Zadatak 401 (2B, TUPŠ)

Riješi jednadžbu: $2^x - 2^{x-2} = 3^{x-1} - 3^{x-2}.$

Rješenje 401

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

$$\frac{a}{b} \cdot \frac{b}{a} = 1 \quad , \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$2^x - 2^{x-2} = 3^{x-1} - 3^{x-2} \Rightarrow 2^x - 2^x \cdot 2^{-2} = 3^x \cdot 3^{-1} - 3^x \cdot 3^{-2} \Rightarrow$$

$$\Rightarrow 2^x - 2^x \cdot 2^{-2} = 3^x \cdot 3^{-1} - 3^x \cdot 3^{-2} \Rightarrow 2^x \cdot \left(1 - 2^{-2}\right) = 3^x \cdot \left(3^{-1} - 3^{-2}\right) \Rightarrow$$

$$\Rightarrow 2^x \cdot \left(1 - \frac{1}{2^2}\right) = 3^x \cdot \left(\frac{1}{3^1} - \frac{1}{3^2}\right) \Rightarrow 2^x \cdot \left(1 - \frac{1}{4}\right) = 3^x \cdot \left(\frac{1}{3} - \frac{1}{9}\right) \Rightarrow$$

$$\Rightarrow 2^x \cdot \left(\frac{1}{1} - \frac{1}{4}\right) = 3^x \cdot \left(\frac{1}{3} - \frac{1}{9}\right) \Rightarrow 2^x \cdot \frac{4-1}{4} = 3^x \cdot \frac{3-1}{9} \Rightarrow 2^x \cdot \frac{3}{4} = 3^x \cdot \frac{2}{9} \Rightarrow$$

$$\Rightarrow 2^x \cdot \frac{3}{4} = 3^x \cdot \frac{2}{9} \; / \cdot \frac{4}{3 \cdot 3^x} \Rightarrow \frac{2^x}{3^x} = \frac{2}{9} \cdot \frac{4}{3} \Rightarrow \left(\frac{2}{3}\right)^x = \frac{8}{27} \Rightarrow \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^3 \Rightarrow x = 3.$$

Vježba 401

Riješi jednadžbu: $2^x - 2^{x-2} = \frac{1}{3} \cdot 3^x - 3^{x-2}.$

Rezultat: x = 3.

Zadatak 402 (2B, TUPŠ)

Riješi jednadžbu: $4 \cdot 10^{x-1} + 10^{x+1} = 1040.$

Rješenje 402

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$a \cdot \frac{b}{c} = \frac{a \cdot b}{c} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \neq 0 \; , \; n \neq 1.$$

$$4 \cdot 10^{x-1} + 10^{x+1} = 1040 \Rightarrow 4 \cdot 10^x \cdot 10^{-1} + 10^x \cdot 10^1 = 1040 \Rightarrow$$

$$\Rightarrow 4\cdot 10^x\cdot\frac{1}{10^1}+10^x\cdot 10=1040 \Rightarrow 4\cdot 10^x\cdot\frac{1}{10}+10^x\cdot 10=1040 \Rightarrow$$

$$\Rightarrow 4\cdot 10^x\cdot\frac{1}{10}+10^x\cdot 10=1040 \Rightarrow 10^x\cdot\left(\frac{4}{10}+10\right)=1040 \Rightarrow 10^x\cdot\left(\frac{4}{10}+\frac{10}{1}\right)=1040 \Rightarrow$$

$$\Rightarrow 10^x\cdot\frac{4+100}{10}=1040 \Rightarrow 10^x\cdot\frac{104}{10}=1040 \Rightarrow 10^x\cdot\frac{104}{10}=1040\ /\cdot\frac{10}{104} \Rightarrow$$

$$\Rightarrow 10^x=1040\cdot\frac{10}{104} \Rightarrow 10^x=1040\cdot\frac{10}{104} \Rightarrow 10^x=10\cdot 10 \Rightarrow 10^x=10^2 \Rightarrow x=2.$$

Vježba 402

Riješi jednadžbu: $4\cdot 10^x+10^{x+2}=10\,400.$

Rezultat: x = 2.

Zadatak 403 (Arlo, gimnazija)

Riješi jednadžbu: $4^x+6^x=2\cdot 9^x.$

Rješenje 403

Ponovimo!

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^{-n}=\frac{1}{a^n}.$$

$$a^1=a \quad , \quad a^0=1 \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad b\cdot\frac{a}{b}=a.$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$4^x+6^x=2\cdot 9^x \Rightarrow 4^x-2\cdot 9^x=-6^x \Rightarrow 4^x-2\cdot 9^x=-6^x\ /:6^x \Rightarrow$$

$$\Rightarrow \frac{4^x}{6^x}-2\cdot\frac{9^x}{6^x}=-1 \Rightarrow \left(\frac{4}{6}\right)^x-2\cdot\left(\frac{9}{6}\right)^x=-1 \Rightarrow \left(\frac{4}{6}\right)^x-2\cdot\left(\frac{9}{6}\right)^x=-1 \Rightarrow$$

$$\Rightarrow \left(\frac{2}{3}\right)^x-2\cdot\left(\frac{3}{2}\right)^x=-1 \Rightarrow \left(\frac{2}{3}\right)^x-2\cdot\left(\frac{2}{3}\right)^{-x}=-1 \Rightarrow \left(\frac{2}{3}\right)^x-2\cdot\left(\left(\frac{2}{3}\right)^x\right)^{-1}=-1 \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{zamjena}\\ t=\left(\frac{2}{3}\right)^x\end{bmatrix} \Rightarrow t-2\cdot t^{-1}=-1 \Rightarrow t-\frac{2}{t}=-1 \Rightarrow t-\frac{2}{t}=-1\ /\cdot t \Rightarrow t^2-2=-t \Rightarrow$$

$$\Rightarrow t^2-2+t=0 \Rightarrow t^2+t-2=0 \Rightarrow \left.\begin{matrix}t^2+t-2=0\\ a=1\;,\; b=1\;,\; c=-2\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix}a=1\;,\; b=1\;,\; c=-2\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\} \Rightarrow t_{1,2}=\frac{-1\pm\sqrt{1^2-4\cdot 1\cdot(-2)}}{2\cdot 1} \Rightarrow t_{1,2}=\frac{-1\pm\sqrt{1+8}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{-1\pm\sqrt{9}}{2} \Rightarrow t_{1,2}=\frac{-1\pm 3}{2} \Rightarrow \left.\begin{aligned} t_1&=\frac{-1+3}{2}\\ t_2&=\frac{-1-3}{2}\end{aligned}\right\} \Rightarrow \left.\begin{aligned} t_1&=\frac{2}{2}\\ t_2&=-\frac{4}{2}\end{aligned}\right\} \Rightarrow \left.\begin{aligned} t_1&=\frac{2}{2}\\ t_2&=-\frac{4}{2}\end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{aligned} t_1&=1\\ t_2&=-2\end{aligned}\right\}.$$

Vraćamo se na zamjenu:

- $\left.\begin{aligned}\left(\frac{2}{3}\right)^x&=t\\ t&=1\end{aligned}\right\} \Rightarrow \left(\frac{2}{3}\right)^x=1 \Rightarrow \left(\frac{2}{3}\right)^x=\left(\frac{2}{3}\right)^0 \Rightarrow x=0$
- $\left.\begin{aligned}\left(\frac{2}{3}\right)^x&=t\\ t&=-2\end{aligned}\right\} \Rightarrow \left(\frac{2}{3}\right)^x=-2$ nema smisla.

