\frac{3}{4}\right)^{x} \cdot \left(\frac{3}{4}\right)^{-1} > \frac{\sqrt{3}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{3}{4}\right)^{x} \cdot \left(\frac{4}{3}\right)^{1} > \frac{\sqrt{3}}{\sqrt{4}} \Rightarrow \left(\frac{3}{4}\right)^{x} \cdot \frac{4}{3} > \frac{\sqrt{3}}{\sqrt{4}} \; / \cdot \frac{3}{4} \Rightarrow \left(\frac{3}{4}\right)^{x} > \frac{3 \cdot \sqrt{3}}{4 \cdot \sqrt{4}} \Rightarrow \left(\frac{3}{4}\right)^{x} > \frac{\sqrt{3^2 \cdot 3}}{\sqrt{4^2 \cdot 4}} \Rightarrow$$

$$\Rightarrow \left(\frac{3}{4}\right)^{x} > \frac{\sqrt{3^3}}{\sqrt{4^3}} \Rightarrow \left(\frac{3}{4}\right)^{x} > \frac{3^{\frac{3}{2}}}{4^{\frac{3}{2}}} \Rightarrow \left(\frac{3}{4}\right)^{x} > \left(\frac{3}{4}\right)^{\frac{3}{2}} \Rightarrow x < \frac{3}{2} \Rightarrow x \in \left\langle -\infty, \frac{3}{2} \right\rangle.$$

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**Vježba 184**

Riješi nejednadžbu: $0.75^{x} > \frac{\sqrt{3}}{2}$

**Rezultat:** $x \in \left\langle -\infty, \frac{1}{2} \right\rangle.$

**Zadatak 185 (Riki, gimnazija)**

Ako je $2 \cdot \log \frac{x+2 \cdot y}{\sqrt{5}} = \log(x+y) + \log(x-y)$, onda $x$ i $y$ zadovoljavaju jednadžbu

*A)* $4 \cdot x^2 - 9 \cdot y^2 = 4 \cdot x \cdot y$ , *B)* $x^2 - 4 \cdot y^2 + 4 \cdot x \cdot y = 0$ , *C)* $(x-y)^2 = 5 \cdot (x - 2 \cdot y)$ , *D)* $x^2 - y^2 = 1$

**Rješenje 185**

Ponovimo!

$\log a^n = n \cdot \log a$ , $\log(a \cdot b) = \log a + \log b$ , $a^2 - b^2 = (a-b) \cdot (a+b)$.

$\log f(x) = \log g(x) \Rightarrow f(x) = g(x)$ , $(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2$ , $\left(\sqrt{a}\right)^2 = a$.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

$$2 \cdot \log \frac{x+2 \cdot y}{\sqrt{5}} = \log(x+y) + \log(x-y) \Rightarrow \log\left(\frac{x+2 \cdot y}{\sqrt{5}}\right)^2 = \log(x+y) \cdot (x-y) \Rightarrow$$

$$\Rightarrow \left(\frac{x+2\cdot y}{\sqrt{5}}\right)^2=(x+y)\cdot(x-y) \Rightarrow \frac{(x+2\cdot y)^2}{(\sqrt{5})^2}=x^2-y^2 \Rightarrow \frac{x^2+4\cdot x\cdot y+4\cdot y^2}{5}=x^2-y^2 \Rightarrow$$

$$\Rightarrow \frac{x^2+4\cdot x\cdot y+4\cdot y^2}{5}=x^2-y^2 \;/\cdot 5 \Rightarrow x^2+4\cdot x\cdot y+4\cdot y^2=5\cdot\left(x^2-y^2\right) \Rightarrow$$

$$\Rightarrow x^2+4\cdot x\cdot y+4\cdot y^2=5\cdot x^2-5\cdot y^2 \Rightarrow x^2+4\cdot x\cdot y+4\cdot y^2-5\cdot x^2+5\cdot y^2=0 \Rightarrow$$

$$\Rightarrow -4\cdot x^2+4\cdot x\cdot y+9\cdot y^2=0 \;/\cdot(-1) \Rightarrow 4\cdot x^2-4\cdot x\cdot y-9\cdot y^2=0 \Rightarrow 4\cdot x^2-9\cdot y^2=4\cdot x\cdot y.$$

Odgovor je pod A.

**Vježba 185**

Ako je $\log\frac{x+2\cdot y}{\sqrt{5}}-\frac{1}{2}\cdot\log(x+y)=\frac{1}{2}\cdot\log(x-y)$, onda $x$ i $y$ zadovoljavaju jednadžbu

$A)\ 4\cdot x^2-9\cdot y^2=4\cdot x\cdot y$ , $B)\ x^2-4\cdot y^2+4\cdot x\cdot y=0$ , $C)\ (x-y)^2=5\cdot(x-2\cdot y)$ , $D)\ x^2-y^2=1$

**Rezultat:** A.

**Zadatak 186 (Monika, Ivana, Matija, HTT)**

Riješi jednadžbu: $10^x\cdot 2^{2-x}+5^x=625.$

**Rješenje 186**

Ponovimo!

$$a^n\cdot b^n=(a\cdot b)^n \quad , \quad a^n\cdot a^m=a^{n+m} , \quad a^{-n}=\frac{1}{a^n} \quad , \quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n .$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

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1.inačica

$$10^x\cdot 2^{2-x}+5^x=625 \Rightarrow (2\cdot 5)^x\cdot 2^{2-x}+5^x=625 \Rightarrow 2^x\cdot 5^x\cdot 2^{2-x}+5^x=625 \Rightarrow$$

$$\Rightarrow 2^{x+2-x}\cdot 5^x+5^x=625 \Rightarrow 2^{x+2-x}\cdot 5^x+5^x=625 \Rightarrow 2^2\cdot 5^x+5^x=625 \Rightarrow$$

$$\Rightarrow 4\cdot 5^x+5^x=625 \Rightarrow 5^x\cdot(4+1)=625 \Rightarrow 5^x\cdot 5=625 \Rightarrow 5^x\cdot 5=625 \;/:5 \Rightarrow$$

$$\Rightarrow 5^x=125 \Rightarrow 5^x=5^3 \Rightarrow x=3.$$

2.inačica

$$10^x\cdot 2^{2-x}+5^x=625 \Rightarrow 10^x\cdot 2^2\cdot 2^{-x}+5^x=625 \Rightarrow 10^x\cdot 2^2\cdot\frac{1}{2^x}+5^x=625 \Rightarrow$$

$$\Rightarrow \frac{10^x}{2^x}\cdot 2^2+5^x=625 \Rightarrow \left(\frac{10}{2}\right)^x\cdot 4+5^x=625 \Rightarrow 5^x\cdot 4+5^x=625 \Rightarrow 5^x\cdot(4+1)=625 \Rightarrow$$

$$\Rightarrow 5^x\cdot 5=625 \Rightarrow 5^x\cdot 5=625 \;/:5 \Rightarrow 5^x=125 \Rightarrow 5^x=5^3 \Rightarrow x=3.$$

**Vježba 186**

Riješi jednadžbu: $10^x\cdot 2^{2-x}+5^x=125.$

**Rezultat:** x = 2.

**Zadatak 187 (Mirza, elektrotehnička škola)**

Riješi jednadžbu: $\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{0.125}}}=4\cdot\sqrt[3]{2}.$

**Rješenje 187**

Ponovimo!

