Zadatak 061 (Mira, gimnazija)

Ako je $\sin x + \cos x = \frac{4}{3}$, koliko je $\sin x \cdot \cos x$?

Rješenje 061

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad \cos^2 x + \sin^2 x = 1$$

$$\sin x + \cos x = \frac{4}{3} \Rightarrow \sin x + \cos x = \frac{4}{3} \ /^2 \Rightarrow \sin^2 x + 2 \cdot \sin x \cdot \cos x + \cos^2 x = \frac{16}{9} \Rightarrow 1 + 2 \cdot \sin x \cdot \cos x = \frac{16}{9} \Rightarrow$$

$$\Rightarrow 2 \cdot \sin x \cdot \cos x = \frac{16}{9} - 1 \Rightarrow 2 \cdot \sin x \cdot \cos x = \frac{7}{9} \ /:2 \Rightarrow \sin x \cdot \cos x = \frac{7}{18}.$$

Vježba 061

Ako je $\sin x - \cos x = \frac{4}{3}$, koliko je $\sin x \cdot \cos x$?

Rezultat: $-\frac{7}{18}$.

Zadatak 062 (Martina, farmaceutska škola)

Izračunaj: $\sqrt{\frac{1-\cos x}{1+\cos x}} + \sqrt{\frac{1+\cos x}{1-\cos x}}, \ 0 < x < 90^0.$

Rješenje 062

1.inačica

Ponovimo!

$$(a-b) \cdot (a+b) = a^2 - b^2 \quad , \quad \cos^2 x + \sin^2 x = 1$$

$$\sqrt{\frac{1-\cos x}{1+\cos x}} + \sqrt{\frac{1+\cos x}{1-\cos x}} = \sqrt{\frac{1-\cos x}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x}} + \sqrt{\frac{1+\cos x}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x}} = \sqrt{\frac{(1-\cos x)^2}{1-\cos^2 x}} + \sqrt{\frac{(1+\cos x)^2}{1-\cos^2 x}} =$$

$$= \sqrt{\frac{(1-\cos x)^2}{\sin^2 x}} + \sqrt{\frac{(1+\cos x)^2}{\sin^2 x}} = \frac{1-\cos x}{\sin x} + \frac{1+\cos x}{\sin x} = \frac{1-\cos x + 1 + \cos x}{\sin x} = \frac{2}{\sin x}.$$

2.inačica

Ponovimo!

$$\left(\sqrt{a} + \sqrt{b}\right)^2 = a + 2 \cdot \sqrt{a \cdot b} + b \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2$$

$$(a-b) \cdot (a+b) = a^2 - b^2 \quad , \quad \cos^2 x + \sin^2 x = 1$$

Označimo zadani izraz slovom x:

$$x = \sqrt{\frac{1-\cos x}{1+\cos x}} + \sqrt{\frac{1+\cos x}{1-\cos x}} \Rightarrow x = \sqrt{\frac{1-\cos x}{1+\cos x}} + \sqrt{\frac{1+\cos x}{1-\cos x}} \ /^2 \Rightarrow$$

$$\Rightarrow x^2 = \frac{1-\cos x}{1+\cos x} + 2 \cdot \sqrt{\frac{1-\cos x}{1+\cos x} \cdot \frac{1+\cos x}{1-\cos x}} + \frac{1+\cos x}{1-\cos x} \Rightarrow x^2 = \frac{1-\cos x}{1+\cos x} + 2 \cdot 1 + \frac{1+\cos x}{1-\cos x} \Rightarrow$$

$$\Rightarrow x^2 = \frac{1-\cos x}{1+\cos x} + 2 + \frac{1+\cos x}{1-\cos x} \Rightarrow x^2 = \frac{(1-\cos x)^2 + 2 \cdot (1+\cos x) \cdot (1-\cos x) + (1+\cos x)^2}{(1+\cos x) \cdot (1-\cos x)} \Rightarrow$$

$$\Rightarrow x^2 = \frac{1 - 2 \cdot \cos x + \cos^2 x + 2 \cdot \left(1 - \cos^2 x\right) + 1 + 2 \cdot \cos x + \cos^2 x}{1 - \cos^2 x} \Rightarrow x^2 = \frac{2 + 2 \cdot \cos^2 x + 2 \cdot \sin^2 x}{\sin^2 x} \Rightarrow$$

$$\Rightarrow x^2 = \frac{2+2\cdot\left(\cos^2 x+\sin^2 x\right)}{\sin^2 x} \Rightarrow x^2 = \frac{2+2\cdot 1}{\sin^2 x} \Rightarrow x^2 = \frac{4}{\sin^2 x} \; / \sqrt{\;} \quad \Rightarrow x = \frac{2}{\sin x}.$$