Vježba 403

Riješi jednadžbu: $6^x-2\cdot 9^x+4^x=0.$

Rezultat: x = 0.

Zadatak 404 (Tomislav, gimnazija)

Ako je log 2 = m, rješenje jednadžbe $4^{x+1}=320$ jednako je:

$A.\ x=\frac{3\cdot m+1}{2\cdot m}$ $B.\ x=\frac{5\cdot m-1}{2}$ $C.\ x=\frac{m+1}{5\cdot m}$ $D.\ x=\frac{3\cdot m-1}{2}$

Rješenje 404

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=b^c \qquad a=b^c$$
$$\rightarrow$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log *x* zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x=\log x.$$

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad \log a^n=n\cdot\log a \quad , \quad \log 10=1 \quad , \quad \log(a\cdot b)=\log a+\log b.$$

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$4^{x+1}=320 \Rightarrow \left(2^2\right)^{x+1}=320 \Rightarrow 2^{2\cdot(x+1)}=320 \Rightarrow 2^{2\cdot x+2}=320 \Rightarrow$$

$$\Rightarrow\ 2^{2\cdot x}\cdot 2^{2}=320\ \Rightarrow\ 2^{2\cdot x}\cdot 4=320\ \Rightarrow\ 2^{2\cdot x}\cdot 4=320\ /:4\ \Rightarrow\ 2^{2\cdot x}=80\ \Rightarrow$$

$$\Rightarrow\ \begin{bmatrix}\text{logaritmiramo}\\ \text{jednadžbu}\end{bmatrix}\ \Rightarrow\ 2^{2\cdot x}=80\ /\log\ \Rightarrow\ \log 2^{2\cdot x}=\log 80\ \Rightarrow\ 2\cdot x\cdot\log 2=\log 80\ \Rightarrow$$

$$\Rightarrow\ 2\cdot x\cdot\log 2=\log(8\cdot 10)\ \Rightarrow\ 2\cdot x\cdot\log 2=\log 8+\log 10\ \Rightarrow\ 2\cdot x\cdot\log 2=\log 2^{3}+1\ \Rightarrow$$

$$\Rightarrow\ 2\cdot x\cdot\log 2=3\cdot\log 2+1\ \Rightarrow\ \begin{bmatrix}\text{uvjet}\\ \log 2=m\end{bmatrix}\ \Rightarrow\ 2\cdot x\cdot m=3\cdot m+1\ \Rightarrow$$

$$\Rightarrow\ 2\cdot x\cdot m=3\cdot m+1\ /\cdot\frac{1}{2\cdot m}\ \Rightarrow\ x=\frac{3\cdot m+1}{2\cdot m}.$$

Odgovor je pod A.

2.inačica

$$4^{x+1}=320\ \Rightarrow\ \left(2^{2}\right)^{x+1}=320\ \Rightarrow\ 2^{2\cdot(x+1)}=320\ \Rightarrow\ 2^{2\cdot x+2}=320\ \Rightarrow$$

$$\Rightarrow\ \begin{bmatrix}\text{logaritmiramo}\\ \text{jednadžbu}\end{bmatrix}\ \Rightarrow\ 2^{2\cdot x+2}=320\ /\log\ \Rightarrow\ \log 2^{2\cdot x+2}=\log 320\ \Rightarrow$$

$$\Rightarrow\ (2\cdot x+2)\cdot\log 2=\log 320\ \Rightarrow\ (2\cdot x+2)\cdot\log 2=\log(32\cdot 10)\ \Rightarrow$$

$$\Rightarrow\ (2\cdot x+2)\cdot\log 2=\log 32+\log 10\ \Rightarrow\ (2\cdot x+2)\cdot\log 2=\log 2^{5}+1\ \Rightarrow$$

$$\Rightarrow\ (2\cdot x+2)\cdot\log 2=5\cdot\log 2+1\ \Rightarrow\ \begin{bmatrix}\text{uvjet}\\ \log 2=m\end{bmatrix}\ \Rightarrow\ (2\cdot x+2)\cdot m=5\cdot m+1\ \Rightarrow$$

$$\Rightarrow\ (2\cdot x+2)\cdot m=5\cdot m+1\ /\cdot\frac{1}{m}\ \Rightarrow\ 2\cdot x+2=\frac{5\cdot m+1}{m}\ \Rightarrow\ 2\cdot x=\frac{5\cdot m+1}{m}-2\ \Rightarrow$$

$$\Rightarrow\ 2\cdot x=\frac{5\cdot m+1}{m}-\frac{2}{1}\ \Rightarrow\ 2\cdot x=\frac{5\cdot m+1-2\cdot m}{m}\ \Rightarrow\ 2\cdot x=\frac{3\cdot m+1}{m}\ \Rightarrow$$

$$\Rightarrow\ 2\cdot x=\frac{3\cdot m+1}{m}\ /\cdot\frac{1}{2}\ \Rightarrow\ x=\frac{3\cdot m+1}{2\cdot m}.$$

Odgovor je pod A.

Vježba 404

Ako je log 2 = m, rješenje jednadžbe $4^{x+2}=1280$ jednako je:

$A.\ x=\frac{3\cdot m+1}{2\cdot m}$ $B.\ x=\frac{5\cdot m-1}{2}$ $C.\ x=\frac{m+1}{5\cdot m}$ $D.\ x=\frac{3\cdot m-1}{2}$

Rezultat: A.

Zadatak 405 (4A, TUPŠ)

Riješi nejednadžbu: $8\cdot 16^{x}\geq 7\cdot 14^{x}$.

Rješenje 405

Ponovimo!

$$\frac{a^{n}}{b^{n}}=\left(\frac{a}{b}\right)^{n}\ ,\quad a^{1}=a\ ,\quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}\ ,\quad a\geq b\ ,\ c>0\ \Rightarrow\ \frac{a}{c}\geq\frac{b}{c}.$$

$$a^{f(x)}\geq a^{g(x)}\ ,\ a>1\ \Rightarrow\ f(x)\geq g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$8\cdot 16^{x}\geq 7\cdot 14^{x}\ \Rightarrow\ 8\cdot 16^{x}\geq 7\cdot 14^{x}\ /\cdot\frac{1}{8\cdot 14^{x}}\ \Rightarrow\ \frac{16^{x}}{14^{x}}\geq\frac{7}{8}\ \Rightarrow\ \left(\frac{16}{14}\right)^{x}\geq\left(\frac{7}{8}\right)^{1}\ \Rightarrow$$

$$\Rightarrow\ \left(\frac{16}{14}\right)^{x}\geq\left(\frac{7}{8}\right)^{1}\ \Rightarrow\ \left(\frac{8}{7}\right)^{x}\geq\left(\frac{8}{7}\right)^{-1}\ \Rightarrow\ x\geq -1.$$

Vježba 405

Riješi nejednadžbu: $8\cdot 16^{x}\leq 7\cdot 14^{x}$.