$$a^{-n}=\frac{1}{a^{n}}\quad,\quad\sqrt[n]{a^{m}}=a^{\frac{m}{n}}\quad,\quad\left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad(a\cdot b)^{n}=a^{n}\cdot b^{n}.$$

$$a^{n}\cdot a^{m}=a^{n+m}.$$

$$a^{f(x)}=a^{g(x)}\;\Rightarrow\;f(x)=g(x)\quad,\quad\left(\sqrt{a}\right)^{2}=a\quad,\quad\left(\sqrt[3]{a}\right)^{3}=a\quad,\quad\left(\sqrt[n]{a}\right)^{n}=a.$$

1.inačica

$$\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{0.125}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{\frac{125}{1000}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{\frac{1}{8}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow$$

$$\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{\frac{1}{2^{3}}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{2^{-3}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot 2^{-\frac{3}{x}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow$$

$$\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{\left(2^{2}\right)^{x}\cdot 2^{-\frac{3}{x}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{2^{2\cdot x}\cdot 2^{-\frac{3}{x}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{2^{2\cdot x-\frac{3}{x}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow$$

$$\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{2^{\frac{2\cdot x^{2}-3}{x}}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x}\cdot 2^{\frac{2\cdot x^{2}-3}{3\cdot x}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x+\frac{2\cdot x^{2}-3}{3\cdot x}}}=4\cdot\sqrt[3]{2}\;\Rightarrow$$

$$\Rightarrow\;\sqrt{2^{\frac{3\cdot x^{2}+2\cdot x^{2}-3}{3\cdot x}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{\frac{5\cdot x^{2}-3}{3\cdot x}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;2^{\frac{5\cdot x^{2}-3}{6\cdot x}}=2^{2}\cdot 2^{\frac{1}{3}}\;\Rightarrow$$

$$\Rightarrow\;2^{\frac{5\cdot x^{2}-3}{6\cdot x}}=2^{2+\frac{1}{3}}\;\Rightarrow\;2^{\frac{5\cdot x^{2}-3}{6\cdot x}}=2^{\frac{7}{3}}\;\Rightarrow\;\frac{5\cdot x^{2}-3}{6\cdot x}=\frac{7}{3}\;/\cdot 6\cdot x\;\Rightarrow\;5\cdot x^{2}-3=14\cdot x\;\Rightarrow$$

$$\Rightarrow\;5\cdot x^{2}-14\cdot x-3=0\;\Rightarrow\;\left.\begin{aligned}&5\cdot x^{2}-14\cdot x-3=0\\&a=5\;,\;b=-14\;,\;c=-3\end{aligned}\right\}\;\Rightarrow\;\left.\begin{aligned}&a=5\;,\;b=-14\;,\;c=-3\\&x_{1,2}=\frac{-b\pm\sqrt{b^{2}-4\cdot a\cdot c}}{2\cdot a}\end{aligned}\right\}\;\Rightarrow$$

$$\Rightarrow\;x_{1,2}=\frac{14\pm\sqrt{196-4\cdot 5\cdot(-3)}}{2\cdot 5}\;\Rightarrow\;x_{1,2}=\frac{14\pm\sqrt{196+60}}{10}\;\Rightarrow\;x_{1,2}=\frac{14\pm\sqrt{256}}{10}\;\Rightarrow$$

$$\Rightarrow\;x_{1,2}=\frac{14\pm 16}{10}\;\Rightarrow\;\left.\begin{aligned}&x_{1}=\frac{14+16}{10}\\&x_{2}=\frac{14-16}{10}\end{aligned}\right\}\;\Rightarrow\;\left.\begin{aligned}&x_{1}=\frac{30}{10}\\&x_{2}=-\frac{2}{10}\end{aligned}\right\}\;\Rightarrow\;\left.\begin{aligned}&x_{1}=3\\&x_{2}=-\frac{1}{5}\end{aligned}\right\}.$$

2.inačica

$$\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{0.125}}}=4\cdot\sqrt[3]{2}\;\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{\frac{125}{1000}}}}=2^{2}\cdot 2^{\frac{1}{3}}\;\Rightarrow\;\sqrt{2^{x}\cdot\sqrt[3]{4^{x}\cdot\sqrt[x]{\frac{1}{8}}}}=2^{2+\frac{1}{3}}\;\Rightarrow$$

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$$\Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot \sqrt[x]{\frac{1}{2^3}}}} = 2^{\frac{7}{3}} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot \sqrt[x]{2^{-3}}}} = 2^{\frac{7}{3}} \Rightarrow \left[\begin{matrix}\text{kvadriramo} \\ \text{jednadžbu}\end{matrix}\right] \Rightarrow$$

$$\Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot \sqrt[x]{2^{-3}}}} = 2^{\frac{7}{3}} \,/^2 \Rightarrow \left(\sqrt{2^x \cdot \sqrt[3]{4^x \cdot \sqrt[x]{2^{-3}}}}\right)^2 = \left(2^{\frac{7}{3}}\right)^2 \Rightarrow$$

$$\Rightarrow 2^x \cdot \sqrt[3]{4^x \cdot \sqrt[x]{2^{-3}}} = 2^{\frac{14}{3}} \Rightarrow \left[\begin{matrix}\text{kubiramo} \\ \text{jednadžbu}\end{matrix}\right] \Rightarrow 2^x \cdot \sqrt[3]{4^x \cdot \sqrt[x]{2^{-3}}} = 2^{\frac{14}{3}} \,/^3 \Rightarrow$$

$$\Rightarrow \left(2^x \cdot \sqrt[3]{4^x \cdot \sqrt[x]{2^{-3}}}\right)^3 = \left(2^{\frac{14}{3}}\right)^3 \Rightarrow \left(2^x\right)^3 \cdot \left(\sqrt[3]{4^x \cdot \sqrt[x]{2^{-3}}}\right)^3 = \left(2^{\frac{14}{3}}\right)^3 \Rightarrow$$

$$\Rightarrow 2^{3\cdot x} \cdot 4^x \cdot \sqrt[x]{2^{-3}} = 2^{14} \Rightarrow 2^{3\cdot x} \cdot \left(2^2\right)^x \cdot \sqrt[x]{2^{-3}} = 2^{14} \Rightarrow 2^{3\cdot x} \cdot 2^{2\cdot x} \cdot \sqrt[x]{2^{-3}} = 2^{14} \Rightarrow$$

$$\Rightarrow 2^{3\cdot x + 2\cdot x} \cdot \sqrt[x]{2^{-3}} = 2^{14} \Rightarrow 2^{5\cdot x} \cdot \sqrt[x]{2^{-3}} = 2^{14} \Rightarrow 2^{5\cdot x} \cdot 2^{-\frac{3}{x}} = 2^{14} \Rightarrow 2^{5\cdot x - \frac{3}{x}} = 2^{14} \Rightarrow$$

$$\Rightarrow 5\cdot x - \frac{3}{x} = 14 \Rightarrow 5\cdot x - \frac{3}{x} = 14 \,/\cdot x \Rightarrow 5\cdot x^2 - 3 = 14\cdot x \Rightarrow$$

$$\Rightarrow 5\cdot x^2 - 14\cdot x - 3 = 0 \Rightarrow \left.\begin{matrix} 5\cdot x^2 - 14\cdot x - 3 = 0 \\ a = 5 \;,\; b = -14 \;,\; c = -3 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a = 5 \;,\; b = -14 \;,\; c = -3 \\ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow x_{1,2} = \frac{14 \pm \sqrt{196 - 4\cdot 5\cdot(-3)}}{2\cdot 5} \Rightarrow x_{1,2} = \frac{14 \pm \sqrt{196+60}}{10} \Rightarrow x_{1,2} = \frac{14 \pm \sqrt{256}}{10} \Rightarrow$$

$$\Rightarrow x_{1,2} = \frac{14 \pm 16}{10} \Rightarrow \left.\begin{matrix} x_1 = \frac{14+16}{10} \\ x_2 = \frac{14-16}{10} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1 = \frac{30}{10} \\ x_2 = -\frac{2}{10} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1 = 3 \\ x_2 = -\frac{1}{5} \end{matrix}\right\}.$$

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**Vježba 187**

Riješi jednadžbu: $2^x \cdot \sqrt[3]{4^x \cdot \sqrt[x]{0.125}} = 16 \cdot \sqrt[3]{4}.$

**Rezultat:** $x_1 = 3 \;,\; x_2 = -\frac{1}{5}.$

**Zadatak 188 (Mirza, elektrotehnička škola)**

Riješi nejednadžbu: $\frac{1}{2^{2\cdot x} + 3} \geq \frac{1}{2^{x+2} - 1}.$

**Rješenje 188**

Ponovimo!

$$a^0 = 1 \quad,\quad a^1 = a \quad,\quad \left(a^n\right)^m = a^{n\cdot m} \quad,\quad \frac{a}{b} \leq 0 \Rightarrow \left.\begin{matrix} a \leq 0 \\ b > 0 \end{matrix}\right\} \text{ i } \left.\begin{matrix} a \geq 0 \\ b < 0 \end{matrix}\right\}.$$

$$a^{-n} = \frac{1}{a^n} \quad,\quad \frac{a}{b} = 0 \Rightarrow a = 0 \quad,\quad a^{f(x)} < a^{g(x)} \Rightarrow f(x) < g(x) \;,\; a > 1.$$

$$(a-b)^2 = a^2 - 2\cdot a\cdot b + b^2 \quad,\quad a \in R \Rightarrow a^2 \geq 0 \quad,\quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Budući da kod razlomaka nazivnici ne smiju biti jednaki nuli, najprije svaki nazivnik izjednačimo sa nulom, riješimo dobivene jednadžbe i rezultate isključimo iz skupa rješenja zadane nejednadžbe.

| | |
|---|---|
| $2^{2\cdot x}+3=0$<br>$2^{2\cdot x}=-3$<br>Nema rješenja.<br>Za bilo koji x potencija je pozitivna. | $2^{x+2}-1=0$<br>$2^{x+2}=1$<br>$2^{x+2}=2^0$<br>$x+2=0$<br>$x=-2$<br>Znači x = – 2 ne može biti u skupu rješenja. |

$$\frac{1}{2^{2\cdot x}+3}\ge\frac{1}{2^{x+2}-1}\;\Rightarrow\;\frac{1}{\left(2^x\right)^2+3}\ge\frac{1}{2^x\cdot 2^2-1}\;\Rightarrow\;\frac{1}{\left(2^x\right)^2+3}\ge\frac{1}{4\cdot 2^x-1}\;\Rightarrow\;\left[\begin{array}{c}\text{supstitucija}\\ 2^x=t\end{array}\right]\Rightarrow$$

$$\Rightarrow\;\frac{1}{t^2+3}\ge\frac{1}{4\cdot t-1}\;\Rightarrow\;\frac{1}{t^2+3}-\frac{1}{4\cdot t-1}\ge 0\;\Rightarrow\;\frac{4\cdot t-1-\left(t^2+3\right)}{\left(t^2+3\right)\cdot\left(4\cdot t-1\right)}\ge 0\;\Rightarrow\;\frac{4\cdot t-1-t^2-3}{\left(t^2+3\right)\cdot\left(4\cdot t-1\right)}\ge 0\;\Rightarrow$$

$$\Rightarrow\;\frac{-t^2+4\cdot t-4}{\left(t^2+3\right)\cdot\left(4\cdot t-1\right)}\ge 0\;\Rightarrow\;\frac{-t^2+4\cdot t-4}{\left(t^2+3\right)\cdot\left(4\cdot t-1\right)}\ge 0\;/\cdot\left(-1\right)\;\Rightarrow\;\frac{t^2-4\cdot t+4}{\left(t^2+3\right)\cdot\left(4\cdot t-1\right)}\le 0\;\Rightarrow$$

$$\Rightarrow\;\frac{\left(t-2\right)^2}{\left(t^2+3\right)\cdot\left(4\cdot t-1\right)}\le 0.$$

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Razlomak je jednak nuli samo ako je brojnik jednak nuli.

$$t-2=0\;\Rightarrow\;t=2.$$

Vraćamo se supstituciji.

$$\left.\begin{array}{l}t=2^x\\ t=2\end{array}\right\}\Rightarrow\;2^x=2\;\Rightarrow\;2^x=2^1\;\Rightarrow\;x=1.$$

Dobili smo jedno rješenje nejednadžbe

x = 1.

Uočimo da su izrazi $(t-2)^2$ i $t^2+3$ pozitivni za svaki t, osim što je prvi izraz za t = 2 jednak nuli. Budući da je razlomak negativan, znači da izraz 4 · t – 1 mora biti negativan. Računamo za koje vrijednosti od t je izraz 4 · t – 1 manji od nule.

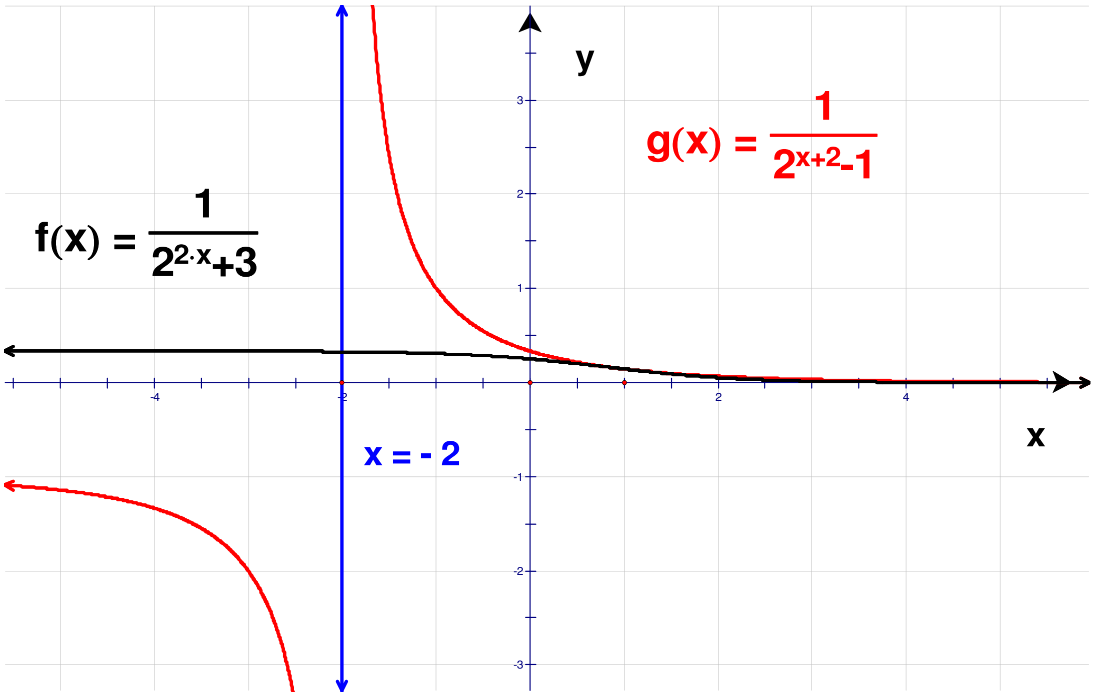
$$\frac{\left(t-2\right)^2}{\left(t^2+3\right)\cdot\left(4\cdot t-1\right)}\le 0\;\Rightarrow\;\frac{\overbrace{\left(t-2\right)^2}^{+}}{\underbrace{\left(t^2+3\right)}_{+}\cdot\left(4\cdot t-1\right)}\le 0\;\Rightarrow\;4\cdot t-1<0\;\Rightarrow\;4\cdot t<1\;/:4\;\Rightarrow\;t<\frac{1}{4}.$$