Rezultat: $x\leq -1.$

Zadatak 406 (4A, TUPŠ)

Rješenje jednadžbe $5\cdot 9^{x+1}=15$ nalazi se u intervalu:

$A.\ \langle -\infty,\ -2]$ $B.\ \langle -2,\ -1]$ $C.\ \langle -1,\ 2]$ $D.\ \langle 2,\ +\infty]$

Rješenje 406

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m}\quad ,\quad a^{1}=a\quad ,\quad a^{f(x)}=a^{g(x)}\ \Rightarrow\ f(x)=g(x).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad ,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$5\cdot 9^{x+1}=15\ \Rightarrow\ 5\cdot 9^{x+1}=15\ /:5\ \Rightarrow\ 9^{x+1}=3\ \Rightarrow\ \left(3^{2}\right)^{x+1}=3\ \Rightarrow\ 3^{2\cdot(x+1)}=3^{1}\ \Rightarrow$$

$$\Rightarrow\ 3^{2\cdot x+2}=3^{1}\ \Rightarrow\ 2\cdot x+2=1\ \Rightarrow\ 2\cdot x=1-2\ \Rightarrow\ 2\cdot x=-1\ \Rightarrow\ 2\cdot x=-1\ /:2\ \Rightarrow\ x=-\frac{1}{2}.$$

Vrijedi:

$$-\frac{1}{2}\in\langle -1,\ 2].$$

Odgovor je pod C.

Vježba 406

Rješenje jednadžbe $4\cdot 9^{x+1}=12$ nalazi se i intervalu:

$A.\ \langle -\infty,\ -2]$ $B.\ \langle -2,\ -1]$ $C.\ \langle -1,\ 2]$ $D.\ \langle 2,\ +\infty]$

Rezultat: C.

Zadatak 407 (4A, TUPŠ)

Izraz $\log_{2}(4\cdot a)+\log_{2}\left(2\cdot a^{2}\right)$ jednak je:

$A.\ 3+3\cdot\log_{2}a$ $B.\ 2\cdot a+2$ $C.\ 4+3\cdot\log_{2}a$ $D.\ 4\cdot a+3$

Rješenje 407

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \quad \rightarrow \qquad a = b^c$$

$$\log_b (x \cdot y) = \log_b x + \log_b y \quad , \quad \log_b b = 1 \quad , \quad \log_b a^n = n \cdot \log_b a.$$

$$\log_2 (4 \cdot a) + \log_2 \left(2 \cdot a^2\right) = \log_2 4 + \log_2 a + \log_2 2 + \log_2 a^2 =$$

$$= \log_2 2^2 + \log_2 a + \log_2 2 + \log_2 a^2 = 2 \cdot \log_2 2 + \log_2 a + \log_2 2 + 2 \cdot \log_2 a =$$

$$= 2 \cdot 1 + \log_2 a + 1 + 2 \cdot \log_2 a = 2 + \log_2 a + 1 + 2 \cdot \log_2 a = 3 + 3 \cdot \log_2 a.$$

Odgovor je pod A.

Vježba 407

Izraz $\log_2 \left(2 \cdot a^2\right) + \log_2 (4 \cdot a)$ jednak je:

A. $3 + 3 \cdot \log_2 a$ *B.* $2 \cdot a + 2$ *C.* $4 + 3 \cdot \log_2 a$ *D.* $4 \cdot a + 3$

Rezultat: A.

Zadatak 408 (Željko, Höhere Technische Lehranstalt)

Riješi jednadžbu: $7^x + 7^{1-x} = 8.$

Rješenje 408

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^1 = a \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a \cdot \frac{b}{c} = \frac{a \cdot b}{c} \quad , \quad a^0 = 1.$$

$$a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$7^x + 7^{1-x} = 8 \Rightarrow 7^x + 7^1 \cdot 7^{-x} = 8 \Rightarrow 7^x + 7 \cdot \frac{1}{7^x} = 8 \Rightarrow \begin{bmatrix} \text{zamjena} \\ t = 7^x \end{bmatrix} \Rightarrow t + 7 \cdot \frac{1}{t} = 8 \Rightarrow$$

$$\Rightarrow t + \frac{7}{t} = 8 \Rightarrow t + \frac{7}{t} = 8 \;/ \cdot t \Rightarrow t^2 + 7 = 8 \cdot t \Rightarrow t^2 + 7 - 8 \cdot t = 0 \Rightarrow t^2 - 8 \cdot t + 7 = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} t^2 - 8 \cdot t + 7 = 0 \\ a = 1 \ , \ b = -8 \ , \ c = 7 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a = 1 \ , \ b = -8 \ , \ c = 7 \\ t_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1} \Rightarrow t_{1,2} = \frac{8 \pm \sqrt{64 - 28}}{2} \Rightarrow t_{1,2} = \frac{8 \pm \sqrt{36}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{8 \pm 6}{2} \Rightarrow \left.\begin{matrix} t_1 = \dfrac{8+6}{2} \\ t_2 = \dfrac{8-6}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = \dfrac{14}{2} \\ t_2 = \dfrac{2}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = \dfrac{14}{2} \\ t_2 = \dfrac{2}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = 7 \\ t_2 = 1 \end{matrix}\right\}.$$

Vraćamo se zamjeni.

- $\left.\begin{matrix} t = 7^x \\ t = 7 \end{matrix}\right\} \Rightarrow 7^x = 7 \Rightarrow 7^x = 7^1 \Rightarrow x_1 = 1$
- $\left.\begin{matrix} t = 7^x \\ t = 1 \end{matrix}\right\} \Rightarrow 7^x = 1 \Rightarrow 7^x = 7^0 \Rightarrow x_2 = 0.$

Vježba 408

Riješi jednadžbu: $7^x - 7^{x-1} = 42.$

Rezultat: x = 2.

Zadatak 409 (Cedric, Sean, Željko, Medox, Höhere Technische Lehranstalt)

Riješi jednadžbu: $\left(\frac{3}{7}\right)^x \cdot \left(\frac{49}{27}\right)^x = \frac{49}{81}.$

Rješenje 409

Ponovimo!

$$a^n \cdot b^n = (a \cdot b)^n \quad , \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

$$a^1 = a \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\left(\frac{3}{7}\right)^x \cdot \left(\frac{49}{27}\right)^x = \frac{49}{81} \Rightarrow \left(\frac{3}{7} \cdot \frac{49}{27}\right)^x = \frac{49}{81} \Rightarrow \left(\frac{3}{7} \cdot \frac{49}{27}\right)^x = \frac{49}{81} \Rightarrow \left(\frac{1}{1} \cdot \frac{7}{9}\right)^x = \frac{49}{81} \Rightarrow$$

$$\Rightarrow \left(\frac{7}{9}\right)^x = \frac{49}{81} \Rightarrow \left(\frac{7}{9}\right)^x = \left(\frac{7}{9}\right)^2 \Rightarrow x = 2.$$

Vježba 409

Riješi jednadžbu: $\left(\frac{3}{7}\right)^x \cdot \left(\frac{49}{27}\right)^x = \frac{7}{9}.$

Rezultat: x = 1.