Vraćamo se supstituciji.

$$\left.\begin{array}{l}t=2^x\\ t<\frac{1}{4}\end{array}\right\}\Rightarrow\;2^x<\frac{1}{4}\;\Rightarrow\;2^x=\frac{1}{2^2}\;\Rightarrow\;2^x<2^{-2}\;\Rightarrow\;x<-2.$$

Dakle, rješenje zadane nejdnadžbe je

$$x\in\left\langle-\infty,-2\right\rangle\cup\left\{1\right\}.$$




**Vježba 188**

Riješi nejednadžbu: $\frac{1}{2^{2\cdot x}+3} \ge \frac{2}{2^{x+3}-2}$.

**Rezultat:** $x \in \langle -\infty, -2 \rangle \cup \{1\}$.

**Zadatak 189 (Mirza, elektrotehnička škola)**

Riješi jednadžbu: $2^x - 2 = 15 \cdot 2^{\frac{x-3}{2}}$.

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**Rješenje 189**

Ponovimo!

$$\frac{a-b}{n} = \frac{a}{n} - \frac{b}{n} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}.$$

$$\left(\sqrt{a}\right)^2 = a \quad , \quad \sqrt[n]{a^m} = a^{\frac{m}{n}} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

$$2^x - 2 = 15 \cdot 2^{\frac{x-3}{2}} \Rightarrow 2^x - 2 = 15 \cdot 2^{\frac{x}{2} - \frac{3}{2}} \Rightarrow 2^x - 2 = 15 \cdot 2^{\frac{x}{2}} \cdot 2^{-\frac{3}{2}} \Rightarrow \left[\begin{array}{l} \text{supstitucija} \\ 2^{\frac{x}{2}} = t \\ 2^x = t^2 \end{array}\right] \Rightarrow$$

$$\Rightarrow t^2 - 2 = 15 \cdot t \cdot 2^{-\frac{3}{2}} \Rightarrow t^2 - 2 = 15 \cdot t \cdot \frac{1}{2^{\frac{3}{2}}} \Rightarrow t^2 - 2 = 15 \cdot t \cdot \frac{1}{\sqrt{2^3}} \Rightarrow \left[\begin{array}{l} \text{djelomično} \\ \text{korjenovanje} \end{array}\right] \Rightarrow$$

$$\Rightarrow t^2 - 2 = 15 \cdot t \cdot \frac{1}{\sqrt{2^2 \cdot 2}} \Rightarrow t^2 - 2 = 15 \cdot t \cdot \frac{1}{2 \cdot \sqrt{2}} \Rightarrow t^2 - 2 = 15 \cdot t \cdot \frac{1}{2 \cdot \sqrt{2}} \;/\cdot 2 \cdot \sqrt{2} \Rightarrow$$

$$\Rightarrow 2 \cdot \sqrt{2} \cdot t^2 - 4 \cdot \sqrt{2} = 15 \cdot t \Rightarrow 2 \cdot \sqrt{2} \cdot t^2 - 15 \cdot t - 4 \cdot \sqrt{2} = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 2\cdot\sqrt{2}\cdot t^2-15\cdot t-4\cdot\sqrt{2}=0 \\ a=2\cdot\sqrt{2}\ ,\ b=-15\ ,\ c=-4\cdot\sqrt{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=2\cdot\sqrt{2}\ ,\ b=-15\ ,\ c=-4\cdot\sqrt{2} \\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{15\pm\sqrt{225-4\cdot 2\cdot\sqrt{2}\cdot\left(-4\cdot\sqrt{2}\right)}}{2\cdot 2\cdot\sqrt{2}} \Rightarrow t_{1,2}=\frac{15\pm\sqrt{225+32\cdot\left(\sqrt{2}\right)^2}}{4\cdot\sqrt{2}} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{15\pm\sqrt{225+32\cdot 2}}{4\cdot\sqrt{2}} \Rightarrow t_{1,2}=\frac{15\pm\sqrt{225+64}}{4\cdot\sqrt{2}} \Rightarrow t_{1,2}=\frac{15\pm\sqrt{289}}{4\cdot\sqrt{2}} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{15\pm 17}{4\cdot\sqrt{2}} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{15+17}{4\cdot\sqrt{2}} \\ t_2=\dfrac{15-17}{4\cdot\sqrt{2}} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{32}{4\cdot\sqrt{2}} \\ t_2=\dfrac{-2}{4\cdot\sqrt{2}} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{8}{\sqrt{2}} \\ t_2=\dfrac{-1}{2\cdot\sqrt{2}}\ \text{nema smisla negativno je} \end{array}\right\} \Rightarrow$$

$$\Rightarrow t=\frac{8}{\sqrt{2}} \Rightarrow t=\frac{2^3}{2^{\frac{1}{2}}} \Rightarrow t=2^3\cdot 2^{-\frac{1}{2}} \Rightarrow t=2^{3-\frac{1}{2}} \Rightarrow t=2^{\frac{5}{2}}.$$

Vraćamo se na supstituciju.

$$\left.\begin{array}{l} t=2^{\frac{x}{2}} \\ t=2^{\frac{5}{2}} \end{array}\right\} \Rightarrow 2^{\frac{x}{2}}=2^{\frac{5}{2}} \Rightarrow \frac{x}{2}=\frac{5}{2}\ /\cdot 2 \Rightarrow x=5.$$

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**Vježba 189**

Riješi jednadžbu: $2^x-15\cdot 2^{\frac{2\cdot x-6}{4}}-2=0.$

**Rezultat:** 5.

**Zadatak 190 (Mirza, elektrotehnička škola)**

Riješi jednadžbu: $2\cdot\log_8(2\cdot x)+\log_8\left(x^2-2\cdot x+1\right)=\frac{4}{3}.$

**Rješenje 190**

Ponovimo!

$$\log_b a=c \Leftrightarrow b^c=a \quad , \quad \log_b a^n=n\cdot\log_b a \quad , \quad \log_b x+\log_b y=\log_b(x\cdot y).$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad a\in R\setminus\{0\} \Rightarrow a^2>0 \quad , \quad a^n\cdot b^n=(a\cdot b)^n.$$

$$\left(a^n\right)^m=a^{n\cdot m}.$$

$$2\cdot\log_8(2\cdot x)+\log_8\left(x^2-2\cdot x+1\right)=\frac{4}{3}.$$

Najprije moramo napraviti diskusiju rješenja zadatka.
Budući da logaritmandi (brojevi ili izrazi pod znakom logaritma) ne smiju biti negativni (logaritamska funkcija definirana je samo za pozitivne realne brojeve!), postavit ćemo sljedeći sustav nejednadžbi koje tražena rješenja moraju zadovoljiti.

| | |
|---|---|
| $2\cdot x>0$<br>$2\cdot x>0 \ /:2$<br>$x>0$ | $x^2-2\cdot x+1>0$<br>$(x-1)^2>0$<br>$x\in\langle-\infty,+\infty\rangle\setminus\{0\}$ |

Rješenje sustava nejednadžbi je njihov presjek, zajednički dio, a to je

$$x\in\langle 0,+\infty\rangle\setminus\{1\}.$$

1.inačica

$$2\cdot\log_8(2\cdot x)+\log_8\left(x^2-2\cdot x+1\right)=\frac{4}{3} \Rightarrow \log_8(2\cdot x)^2+\log_8(x-1)^2=\frac{4}{3} \Rightarrow$$

$$\Rightarrow \log_8\left((2\cdot x)^2\cdot(x-1)^2\right)=\frac{4}{3} \Rightarrow \log_8((2\cdot x)\cdot(x-1))^2=\frac{4}{3} \Rightarrow 2\cdot\log_8((2\cdot x)\cdot(x-1))=\frac{4}{3} \ /\cdot\frac{1}{2} \Rightarrow$$

$$\Rightarrow \log_8((2\cdot x)\cdot(x-1))=\frac{2}{3} \Rightarrow 2\cdot x\cdot(x-1)=8^{\frac{2}{3}} \Rightarrow 2\cdot x\cdot(x-1)=\left(2^3\right)^{\frac{2}{3}} \Rightarrow$$

$$\Rightarrow 2\cdot x\cdot(x-1)=2^2 \Rightarrow 2\cdot x\cdot(x-1)=4 \ /:2 \Rightarrow x\cdot(x-1)=2 \Rightarrow x^2-x-2=0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x^2-x-2=0 \\ a=1\ ,\ b=-1\ ,\ c=-2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=1\ ,\ b=-1\ ,\ c=-2 \\ x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1-4\cdot 1\cdot(-2)}}{2\cdot 1} \Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+8}}{2} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{9}}{2} \Rightarrow x_{1,2}=\frac{1\pm 3}{2} \Rightarrow \left.\begin{array}{l} x_1=\dfrac{1+3}{2} \\ x_2=\dfrac{1-3}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1=\dfrac{4}{2} \\ x_2=\dfrac{-2}{2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x_1=2 \\ x_2=-1 \ \text{nema smisla} \end{array}\right\} \Rightarrow x=2.$$

2.inačica

$$2\cdot\log_8(2\cdot x)+\log_8\left(x^2-2\cdot x+1\right)=\frac{4}{3} \Rightarrow 2\cdot\log_8(2\cdot x)+\log_8(x-1)^2=\frac{4}{3} \Rightarrow$$

$$\Rightarrow 2\cdot\log_8(2\cdot x)+2\cdot\log_8(x-1)=\frac{4}{3} \Rightarrow 2\cdot\log_8(2\cdot x)+2\cdot\log_8(x-1)=\frac{4}{3} \ /\cdot\frac{1}{2} \Rightarrow$$

$$\Rightarrow \log_8(2\cdot x)+\log_8(x-1)=\frac{2}{3} \Rightarrow \log_8((2\cdot x)\cdot(x-1))=\frac{2}{3} \Rightarrow 2\cdot x\cdot(x-1)=8^{\frac{2}{3}} \Rightarrow$$

$$\Rightarrow 2\cdot x\cdot(x-1)=\left(2^3\right)^{\frac{2}{3}} \Rightarrow 2\cdot x\cdot(x-1)=2^2 \Rightarrow 2\cdot x\cdot(x-1)=4 \ /:2 \Rightarrow x\cdot(x-1)=2 \Rightarrow$$

$$\Rightarrow x^2-x-2=0 \Rightarrow \left.\begin{array}{l} x^2-x-2=0 \\ a=1\ ,\ b=-1\ ,\ c=-2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=1\ ,\ b=-1\ ,\ c=-2 \\ x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{1\pm\sqrt{1-4\cdot 1\cdot(-2)}}{2\cdot 1} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+8}}{2} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{9}}{2} \Rightarrow x_{1,2}=\frac{1\pm 3}{2} \Rightarrow$$

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$$\Rightarrow \left.\begin{matrix} x_1 = \frac{1+3}{2} \\ x_2 = \frac{1-3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1 = \frac{4}{2} \\ x_2 = \frac{-2}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1 = 2 \\ x_2 = -1 \ \text{nema smisla} \end{matrix}\right\} \Rightarrow x = 2.$$

**Vježba 190**

Riješi jednadžbu: $6 \cdot \log_8 (2 \cdot x) + 3 \cdot \log_8 \left(x^2 - 2 \cdot x + 1\right) - 4 = 0.$

**Rezultat:** 2.

**Zadatak 191 (Xena, gimnazija)**

Riješi jednadžbu: $16^{\frac{1}{x}} = 4^{\frac{x}{2}}.$

**Rješenje 191**

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x) \quad , \quad \frac{a}{b} = \frac{c}{d} \Rightarrow a \cdot d = b \cdot c.$$

1.inačica

$$16^{\frac{1}{x}} = 4^{\frac{x}{2}} \Rightarrow \left(4^2\right)^{\frac{1}{x}} = 4^{\frac{x}{2}} \Rightarrow 4^{\frac{2}{x}} = 4^{\frac{x}{2}} \Rightarrow \frac{2}{x} = \frac{x}{2} \Rightarrow x^2 = 4 \Rightarrow x^2 = 4 \ /\sqrt{\ } \Rightarrow$$

$$\Rightarrow x_{1,2} = \pm\sqrt{4} \Rightarrow \left.\begin{matrix} x_1 = 2 \\ x_2 = -2 \end{matrix}\right\}.$$

2.inačica

$$16^{\frac{1}{x}} = 4^{\frac{x}{2}} \Rightarrow \left(2^4\right)^{\frac{1}{x}} = \left(2^2\right)^{\frac{x}{2}} \Rightarrow 2^{\frac{4}{x}} = 2^{\frac{2 \cdot x}{2}} \Rightarrow 2^{\frac{4}{x}} = 2^{\frac{2 \cdot x}{2}} \Rightarrow 2^{\frac{4}{x}} = 2^x \Rightarrow$$

$$\Rightarrow \frac{4}{x} = x \Rightarrow \frac{4}{x} = x \ / \cdot x \Rightarrow x^2 = 4 \Rightarrow x^2 = 4 \ /\sqrt{\ } \Rightarrow x_{1,2} = \pm\sqrt{4} \Rightarrow \left.\begin{matrix} x_1 = 2 \\ x_2 = -2 \end{matrix}\right\}.$$

**Vježba 191**

Riješi jednadžbu: $81^{\frac{1}{x}} = 9^{\frac{x}{2}}.$

**Rezultat:** $x_1 = -2$ , $x_2 = 2$.

**Zadatak 192 (Xena, gimnazija)**

Riješi jednadžbu: $3^{x-1} - 4 \cdot 3^x + 33 = 0.$

**Rješenje 192**

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x) \quad , \quad a^1 = a.$$

1.inačica

$$3^{x-1} - 4 \cdot 3^x + 33 = 0 \Rightarrow 3^x \cdot 3^{-1} - 4 \cdot 3^x + 33 = 0 \Rightarrow 3^x \cdot \frac{1}{3} - 4 \cdot 3^x + 33 = 0 \Rightarrow$$

$$\Rightarrow 3^x \cdot \left(\frac{1}{3} - 4\right) + 33 = 0 \Rightarrow 3^x \cdot \left(-\frac{11}{3}\right) + 33 = 0 \Rightarrow 3^x \cdot \left(-\frac{11}{3}\right) = -33 \Rightarrow$$