Zadatak 410 (Ante, srednja škola)

Riješi jednadžbu: $3^{x-1} + 3^x + 3^{x+1} = 5.$

Rješenje 410

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \qquad a = b^c$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log *x* zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log a^n = n \cdot \log a \quad , \quad \log \frac{a}{b} = \log a - \log b \quad , \quad \log(a \cdot b) = \log a + \log b.$$

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^1 = a \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$\frac{a}{b} \cdot \frac{b}{a} = 1 \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \;,\; n \neq 0 \;,\; n \neq 1.$$

1.inačica

$$3^{x-1} + 3^x + 3^{x+1} = 5 \Rightarrow 3^x \cdot 3^{-1} + 3^x + 3^x \cdot 3^1 = 5 \Rightarrow \frac{3^x}{3^1} + 3^x + 3^x \cdot 3 = 5 \Rightarrow$$

$$\Rightarrow \frac{3^x}{3} + 3^x + 3 \cdot 3^x = 5 \Rightarrow \frac{3^x}{3} + \frac{3^x}{1} + \frac{3 \cdot 3^x}{1} = 5 \Rightarrow \frac{3^x + 3 \cdot 3^x + 3 \cdot 3 \cdot 3^x}{3} = 5 \Rightarrow$$

$$\Rightarrow \frac{3^x + 3 \cdot 3^x + 9 \cdot 3^x}{3} = 5 \Rightarrow \frac{3^x + 3 \cdot 3^x + 9 \cdot 3^x}{3} = 5 \Rightarrow \frac{3^x \cdot (1 + 3 + 9)}{3} = 5 \Rightarrow$$

$$\Rightarrow \frac{3^x \cdot 13}{3} = 5 \Rightarrow \frac{3^x \cdot 13}{3} = 5 \;/ \cdot \frac{3}{13} \Rightarrow 3^x = 5 \cdot \frac{3}{13} \Rightarrow 3^x = \frac{5}{1} \cdot \frac{3}{13} \Rightarrow 3^x = \frac{15}{13} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{logaritmiramo} \\ \text{jednadžbu} \end{bmatrix} \Rightarrow 3^x = \frac{15}{13} \;/ \log \Rightarrow \log 3^x = \log \frac{15}{13} \Rightarrow x \cdot \log 3 = \log 15 - \log 13 \Rightarrow$$

$$\Rightarrow x \cdot \log 3 = \log 15 - \log 13 \;/ \cdot \frac{1}{\log 3} \Rightarrow x = \frac{\log 15 - \log 13}{\log 3} \Rightarrow x = 0.13.$$

2.inačica

$$3^{x-1} + 3^x + 3^{x+1} = 5 \Rightarrow 3^{x-1} + 3^{x-1+1} + 3^{x-1+2} = 5 \Rightarrow$$

$$\Rightarrow 3^{x-1} + 3^{x-1} \cdot 3^1 + 3^{x-1} \cdot 3^2 = 5 \Rightarrow 3^{x-1} + 3^{x-1} \cdot 3 + 3^{x-1} \cdot 9 = 5 \Rightarrow$$

$$\Rightarrow 3^{x-1} + 3^{x-1} \cdot 3 + 3^{x-1} \cdot 9 = 5 \Rightarrow 3^{x-1} \cdot (1 + 3 + 9) = 5 \Rightarrow 3^{x-1} \cdot 13 = 5 \Rightarrow$$

$$\Rightarrow 3^{x-1} \cdot 13 = 5 \;/ : 13 \Rightarrow 3^{x-1} = \frac{5}{13} \Rightarrow \begin{bmatrix} \text{logaritmiramo} \\ \text{jednadžbu} \end{bmatrix} \Rightarrow 3^{x-1} = \frac{5}{13} \;/ \log \Rightarrow$$

$$\Rightarrow \log 3^{x-1} = \log \frac{5}{13} \Rightarrow (x - 1) \cdot \log 3 = \log 5 - \log 13 \Rightarrow x \cdot \log 3 - \log 3 = \log 5 - \log 13 \Rightarrow$$

$$\Rightarrow x \cdot \log 3 = \log 5 - \log 13 + \log 3 \Rightarrow x \cdot \log 3 = (\log 5 + \log 3) - \log 13 \Rightarrow$$

$$\Rightarrow x \cdot \log 3 = \log(5 \cdot 3) - \log 13 \Rightarrow x \cdot \log 3 = \log 15 - \log 13 \Rightarrow$$

$$\Rightarrow x \cdot \log 3 = \log 15 - \log 13 \;/ \cdot \frac{1}{\log 3} \Rightarrow x = \frac{\log 15 - \log 13}{\log 3} \Rightarrow x = 0.13.$$

3.inačica

$$3^{x-1}+3^{x}+3^{x+1}=5 \Rightarrow 3^{x}\cdot 3^{-1}+3^{x}+3^{x}\cdot 3^{1}=5 \Rightarrow 3^{x}\cdot\frac{1}{3^{1}}+3^{x}+3^{x}\cdot 3=5 \Rightarrow$$

$$\Rightarrow 3^{x}\cdot\frac{1}{3}+3^{x}+3\cdot 3^{x}=5 \Rightarrow 3^{x}\cdot\frac{1}{3}+3^{x}+3\cdot 3^{x}=5 \Rightarrow 3^{x}\cdot\left(\frac{1}{3}+1+3\right)=5 \Rightarrow$$

$$\Rightarrow 3^{x}\cdot\left(\frac{1}{3}+\frac{1}{1}+\frac{3}{1}\right)=5 \Rightarrow 3^{x}\cdot\frac{1+3+9}{3}=5 \Rightarrow 3^{x}\cdot\frac{13}{3}=5 \Rightarrow 3^{x}\cdot\frac{13}{3}=5 \,/\cdot\frac{3}{13} \Rightarrow$$

$$\Rightarrow 3^{x}=5\cdot\frac{3}{13} \Rightarrow 3^{x}=\frac{5}{1}\cdot\frac{3}{13} \Rightarrow 3^{x}=\frac{15}{13} \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{logaritmiramo}\\ \text{jednadžbu}\end{bmatrix} \Rightarrow 3^{x}=\frac{15}{13} \,/\log \Rightarrow \log 3^{x}=\log\frac{15}{13} \Rightarrow x\cdot\log 3=\log 15-\log 13 \Rightarrow$$

$$\Rightarrow x\cdot\log 3=\log 15-\log 13 \,/\cdot\frac{1}{\log 3} \Rightarrow x=\frac{\log 15-\log 13}{\log 3} \Rightarrow x=0.13.$$

4.inačica

$$3^{x-1}+3^{x}+3^{x+1}=5 \Rightarrow 3^{x+1-2}+3^{x+1-1}+3^{x+1}=5 \Rightarrow$$

$$\Rightarrow 3^{x+1}\cdot 3^{-2}+3^{x+1}\cdot 3^{-1}+3^{x+1}=5 \Rightarrow 3^{x+1}\cdot\frac{1}{3^{2}}+3^{x+1}\cdot\frac{1}{3^{1}}+3^{x+1}=5 \Rightarrow$$

$$\Rightarrow 3^{x+1}\cdot\frac{1}{9}+3^{x+1}\cdot\frac{1}{3}+3^{x+1}=5 \Rightarrow 3^{x+1}\cdot\frac{1}{9}+3^{x+1}\cdot\frac{1}{3}+3^{x+1}=5 \Rightarrow$$

$$\Rightarrow 3^{x+1}\cdot\left(\frac{1}{9}+\frac{1}{3}+1\right)=5 \Rightarrow 3^{x+1}\cdot\left(\frac{1}{9}+\frac{1}{3}+\frac{1}{1}\right)=5 \Rightarrow 3^{x+1}\cdot\frac{1+3+9}{9}=5 \Rightarrow$$

$$\Rightarrow 3^{x+1}\cdot\frac{13}{9}=5 \Rightarrow 3^{x+1}\cdot\frac{13}{9}=5 \,/\cdot\frac{9}{13} \Rightarrow 3^{x+1}=5\cdot\frac{9}{13} \Rightarrow 3^{x+1}=\frac{5}{1}\cdot\frac{9}{13} \Rightarrow$$