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$$\Rightarrow 3^x \cdot \left(-\frac{11}{3}\right) = -33 \; / \cdot \left(-\frac{3}{11}\right) \Rightarrow 3^x = \frac{99}{11} \Rightarrow 3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2.$$

2.inačica

$$3^{x-1} - 4 \cdot 3^x + 33 = 0 \Rightarrow 3^x \cdot 3^{-1} - 4 \cdot 3^x + 33 = 0 \Rightarrow 3^x \cdot \frac{1}{3} - 4 \cdot 3^x + 33 = 0 \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{supstitucija} \\ 3^x = t \end{bmatrix} \Rightarrow \frac{1}{3} \cdot t - 4 \cdot t + 33 = 0 \Rightarrow \frac{1}{3} \cdot t - 4 \cdot t + 33 = 0 \; / \cdot 3 \Rightarrow t - 12 \cdot t + 99 = 0 \Rightarrow$$

$$\Rightarrow -11 \cdot t = -99 \Rightarrow -11 \cdot t = -99 \; /: (-11) \Rightarrow t = 9.$$

Vraćamo se supstituciji (zamjeni).

$$\left. \begin{array}{l} 3^x = t \\ t = 9 \end{array} \right\} \Rightarrow 3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2.$$

3.inačica

$$3^{x-1} - 4 \cdot 3^x + 33 = 0 \Rightarrow 3^{x-1} - 4 \cdot 3^{x-1} \cdot 3^1 + 33 = 0 \Rightarrow 3^{x-1} - 4 \cdot 3^{x-1} \cdot 3^1 + 33 = 0 \Rightarrow$$

$$\Rightarrow 3^{x-1} - 4 \cdot 3^{x-1} \cdot 3 + 33 = 0 \Rightarrow 3^{x-1} - 12 \cdot 3^{x-1} + 33 = 0 \Rightarrow 3^{x-1} \cdot (1 - 12) + 33 = 0 \Rightarrow$$

$$\Rightarrow 3^{x-1} \cdot (-11) + 33 = 0 \Rightarrow 3^{x-1} \cdot (-11) = -33 \Rightarrow 3^{x-1} \cdot (-11) = -33 \; /: (-11) \Rightarrow$$

$$\Rightarrow 3^{x-1} = 3 \Rightarrow 3^{x-1} = 3^1 \Rightarrow x - 1 = 1 \Rightarrow x = 1 + 1 \Rightarrow x = 2.$$

**Vježba 192**

Riješi jednadžbu: $3^{x-2} - 4 \cdot 3^{x-1} + 11 = 0.$

**Rezultat:** x = 2.

**Zadatak 193 (Xena, gimnazija)**

Riješi jednadžbu: $\left(x^2 + 1\right)^{2 \cdot x - 3} = 1.$

**Rješenje 193**

Ponovimo!

$$a^0 = 1 \quad , \quad 1^n = 1 \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x) \quad , \quad a^{f(x)} = b^{f(x)} \Rightarrow a = b.$$

- prvo rješenje

Budući da za potenciju kojoj je eksponent nula vrijedi

$$a^0 = 1 \; , \; a \neq 0,$$

prvo rješenje glasi:

$$\left(x^2 + 1\right)^{2 \cdot x - 3} = 1 \Rightarrow \left(x^2 + 1\right)^{2 \cdot x - 3} = \left(x^2 + 1\right)^0 \Rightarrow 2 \cdot x - 3 = 0 \Rightarrow 2 \cdot x = 3 \Rightarrow$$

$$\Rightarrow 2 \cdot x = 3 \; /: 2 \Rightarrow x_1 = \frac{3}{2}.$$

- drugo rješenje

Budući da za potenciju kojoj je baza 1 vrijedi

$$1^n = 1,$$

drugo rješenje glasi:

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$$\left(x^2+1\right)^{2\cdot x-3}=1 \Rightarrow \left(x^2+1\right)^{2\cdot x-3}=1^{2\cdot x-3} \Rightarrow \left(x^2+1\right)^{2\cdot x-3}=1^{2\cdot x-3} \Rightarrow$$

$$\Rightarrow x^2+1=1 \Rightarrow x^2=1-1 \Rightarrow x^2=0 \Rightarrow x^2=0 \; / \sqrt{\;} \Rightarrow x_2=0.$$

**Vježba 193**

Riješi jednadžbu: $\left(x^2+1\right)^{x-3}=1.$

**Rezultat:** $x_1 = 3$ , $x_2 = 0$.

**Zadatak 194 (Vlado, gimnazija)**

Nađi zbroj rješenja jednadžbe: $\dfrac{3}{\log x-1}=\log x+1.$

**Rješenje 194**

Ponovimo!

$$(a+b)\cdot(a-b)=a^2-b^2 \quad , \quad \log_{10} x=\log x \quad , \quad a^1=a \quad , \quad a^{-n}=\frac{1}{a^n}.$$

Definicija:

$$\log_b a=c \Leftrightarrow b^c=a.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=b^c \qquad a=b^c$$
$$\rightarrow$$

Prvo moramo napraviti diskusiju rješenja zadatka. Budući da logaritmandi (brojevi ili izrazi pod znakom logaritma) ne smiju biti negativni (logaritamska funkcija definirana je samo za pozitivne realne brojeve!), postavit ćemo sljedeću nejednadžbu koju traženo rješenje mora zadovoljiti:

$$x > 0.$$

Kod razlomka nazivnik ne smije biti jednak nuli. Zato nazivnik izjednačimo sa nulom, riješimo dobivenu jednadžbu i rezultat isključimo iz skupa rješenja zadane jednadžbe.

$$\log x-1=0 \Rightarrow \log x=1 \Rightarrow x=10^1 \Rightarrow x=10 \text{ taj broj izbacuje se iz rezultata, } x\neq 10.$$

Računamo rješenja jednadžbe:

$$\frac{3}{\log x-1}=\log x+1 \Rightarrow \frac{3}{\log x-1}=\log x+1 \; /\cdot(\log x-1) \Rightarrow 3=(\log x+1)\cdot(\log x-1) \Rightarrow$$

$$\Rightarrow (\log x+1)\cdot(\log x-1)=3 \Rightarrow \log^2 x-1=3 \Rightarrow \log^2 x=3+1 \Rightarrow \log^2 x=4 \Rightarrow$$

$$\Rightarrow \log^2 x=4 \; / \sqrt{\;} \Rightarrow \log x=\pm\sqrt{4} \Rightarrow \left.\begin{matrix}\log x=2\\ \log x=-2\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x_1=10^2\\ x_2=10^{-2}\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x_1=100\\ x_2=\dfrac{1}{10^2}\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix}x_1=100\\ x_2=0.01\end{matrix}\right\}.$$

Zbroj rješenja iznosi:

$$x_1+x_2=100+0.01 \Rightarrow x_1+x_2=100.01.$$

**Vježba 194**

Nađi umnožak rješenja jednadžbe: $\frac{3}{\log x+1}=\log x-1.$

**Rezultat:** 1.

**Zadatak 195 (Maturanti, HTT)**

Nađi rješenje jednadžbe: $12\cdot\left(\frac{1}{3}\right)^{x}=\frac{4}{3}.$

**Rješenje 195**

Ponovimo!