$$\Rightarrow 3^{x+1}=\frac{45}{13} \Rightarrow \begin{bmatrix}\text{logaritmiramo}\\ \text{jednadžbu}\end{bmatrix} \Rightarrow 3^{x+1}=\frac{45}{13} \,/\log \Rightarrow \log 3^{x+1}=\log\frac{45}{13} \Rightarrow$$

$$\Rightarrow (x+1)\cdot\log 3=\log 45-\log 13 \Rightarrow x\cdot\log 3+\log 3=\log 45-\log 13 \Rightarrow$$

$$\Rightarrow x\cdot\log 3=\log 45-\log 13-\log 3 \Rightarrow x\cdot\log 3=(\log 45-\log 3)-\log 13 \Rightarrow$$

$$\Rightarrow x\cdot\log 3=\log\frac{45}{3}-\log 13 \Rightarrow x\cdot\log 3=\log\frac{45}{3}-\log 13 \Rightarrow x\cdot\log 3=\log 15-\log 13 \Rightarrow$$

$$\Rightarrow x\cdot\log 3=\log 15-\log 13 \,/\cdot\frac{1}{\log 3} \Rightarrow x=\frac{\log 15-\log 13}{\log 3} \Rightarrow x=0.13.$$

Vježba 410

Riješi jednadžbu: $3^{x}+3^{x+1}+3^{x+2}=15.$

Rezultat: $x=\frac{\log 15-\log 13}{\log 3}=0.13.$

Zadatak 411 (Ivan, gimnazija)

Riješi jednadžbu: $10^{x}-5^{x-1}\cdot 2^{x-2}=950.$

Rješenje 411

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^1 = a \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^n \cdot b^n = (a \cdot b)^n \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad \frac{a}{b} \cdot \frac{b}{a} = 1 \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$10^x - 5^{x-1} \cdot 2^{x-2} = 950 \Rightarrow 10^x - 5^x \cdot 5^{-1} \cdot 2^x \cdot 2^{-2} = 950 \Rightarrow$$

$$\Rightarrow 10^x - 5^x \cdot \frac{1}{5^1} \cdot 2^x \cdot \frac{1}{2^2} = 950 \Rightarrow 10^x - 5^x \cdot \frac{1}{5} \cdot 2^x \cdot \frac{1}{4} = 950 \Rightarrow$$

$$\Rightarrow 10^x - 5^x \cdot 2^x \cdot \frac{1}{5} \cdot \frac{1}{4} = 950 \Rightarrow 10^x - (5 \cdot 2)^x \cdot \frac{1}{20} = 950 \Rightarrow 10^x - 10^x \cdot \frac{1}{20} = 950 \Rightarrow$$

$$\Rightarrow 10^x \cdot \left(1 - \frac{1}{20}\right) = 950 \Rightarrow 10^x \cdot \left(\frac{1}{1} - \frac{1}{20}\right) = 950 \Rightarrow 10^x \cdot \frac{20-1}{20} = 950 \Rightarrow$$

$$\Rightarrow 10^x \cdot \frac{19}{20} = 950 \Rightarrow 10^x \cdot \frac{19}{20} = 950 \ / \cdot \frac{20}{19} \Rightarrow 10^x = 950 \cdot \frac{20}{19} \Rightarrow 10^x = \frac{950}{1} \cdot \frac{20}{19} \Rightarrow$$

$$\Rightarrow 10^x = \frac{950}{1} \cdot \frac{20}{19} \Rightarrow 10^x = \frac{50}{1} \cdot \frac{20}{1} \Rightarrow 10^x = \frac{1000}{1} \Rightarrow 10^x = 1000 \Rightarrow 10^x = 10^3 \Rightarrow x = 3.$$

Vježba 411

Riješi jednadžbu: $5 \cdot 10^x - 5^x \cdot 2^{x-2} = 4750.$

Rezultat: x = 3.

Zadatak 412 (Tea, gimnazija)

Riješi sustav jednadžbi: $\begin{cases} 2^x + y = 5 \\ x - 2 = \log_2 y. \end{cases}$

Rješenje 412

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \ \rightarrow \qquad a = b^c$$

$$a^n \cdot a^m = a^{n+m} \quad , \quad b^{\log_b a} = a \quad , \quad \log_b 1 = 0 \quad , \quad a^{-n} = \frac{1}{a^n}.$$

$$n = \frac{n}{1} \quad , \quad b \cdot \frac{a}{b} = a \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \frac{a}{b} \cdot \frac{b}{a} = 1.$$

$$a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$\left.\begin{array}{l}2^{x}+y=5\\x-2=\log_2 y\end{array}\right\} \Rightarrow \left.\begin{array}{l}2^{x}+y=5\\x=\log_2 y+2\end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{metoda}\\\text{zamjene}\end{array}\right] \Rightarrow 2^{\log_2 y+2}+y=5 \Rightarrow$$

$$\Rightarrow 2^{\log_2 y}\cdot 2^{2}+y=5 \Rightarrow y\cdot 4+y=5 \Rightarrow 4\cdot y+y=5 \Rightarrow 5\cdot y=5 \Rightarrow 5\cdot y=5 \;/:5 \Rightarrow y=1.$$

Računamo x.

$$\left.\begin{array}{l}y=1\\x=\log_2 y+2\end{array}\right\} \Rightarrow x=\log_2 1+2 \Rightarrow x=0+2 \Rightarrow x=2.$$

Rješenje je uređeni par

$$(x,\ y)=(2,\ 1).$$

2.inačica

$$\left.\begin{array}{l}2^{x}+y=5\\x-2=\log_2 y\end{array}\right\} \Rightarrow \left.\begin{array}{l}y=5-2^{x}\\x-2=\log_2 y\end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{metoda}\\\text{zamjene}\end{array}\right] \Rightarrow x-2=\log_2\left(5-2^{x}\right) \Rightarrow \left[\begin{array}{c}\text{uvjet}\\5-2^{x}>0\end{array}\right] \Rightarrow$$

$$\Rightarrow \log_2\left(5-2^{x}\right)=x-2 \Rightarrow 5-2^{x}=2^{x-2} \Rightarrow 5-2^{x}=2^{x}\cdot 2^{-2} \Rightarrow 5-2^{x}=2^{x}\cdot\frac{1}{2^{2}} \Rightarrow$$

$$\Rightarrow 5-2^{x}=2^{x}\cdot\frac{1}{4} \Rightarrow 5=2^{x}\cdot\frac{1}{4}+2^{x} \Rightarrow 5=2^{x}\cdot\left(\frac{1}{4}+1\right) \Rightarrow 5=2^{x}\cdot\left(\frac{1}{4}+\frac{1}{1}\right) \Rightarrow$$

$$\Rightarrow 5=2^{x}\cdot\left(\frac{1}{4}+\frac{1}{1}\right) \Rightarrow 5=2^{x}\cdot\frac{1+4}{4} \Rightarrow 5=2^{x}\cdot\frac{5}{4} \Rightarrow 2^{x}\cdot\frac{5}{4}=5 \Rightarrow$$

$$\Rightarrow 2^{x}\cdot\frac{5}{4}=5 \;/\cdot\frac{4}{5} \Rightarrow 2^{x}=4 \Rightarrow 2^{x}=2^{2} \Rightarrow x=2.$$

Računamo y.

$$\left.\begin{array}{l}x=2\\y=5-2^{x}\end{array}\right\} \Rightarrow y=5-2^{2} \Rightarrow y=5-4 \Rightarrow y=1.$$

Rješenje je uređeni par

$$(x,\ y)=(2,\ 1).$$

Vježba 412

Riješi sustav jednadžbi: $\begin{cases}2^{x}+y=5\\x=\log_2(4y).\end{cases}$

Rezultat: $(x,\ y)=(2,\ 1).$

Zadatak 413 (Max, gimnazija)

Riješi jednadžbu: $5^{x}-2^{x+1}=2^{x}-5^{x-1}.$

Rješenje 413

Ponovimo!

$$a^{n}\cdot a^{m}=a^{n+m}\quad,\quad a^{-n}=\frac{1}{a^{n}}\quad,\quad a^{1}=a\quad,\quad n=\frac{n}{1}.$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}\cdot\frac{b}{a}=1\quad,\quad \frac{a^{n}}{b^{n}}=\left(\frac{a}{b}\right)^{n}\quad,\quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

$$5^{x}-2^{x+1}=2^{x}-5^{x-1} \Rightarrow 5^{x}+5^{x-1}=2^{x}+2^{x+1} \Rightarrow 5^{x}+5^{x}\cdot 5^{-1}=2^{x}+2^{x}\cdot 2^{1} \Rightarrow$$

$$\Rightarrow 5^{x}+5^{x}\cdot\frac{1}{5}=2^{x}+2^{x}\cdot 2 \Rightarrow 5^{x}\cdot\left(1+\frac{1}{5}\right)=2^{x}\cdot(1+2) \Rightarrow 5^{x}\cdot\left(\frac{1}{1}+\frac{1}{5}\right)=2^{x}\cdot 3 \Rightarrow$$

$$\Rightarrow 5^{x}\cdot\frac{5+1}{5}=2^{x}\cdot 3 \Rightarrow 5^{x}\cdot\frac{6}{5}=2^{x}\cdot 3 \Rightarrow 5^{x}\cdot\frac{6}{5}=2^{x}\cdot 3 \;/\cdot\frac{5}{6} \Rightarrow 5^{x}=2^{x}\cdot 3\cdot\frac{5}{6} \Rightarrow$$

$$\Rightarrow 5^{x}=2^{x}\cdot 3\cdot\frac{5}{6} \Rightarrow 5^{x}=2^{x}\cdot\frac{5}{2} \Rightarrow 5^{x}=2^{x}\cdot\frac{5}{2} \;/\cdot\frac{1}{2^{x}} \Rightarrow \frac{5^{x}}{2^{x}}=\frac{5}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{5}{2}\right)^{x}=\left(\frac{5}{2}\right)^{1} \Rightarrow x=1.$$

Vježba 413

Riješi jednadžbu: $5^{x}=3\cdot 2^{x}-5^{x-1}.$

Rezultat: x = 1.

Zadatak 414 (Max, gimnazija)

Riješi jednadžbu: $3^{x}-5^{x+2}=3^{x+4}-5^{x+3}.$

Rješenje 414

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=b^{c} \;(\rightarrow) \qquad a=b^{c}$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log *x* zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10}x=\log x.$$

$$\log a^{n}=n\cdot\log a \quad , \quad \log\frac{a}{b}=\log a-\log b \quad , \quad \frac{a^{n}}{b^{n}}=\left(\frac{a}{b}\right)^{n} \quad , \quad a^{n}\cdot a^{m}=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

$$3^{x}-5^{x+2}=3^{x+4}-5^{x+3} \Rightarrow 3^{x+4}-5^{x+3}=3^{x}-5^{x+2} \Rightarrow$$

$$\Rightarrow 3^{x+4}-3^x=5^{x+3}-5^{x+2} \Rightarrow 3^x\cdot 3^4-3^x=5^x\cdot 5^3-5^x\cdot 5^2 \Rightarrow$$

$$\Rightarrow 3^x\cdot\left(3^4-1\right)=5^x\cdot\left(5^3-5^2\right) \Rightarrow 3^x\cdot(81-1)=5^x\cdot(125-25) \Rightarrow 3^x\cdot 80=5^x\cdot 100 \Rightarrow$$

$$\Rightarrow 3^x\cdot 80=5^x\cdot 100 \;/\cdot\frac{1}{5^x\cdot 80} \Rightarrow \frac{3^x}{5^x}=\frac{100}{80} \Rightarrow \left(\frac{3}{5}\right)^x=\frac{100}{80} \Rightarrow \left(\frac{3}{5}\right)^x=\frac{5}{4} \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{logaritmiramo}\\ \text{jednadžbu}\end{bmatrix} \Rightarrow \left(\frac{3}{5}\right)^x=\frac{5}{4} \;/\log \Rightarrow \log\left(\frac{3}{5}\right)^x=\log\left(\frac{5}{4}\right) \Rightarrow$$

$$\Rightarrow x\cdot\log\left(\frac{3}{5}\right)=\log\left(\frac{5}{4}\right) \Rightarrow x\cdot\log\left(\frac{3}{5}\right)=\log\left(\frac{5}{4}\right) \;/\cdot\frac{1}{\log\left(\frac{3}{5}\right)} \Rightarrow x=\frac{\log\left(\frac{5}{4}\right)}{\log\left(\frac{3}{5}\right)} \Rightarrow$$

$$\Rightarrow x=\frac{\log 5-\log 4}{\log 3-\log 5} \Rightarrow x=-0.43683.$$

Vježba 414

Riješi jednadžbu: $3^x-25\cdot 5^x=3^{x+4}-25\cdot 5^{x+1}$.

Rezultat: -0.43683.

Zadatak 415 (Max, gimnazija)

Riješi jednadžbu: $\log_2\log_2 x+\log_2\left(\log_2 x-1\right)=1$.

Rješenje 415

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=b^c \qquad a=b^c$$
$$\rightarrow$$

$$\log_b(x\cdot y)=\log_b x+\log_b y \quad , \quad \log_b b=1 \quad , \quad \log_b f(x)=\log_b g(x) \Rightarrow f(x)=g(x).$$

$$a^{-n}=\frac{1}{a^n} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$\log_2\log_2 x+\log_2\left(\log_2 x-1\right)=1 \Rightarrow \begin{bmatrix}\text{uvjet}\\ x>0\end{bmatrix} \Rightarrow \log_2\left(\log_2 x\cdot\left(\log_2 x-1\right)\right)=1 \Rightarrow$$

$$\Rightarrow \log_2\left(\log_2 x\cdot\left(\log_2 x-1\right)\right)=\log_2 2 \Rightarrow \log_2 x\cdot\left(\log_2 x-1\right)=2 \Rightarrow$$

$$\Rightarrow \log_2^2 x-\log_2 x=2 \Rightarrow \Rightarrow \log_2^2 x-\log_2 x-2=0 \Rightarrow \begin{bmatrix}\text{zamjena}\\ \log_2 x=t\end{bmatrix} \Rightarrow t^2-t-2=0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t^2 - t - 2 = 0 \\ a = 1 \;,\; b = -1 \;,\; c = -2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = 1 \;,\; b = -1 \;,\; c = -2 \\ t_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array}\right\} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{9}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{1 \pm 3}{2} \Rightarrow \left.\begin{array}{l} t_1 = \dfrac{1+3}{2} \\ t_2 = \dfrac{1-3}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = \dfrac{4}{2} \\ t_2 = -\dfrac{2}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = \dfrac{4}{2} \\ t_2 = -\dfrac{2}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = 2 \\ t_2 = -1 \end{array}\right\}.$$

Vraćamo se zamjeni.