$$a^{-n}=\frac{1}{a^{n}} \quad , \quad \left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{1}=a \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$a\cdot a=a^{2}.$$

1.inačica

$$12\cdot\left(\frac{1}{3}\right)^{x}=\frac{4}{3} \Rightarrow 12\cdot\left(\frac{1}{3}\right)^{x}=\frac{4}{3} \;/\cdot\frac{1}{12} \Rightarrow \left(\frac{1}{3}\right)^{x}=\frac{4}{3}\cdot\frac{1}{12} \Rightarrow \left(\frac{1}{3}\right)^{x}=\frac{4}{3}\cdot\frac{1}{12} \Rightarrow \left(\frac{1}{3}\right)^{x}=\frac{1}{3}\cdot\frac{1}{3} \Rightarrow$$

$$\Rightarrow \left(\frac{1}{3}\right)^{x}=\left(\frac{1}{3}\right)^{2} \Rightarrow x=2.$$

2.inačica

$$12\cdot\left(\frac{1}{3}\right)^{x}=\frac{4}{3} \Rightarrow 12\cdot\left(3^{-1}\right)^{x}=\frac{4}{3} \Rightarrow 12\cdot 3^{-x}=\frac{4}{3} \;/\cdot\frac{1}{12} \Rightarrow 3^{-x}=\frac{4}{3}\cdot\frac{1}{12} \Rightarrow 3^{-x}=\frac{4}{3}\cdot\frac{1}{12} \Rightarrow$$

$$\Rightarrow 3^{-x}=\frac{1}{3^{2}} \Rightarrow 3^{-x}=3^{-2} \Rightarrow -x=-2 \Rightarrow -x=-2 \;/\cdot(-1) \Rightarrow x=2.$$

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**Vježba 195**

Nađi rješenje jednadžbe: $9\cdot\left(\frac{1}{3}\right)^{x}=1.$

**Rezultat:** 2.

**Zadatak 196 (Maturanti, HTT)**

Izračunaj: $49^{0.5-\log_{7}5}.$

**Rješenje 196**

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{n}\cdot a^{m}=a^{n+m} \quad , \quad a^{1}=a \quad , \quad \log_{b}a^{n}=n\cdot\log_{b}a.$$

$$a^{-n}=\frac{1}{a^{n}} \quad , \quad b^{\log_{b}n}=n.$$

1.inačica

$$49^{0.5-\log_{7}5}=\left(7^{2}\right)^{0.5-\log_{7}5}=7^{2\cdot\left(0.5-\log_{7}5\right)}=7^{1-2\cdot\log_{7}5}=7^{1}\cdot 7^{-2\cdot\log_{7}5}=$$

$$=7\cdot 7^{\log_{7}5^{-2}}=7\cdot 5^{-2}=\frac{7}{5^{2}}=\frac{7}{25}.$$

2.inačica

$$49^{0.5-\log_7 5}=49^{0.5}\cdot 49^{-\log_7 5}=\frac{49^{0.5}}{49^{\log_7 5}}=\frac{\left(7^2\right)^{0.5}}{\left(7^2\right)^{\log_7 5}}=\frac{7^1}{7^{2\cdot\log_7 5}}=$$

$$=\frac{7}{7^{\log_7 5^2}}=\frac{7}{5^2}=\frac{7}{25}.$$

**Vježba 196**

Izračunaj: $49^{1-\log_7 5}$.

**Rezultat:** $\frac{49}{25}$.

**Zadatak 197 (Maturanti, HTT)**

Napišite kao jedan logaritam: $\log\frac{x}{x-1}+\log\frac{x+1}{x}-\log\left(x^2-1\right)$.

**Rješenje 197**

Ponovimo!

$$a^2-b^2=(a+b)\cdot(a-b)\quad,\quad\log(a\cdot b)=\log a+\log b\quad,\quad\log\frac{a}{b}=\log a-\log b\quad,\quad a^{-n}=\frac{1}{a^n}.$$

$$\log a^n=n\cdot\log a.$$

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1.inačica

$$\log\frac{x}{x-1}+\log\frac{x+1}{x}-\log\left(x^2-1\right)=\left(\log\frac{x}{x-1}+\log\frac{x+1}{x}\right)-\log\left(x^2-1\right)=\log\left(\frac{x}{x-1}\cdot\frac{x+1}{x}\right)-\log\left(x^2-1\right)=$$

$$=\log\left(\frac{x}{x-1}\cdot\frac{x+1}{x}\right)-\log\left(x^2-1\right)=\log\frac{x+1}{x-1}-\log\left(x^2-1\right)=\log\frac{\frac{x+1}{x-1}}{x^2-1}=\log\frac{\frac{x+1}{x-1}}{\frac{x^2-1}{1}}=$$

$$=\log\frac{\frac{x+1}{x-1}}{\frac{(x-1)\cdot(x+1)}{1}}=\log\frac{\frac{x+1}{x-1}}{\frac{(x-1)\cdot(x+1)}{1}}=\log\frac{\frac{1}{x-1}}{\frac{x-1}{1}}=\log\frac{1}{(x-1)^2}=\log(x-1)^{-2}=-2\cdot\log(x-1).$$

2.inačica

$$\log\frac{x}{x-1}+\log\frac{x+1}{x}-\log\left(x^2-1\right)=\log\frac{x}{x-1}+\log\frac{x+1}{x}-\log(x-1)\cdot(x+1)=$$

$$=\log x-\log(x-1)+\log(x+1)-\log x-\left(\log(x-1)+\log(x+1)\right)=$$

$$=\log x-\log(x-1)+\log(x+1)-\log x-\log(x-1)-\log(x+1)=$$

$$=\log x-\log(x-1)+\log(x+1)-\log x-\log(x-1)-\log(x+1)=-2\cdot\log(x-1).$$

3.inačica

$$\log\frac{x}{x-1}+\log\frac{x+1}{x}-\log\left(x^2-1\right)=\log\frac{x}{x-1}+\log\frac{x+1}{x}+(-1)\cdot\log\left(x^2-1\right)=$$

$$=\log\frac{x}{x-1}+\log\frac{x+1}{x}+\log\left(x^{2}-1\right)^{-1}=\log\frac{x}{x-1}+\log\frac{x+1}{x}+\log\frac{1}{x^{2}-1}=$$

$$=\log\left(\frac{x}{x-1}\cdot\frac{x+1}{x}\cdot\frac{1}{x^{2}-1}\right)=\log\left(\frac{x}{x-1}\cdot\frac{x+1}{x}\cdot\frac{1}{(x-1)\cdot(x+1)}\right)=\log\left(\frac{x}{x-1}\cdot\frac{x+1}{x}\cdot\frac{1}{(x-1)\cdot(x+1)}\right)=$$

$$=\log\left(\frac{1}{x-1}\cdot\frac{1}{1}\cdot\frac{1}{x-1}\right)=\log\frac{1}{(x-1)^{2}}=\log(x-1)^{-2}=-2\cdot\log(x-1).$$

**Vježba 197**

Napišite kao jedan logaritam: $\log\frac{x}{x-1}-\log\frac{x}{x+1}+\log\frac{1}{x^{2}-1}.$

**Rezultat:** $-2\cdot\log(x-1).$

**Zadatak 198 (Maturanti, HTT)**

Riješi jednadžbu: $\log_{4}x^{3}+\log_{2}\sqrt{x}=8.$

**Rješenje 198**

Ponovimo!