- $$\left.\begin{array}{l} \log_2 x = t \\ t = 2 \end{array}\right\} \Rightarrow \log_2 x = 2 \Rightarrow x = 2^2 \Rightarrow x_1 = 4$$
- $$\left.\begin{array}{l} \log_2 x = t \\ t = -1 \end{array}\right\} \Rightarrow \log_2 x = -1 \Rightarrow x = 2^{-1} \Rightarrow x_2 = \frac{1}{2}.$$

Vježba 415

Riješi jednadžbu: $\log_2 \left(\log_2 x - 1 \right) = 1 - \log_2 \log_2 x.$

Rezultat: $x_1 = 4, \; x_2 = \frac{1}{2}.$

Zadatak 416 (Max, gimnazija)

Koliko je: $\dfrac{\log_5 12 - 2 \cdot \log_5 2}{\log_5 18 + \log_5 0.5}$?

Rješenje 416

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \qquad a = b^c$$
$$\rightarrow$$

$$\log_b a^n = n \cdot \log_b a \quad , \quad \log_b (x \cdot y) = \log_b x + \log_b y \quad , \quad \log_b \frac{x}{y} = \log_b x - \log_b y.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \;,\; n \neq 0 \;,\; n \neq 1.$$

$$\frac{\log_5 12 - 2 \cdot \log_5 2}{\log_5 18 + \log_5 0.5} = \frac{\log_5 12 - \log_5 2^2}{\log_5 (18 \cdot 0.5)} = \frac{\log_5 12 - \log_5 4}{\log_5 9} = \frac{\log_5 \frac{12}{4}}{\log_5 9} = \frac{\log_5 \frac{12}{4}}{\log_5 9} =$$

$$= \frac{\log_5 3}{\log_5 9} = \frac{\log_5 3}{\log_5 3^2} = \frac{\log_5 3}{2 \cdot \log_5 3} = \frac{\log_5 3}{2 \cdot \log_5 3} = \frac{1}{2}.$$

Vježba 416

Koliko je: $\frac{\log_5 18+\log_5 0.5}{\log_5 12-2\cdot\log_5 2}$?

Rezultat: 2.

Zadatak 417 (Sonja, Natascha, Bundeshandelsakademie)

Riješi jednadžbu: $25^{\sqrt{x}}-124\cdot 5^{\sqrt{x}}=125.$

Rješenje 417

Ponovimo!

$$\left(a^n\right)^m=\left(a^m\right)^n=a^{n\cdot m} \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad \left(\sqrt{a}\right)^2=a.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$25^{\sqrt{x}}-124\cdot 5^{\sqrt{x}}=125 \Rightarrow \left(5^2\right)^{\sqrt{x}}-124\cdot 5^{\sqrt{x}}=125 \Rightarrow \left(5^{\sqrt{x}}\right)^2-124\cdot 5^{\sqrt{x}}=125 \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{zamjena} \\ 5^{\sqrt{x}}=t \end{bmatrix} \Rightarrow t^2-124\cdot t=125 \Rightarrow t^2-124\cdot t-125=0 \Rightarrow \left.\begin{matrix} t^2-124\cdot t-125=0 \\ a=1 \ , \ b=-124 \ , \ c=-125 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=1 \ , \ b=-124 \ , \ c=-125 \\ t_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{matrix}\right\} \Rightarrow t_{1,2}=\frac{-(-124)\pm\sqrt{(-124)^2-4\cdot 1\cdot(-125)}}{2\cdot 1} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{124\pm\sqrt{15376+500}}{2} \Rightarrow t_{1,2}=\frac{124\pm\sqrt{15876}}{2} \Rightarrow t_{1,2}=\frac{124\pm 126}{2} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} t_1=\frac{124+126}{2} \\ t_2=\frac{124-126}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\frac{250}{2} \\ t_2=-\frac{2}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\frac{250}{2} \\ t_2=-\frac{2}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=125 \\ t_2=-1 \end{matrix}\right\}.$$

Vraćamo se na zamjenu.

- $$\left.\begin{matrix} 5^{\sqrt{x}}=t \\ t=125 \end{matrix}\right\} \Rightarrow 5^{\sqrt{x}}=125 \Rightarrow 5^{\sqrt{x}}=5^3 \Rightarrow \sqrt{x}=3 \Rightarrow \sqrt{x}=3 \ /^2 \Rightarrow$$

$$\Rightarrow \left(\sqrt{x}\right)^2=3^2 \Rightarrow x=9.$$

- $$\left.\begin{matrix} 5^{\sqrt{x}}=t \\ t=-1 \end{matrix}\right\} \Rightarrow 5^{\sqrt{x}}=-1 \text{ nema smisla}.$$

Vježba 417

Riješi jednadžbu: $5^{2\cdot\sqrt{x}}-124\cdot 5^{\sqrt{x}}-125=0.$

Rezultat: x = 9.

Zadatak 418 (Sonja, Natascha, Bundeshandelsakademie)

Riješi jednadžbu: $4^{\frac{2}{x}}-5\cdot 4^{\frac{1}{x}}=-4.$

Rješenje 418

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^1=a \quad , \quad a^0=1 \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$\frac{a\cdot b}{c}=\frac{a}{c}\cdot b=\frac{b}{c}\cdot a \quad , \quad \frac{a}{b}=1 \Rightarrow a=b.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$4^{\frac{2}{x}}-5\cdot 4^{\frac{1}{x}}=-4 \Rightarrow \left(4^{\frac{1}{x}}\right)^2-5\cdot 4^{\frac{1}{x}}=-4 \Rightarrow \left(4^{\frac{1}{x}}\right)^2-5\cdot 4^{\frac{1}{x}}+4=0 \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{zamjena} \\ 4^{\frac{1}{x}}=t \end{bmatrix} \Rightarrow t^2-5\cdot t+4=0 \Rightarrow \left.\begin{matrix} t^2-5\cdot t+4=0 \\ a=1 \ , \ b=-5 \ , \ c=4 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=1 \ , \ b=-5 \ , \ c=4 \\ t_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{matrix}\right\} \Rightarrow t_{1,2}=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 1\cdot 4}}{2\cdot 1} \Rightarrow t_{1,2}=\frac{5\pm\sqrt{25-16}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{5\pm\sqrt{9}}{2} \Rightarrow t_{1,2}=\frac{5\pm 3}{2} \Rightarrow \left.\begin{matrix} t_1=\frac{5+3}{2} \\ t_2=\frac{5-3}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\frac{8}{2} \\ t_2=\frac{2}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\frac{8}{2} \\ t_2=\frac{2}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=4 \\ t_2=1 \end{matrix}\right\}.$$

Vraćamo se na zamjenu.

- $\left.\begin{matrix} 4^{\frac{1}{x}}=t \\ t=4 \end{matrix}\right\} \Rightarrow 4^{\frac{1}{x}}=4 \Rightarrow 4^{\frac{1}{x}}=4^1 \Rightarrow \frac{1}{x}=1 \Rightarrow x=1$
- $\left.\begin{matrix} 4^{\frac{1}{x}}=t \\ t=1 \end{matrix}\right\} \Rightarrow 4^{\frac{1}{x}}=1 \Rightarrow 4^{\frac{1}{x}}=4^0 \Rightarrow \frac{1}{x}=0$ nema smisla.

Vježba 418

Riješi jednadžbu: $16^{\frac{1}{x}}-5\cdot 4^{\frac{1}{x}}=-4.$

Rezultat: x = 1.

Zadatak 419 (Sonja, Natascha, Bundeshandelsakademie)

Riješi jednadžbu: $5^x\cdot\sqrt[x]{8^{x-1}}=500, \ x\in N.$

Rješenje 419

Ponovimo!

$$\left(a^{n}\right)^{m} = a^{n\cdot m} \quad , \quad \sqrt[n]{a^{m}} = a^{\frac{m}{n}} \quad , \quad a^{n} = a^{m} \Rightarrow n = m.$$

$$a^{n}\cdot b^{m} = a^{x}\cdot b^{y} \Rightarrow \left.\begin{matrix} n = x \\ m = y \end{matrix}\right\}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Prosti brojevi (prim – brojevi) su prirodni brojevi djeljivi bez ostatka samo s brojem 1 i sami sa sobom, a veći od broja 1. Prirodni brojevi koji su veći od broja 1, a nisu prosti brojevi nazivaju se složenim brojevima.

Prost broj (prim broj) je prirodan broj veći od jedan koji je djeljiv jedino brojem 1 i samim sobom. Prost broj ima točno dva djelitelja.

Prosti brojevi su: 2, 3, 5, 7 ... . Ima ih beskonačno mnogo.

Složen broj je prirodan broj veći od jedan koji je djeljiv brojem 1, samim sobom i barem još jednim brojem.

Složeni brojevi imaju više od dva djelitelja. Složeni brojevi su: 4, 6, 8, 10, … . Ima ih beskonačno mnogo.

Svaki se složeni broj može rastaviti na proste faktore.

Broj 1 nije ni složen ni prost broj.

$$5^{x}\cdot\sqrt[x]{8^{x-1}} = 500 \Rightarrow 5^{x}\cdot\sqrt[x]{\left(2^{3}\right)^{x-1}} = 500 \Rightarrow 5^{x}\cdot\sqrt[x]{2^{3\cdot(x-1)}} = 500 \Rightarrow$$

$$\Rightarrow 5^{x}\cdot\sqrt[x]{2^{3\cdot x-3}} = 500 \Rightarrow 5^{x}\cdot 2^{\frac{3\cdot x-3}{x}} = 500 \Rightarrow \begin{bmatrix} 500 = 2\cdot 250 = \\ = 2\cdot 2\cdot 125 = \\ = 2\cdot 2\cdot 5\cdot 25 = \\ = 2\cdot 2\cdot 5\cdot 5\cdot 5 = \\ = 2^{2}\cdot 5^{3} \end{bmatrix} \Rightarrow$$

$$\Rightarrow 5^{x}\cdot 2^{\frac{3\cdot x-3}{x}} = 5^{3}\cdot 2^{2} \Rightarrow \left.\begin{matrix} x = 3 \\ \frac{3\cdot x-3}{x} = 2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = 3 \\ \frac{3\cdot x-3}{x} = 2 \;/\cdot x \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = 3 \\ 3\cdot x-3 = 2\cdot x \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x = 3 \\ 3\cdot x-2\cdot x = 3 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = 3 \\ x = 3 \end{matrix}\right\} \Rightarrow x = 3.$$

Vježba 419

Riješi jednadžbu: $2\cdot 5^{x}\cdot\sqrt[x]{8^{x-1}} = 1000,\ x\in N.$

Rezultat: x = 3.

Zadatak 420 (Sonja, Natascha, Bundeshandelsakademie)

Riješi jednadžbu: $(0.4)^{\log^{2} x+1} = (6.25)^{2-\log x^{3}}.$

Rješenje 420

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_{b} a = c \qquad \underset{\rightarrow}{\log_{b} a = b^{c}} \qquad a = b^{c}$$

Dekadski logaritam

Logaritamska funkcija log10 označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log a^n = n \cdot \log a.$$

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Decimalni broj piše se u obliku decimalnog razlomka tako da se u brojnik napiše zadani decimalni broj bez decimalne točke, a u nazivnik se napiše dekadska jedinica (10, 100, 1000, 10000, 100000, …) koja ima toliko nula koliko decimalni broj ima decimala (znamenaka na decimalnom mjestu, tj. iza decimalne točke ili decimalnog zareza).

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$(0.4)^{\log^2 x+1} = (6.25)^{2-\log x^3} \Rightarrow \left(\frac{4}{10}\right)^{\log^2 x+1} = \left(\frac{625}{100}\right)^{2-\log x^3} \Rightarrow$$

$$\Rightarrow \left(\frac{4}{10}\right)^{\log^2 x+1} = \left(\frac{625}{100}\right)^{2-\log x^3} \Rightarrow \left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\frac{25}{4}\right)^{2-\log x^3} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\left(\frac{5}{2}\right)^2\right)^{2-\log x^3} \Rightarrow \left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\frac{5}{2}\right)^{2\cdot\left(2-\log x^3\right)} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\frac{5}{2}\right)^{4-2\cdot\log x^3} \Rightarrow \left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\left(\frac{2}{5}\right)^{-1}\right)^{4-2\cdot\log x^3} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\frac{2}{5}\right)^{-1\cdot\left(4-2\cdot\log x^3\right)} \Rightarrow \left(\frac{2}{5}\right)^{\log^2 x+1} = \left(\frac{2}{5}\right)^{-4+2\cdot\log x^3} \Rightarrow$$

$$\Rightarrow \log^2 x + 1 = -4 + 2 \cdot \log x^3 \Rightarrow \log^2 x + 1 = -4 + 2 \cdot 3 \cdot \log x \Rightarrow$$

$$\Rightarrow \log^2 x + 1 = -4 + 6 \cdot \log x \Rightarrow \log^2 x + 1 + 4 - 6 \cdot \log x = 0 \Rightarrow$$

$$\Rightarrow \log^2 x - 6 \cdot \log x + 5 = 0 \Rightarrow \left[\begin{array}{c}\text{zamjena} \\ \log x = t\end{array}\right] \Rightarrow t^2 - 6 \cdot t + 5 = 0 \Rightarrow \left.\begin{array}{l} t^2 - 6 \cdot t + 5 = 0 \\ a = 1 \ , \ b = -6 \ , \ c = 5 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a = 1 \ , \ b = -6 \ , \ c = 5 \\ t_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array}\right\} \Rightarrow t_{1,2} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} \Rightarrow t_{1,2} = \frac{6 \pm \sqrt{36 - 20}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{6\pm\sqrt{16}}{2} \Rightarrow t_{1,2}=\frac{6\pm4}{2} \Rightarrow \left.\begin{matrix} t_1=\frac{6+4}{2} \\ t_2=\frac{6-4}{2}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\frac{10}{2} \\ t_2=\frac{2}{2}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\frac{10}{2} \\ t_2=\frac{2}{2}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=5 \\ t_2=1\end{matrix}\right\}.$$

Vraćamo se na zamjenu.

- $\left.\begin{matrix}\log x=t \\ t=5\end{matrix}\right\} \Rightarrow \log x=5 \Rightarrow x=10^5 \Rightarrow x_1=100\,000$
- $\left.\begin{matrix}\log x=t \\ t=1\end{matrix}\right\} \Rightarrow \log x=1 \Rightarrow x=10^1 \Rightarrow x_2=10.$

Vježba 420

Riješi jednadžbu: $\frac{2}{5}\cdot(0.4)^{\log^2 x}=(6.25)^{2-\log x^3}$.

Rezultat: $x_1=100\,000,\ x_2=10.$