$\log_{b}a^{n}=n\cdot\log_{b}a$ , $\log_{b}a=\frac{\log_{c}a}{\log_{c}b}$ , $\log_{10}a=\log a$ , $\log_{b}\sqrt[n]{a}=\frac{1}{n}\cdot\log_{b}a.$

$a^{n}\cdot a^{m}=a^{n+m}$ , $\log_{b}(x\cdot y)=\log_{b}x+\log_{b}y$ , $\log_{b^{n}}a=\frac{1}{n}\cdot\log_{b}a.$

$a^{1}=a$ , $a^{f(x)}=a^{g(x)}\Rightarrow f(x)=g(x)$ , $\log f(x)=\log g(x)\Rightarrow f(x)=g(x).$

Definicija:

$$\log_{b}a=c\Leftrightarrow b^{c}=a.$$

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Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$\log_{b}a=c$ $\quad$ $\log_{b}a=b^{c}$ $\quad$ $a=b^{c}$

$\rightarrow$

Prvo moramo napraviti diskusiju rješenja zadatka. Budući da logaritmandi (brojevi ili izrazi pod znakom logaritma) ne smiju biti negativni (logaritamska funkcija definirana je samo za pozitivne realne brojeve!), postavit ćemo sljedeću nejednadžbu koju traženo rješenje mora zadovoljiti:

$$x>0.$$

1.inačica

$$\log_{4}x^{3}+\log_{2}\sqrt{x}=8\Rightarrow 3\cdot\log_{4}x+\frac{1}{2}\cdot\log_{2}x=8\Rightarrow 3\cdot\frac{\log x}{\log 4}+\frac{1}{2}\cdot\frac{\log x}{\log 2}=8\Rightarrow$$

$$\Rightarrow 3\cdot\frac{\log x}{\log 2^{2}}+\frac{\log x}{2\cdot\log 2}=8\Rightarrow 3\cdot\frac{\log x}{2\cdot\log 2}+\frac{\log x}{2\cdot\log 2}=8\Rightarrow 3\cdot\frac{\log x}{2\cdot\log 2}+\frac{\log x}{2\cdot\log 2}=8\ /\cdot 2\cdot\log 2\Rightarrow$$

$$\Rightarrow 3\cdot\log x+\log x=16\cdot\log 2\Rightarrow 4\cdot\log x=16\cdot\log 2\Rightarrow 4\cdot\log x=16\cdot\log 2\ /:4\Rightarrow\log x=4\cdot\log 2\Rightarrow$$

$$\Rightarrow\log x=\log 2^{4}\Rightarrow x=2^{4}\Rightarrow x=16.$$

2.inačica

$$\log_4 x^3 + \log_2 \sqrt{x} = 8 \Rightarrow 3 \cdot \log_4 x + \frac{1}{2} \cdot \log_2 x = 8 \Rightarrow 3 \cdot \log_{2^2} x + \frac{1}{2} \cdot \log_2 x = 8 \Rightarrow$$

$$\Rightarrow 3 \cdot \frac{1}{2} \cdot \log_2 x + \frac{1}{2} \cdot \log_2 x = 8 \Rightarrow \frac{3}{2} \cdot \log_2 x + \frac{1}{2} \cdot \log_2 x = 8 \Rightarrow \frac{4}{2} \cdot \log_2 x = 8 \Rightarrow 2 \cdot \log_2 x = 8 \Rightarrow$$

$$\Rightarrow 2 \cdot \log_2 x = 8 \ /:2 \Rightarrow \log_2 x = 4 \Rightarrow x = 2^4 \Rightarrow x = 16.$$

3.inačica

$$\log_4 x^3 + \log_2 \sqrt{x} = 8 \Rightarrow \log_{2^2} x^3 + \log_2 \sqrt{x} = 8 \Rightarrow \frac{1}{2} \cdot \log_2 x^3 + \frac{1}{2} \cdot \log_2 x = 8 \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot \log_2 x^3 + \frac{1}{2} \cdot \log_2 x = 8 \ / \cdot 2 \Rightarrow \log_2 x^3 + \log_2 x = 16 \Rightarrow \log_2 \left(x^3 \cdot x\right) = 16 \Rightarrow$$

$$\Rightarrow \log_2 x^4 = 16 \Rightarrow 4 \cdot \log_2 x = 16 \Rightarrow 4 \cdot \log_2 x = 16 \ /:4 \Rightarrow \log_2 x = 4 \Rightarrow x = 2^4 \Rightarrow x = 16.$$

**Vježba 198**

Riješi jednadžbu: $\log_4 x^3 + \log_2 \sqrt{x} - \log_2 256 = 0.$

**Rezultat:** 16.

**Zadatak 199 (Maturanti, HTT)**

Neka je $\log_2 P = x$, $\log_2 Q = y$, $\log_2 R = z$. Nađi $\log_2 \left(\frac{R^2 \cdot \sqrt{Q}}{P^3}\right)$.

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**Rješenje 199**

Ponovimo!

$$\log_b (x \cdot y) = \log_b x + \log_b y \quad , \quad \log_b \frac{x}{y} = \log_b x - \log_b y \quad , \quad \log_b x^y = y \cdot \log_b x.$$

$$\log_b \sqrt[n]{x} = \frac{1}{n} \cdot \log_b x.$$

$$\log_2 \left(\frac{R^2 \cdot \sqrt{Q}}{P^3}\right) = \log_2 \left(R^2 \cdot \sqrt{Q}\right) - \log_2 P^3 = \log_2 R^2 + \log_2 \sqrt{Q} - \log_2 P^3 =$$

$$= 2 \cdot \log_2 R + \frac{1}{2} \cdot \log_2 Q - 3 \cdot \log_2 P = \begin{bmatrix} \log_2 P = x \\ \log_2 Q = y \\ \log_2 R = z \end{bmatrix} = 2 \cdot x + \frac{1}{2} \cdot y - 3 \cdot z.$$

**Vježba 199**

Neka je $\log_2 P = x$, $\log_2 Q = y$, $\log_2 R = z$. Nađi $\log_2 \left(\frac{R \cdot \sqrt{Q}}{P^2}\right)$.

**Rezultat:** $x + \frac{1}{2} \cdot y - 2 \cdot z.$

**Zadatak 200 (Maturanti, HTT)**

Neka je $a = \log 3$, $b = \log 5$, $c = \log 2$. Nađi $\log_{40} 9$.

**Rješenje 200**

Ponovimo!

$$\log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log a^n = n \cdot \log a \quad , \quad \log(a \cdot b) = \log a + \log b.$$

$$\log_{40} 9 = \frac{\log 9}{\log 40} = \frac{\log 3^2}{\log(8 \cdot 5)} = \frac{2 \cdot \log 3}{\log 8 + \log 5} = \frac{2 \cdot \log 3}{\log 2^3 + \log 5} = \frac{2 \cdot \log 3}{3 \cdot \log 2 + \log 5} =$$

$$= \begin{bmatrix} a = \log 3 \\ b = \log 5 \\ c = \log 2 \end{bmatrix} = \frac{2 \cdot a}{3 \cdot c + b}.$$

**Vježba 200**

Neka je $a = \log 3$, $b = \log 5$, $c = \log 2$. Nađi $\log_{40} 27$.

**Rezultat:** $\frac{3 \cdot a}{3 \cdot c + b}$.

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