Vježba 062

Izračunaj: $\sqrt{\dfrac{1-\cos x}{1+\cos x}} + ctg\, x,\ 0 < x < 90^0.$

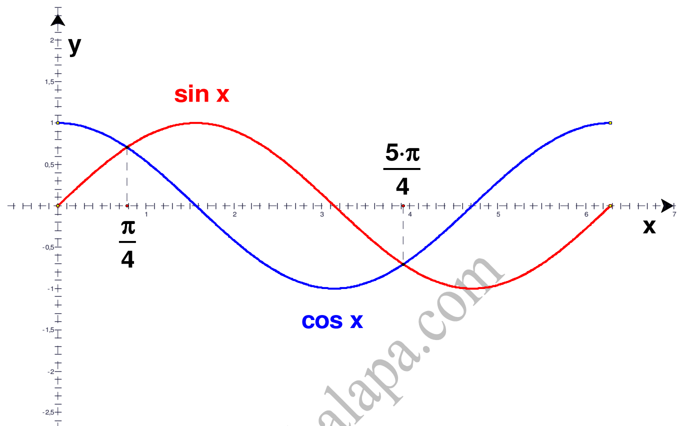
Rezultat: $\dfrac{1}{\sin x}.$

Zadatak 063 (Martina, farmaceutska škola)

Koliko iznosi zbroj rješenja jednadžbe sin x + cos x + $|\sin x - \cos x| = 1$ iz intervala $\langle 0, 2\cdot\pi \rangle$?

Rješenje 063

Nacrtamo grafove funkcija sin x i cos x na segmentu $[0, 2\cdot\pi]$.



Sa slike vidi se:

$$\sin x > \cos x \Rightarrow \sin x - \cos x > 0 \text{ za } x \in \left\langle \frac{\pi}{4}, \frac{5\cdot\pi}{4} \right\rangle.$$

Zato je:

$$\sin x + \cos x + \left| \underbrace{\sin x - \cos x}_{+} \right| = 1 \Rightarrow \sin x + \cos x + \sin x - \cos x = 1 \Rightarrow 2\cdot\sin x = 1 \; /:2 \Rightarrow \sin x = \frac{1}{2} \Rightarrow$$

$$\Rightarrow x = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow x_1 = \frac{5\cdot\pi}{6}.$$

Uočimo da je:

$$\sin x < \cos x \Rightarrow \sin x - \cos x < 0 \text{ za } x \in \left\langle 0, \frac{\pi}{4} \right\rangle \cup \left\langle \frac{5\cdot\pi}{4}, 2\cdot\pi \right\rangle.$$

Zato je:

$$\sin x + \cos x + \left| \underbrace{\sin x - \cos x}_{-} \right| = 1 \Rightarrow \sin x + \cos x - \sin x + \cos x = 1 \Rightarrow 2\cdot\cos x = 1 \; /:2 \Rightarrow \cos x = \frac{1}{2} \Rightarrow$$

$$\Rightarrow x = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow x_2 = \frac{5\cdot\pi}{3}.$$

Zbroj rješenja iznosi:

$$x_1 + x_2 = \frac{5\cdot\pi}{6} + \frac{5\cdot\pi}{3} = \frac{5\cdot\pi + 10\cdot\pi}{6} = \frac{15\cdot\pi}{6} = \frac{5\cdot\pi}{2}.$$

Vježba 063

Koliko iznosi zbroj rješenja jednadžbe sin x + cos x + $|\sin x - \cos x| = 2$ iz intervala $\langle 0, 2\cdot\pi \rangle$?

Rezultat: 90°.

Zadatak 064 (Vedrana, gimnazija)

Ako je $\cos 2x + 2\cdot\cos x = 0$, koliko je $\cos^2 x + \cos x$?

Rješenje 064

Ponovimo!

$$\cos 2x = \cos^2 x - \sin^2 x \quad , \quad \cos^2 x + \sin^2 x = 1$$

$$\cos 2x + 2\cdot\cos x = 0 \Rightarrow \cos^2 x - \sin^2 x + 2\cdot\cos x = 0 \Rightarrow \cos^2 x - \left(1-\cos^2 x\right) + 2\cdot\cos x = 0 \Rightarrow$$

$$\Rightarrow \cos^2 x - 1 + \cos^2 x + 2\cdot\cos x = 0 \Rightarrow 2\cdot\cos^2 x + 2\cdot\cos x - 1 = 0 \Rightarrow 2\cdot\cos^2 x + 2\cdot\cos x = 1 \,/:2 \Rightarrow$$

$$\Rightarrow \cos^2 x + \cos x = \frac{1}{2}.$$

Vježba 064

Ako je $\cos 2x + 2\cdot\sin x = 0$, koliko je $\sin^2 x - \sin x$?

Rezultat: $\frac{1}{2}.$

Zadatak 065 (Maja, gimnazija)

Ako je u trokutu $tg\,\alpha = \frac{2}{3}$ i $\gamma = 135^0$, koliko iznosi $tg\,\beta$?

Rješenje 065

Ponovimo!

$$\alpha + \beta + \gamma = 180^0 \quad , \quad tg\,(x+y) = \frac{tg\,x + tg\,y}{1 - tg\,x\cdot tg\,y}.$$

Odredimo zbroj kutova α i β:

$$\left.\begin{array}{l}\alpha+\beta+\gamma=180^0\\ \gamma=135^0\end{array}\right\} \Rightarrow \left.\begin{array}{l}\alpha+\beta=180^0-\gamma\\ \gamma=135^0\end{array}\right\} \Rightarrow \alpha+\beta=180^0-135^0 \Rightarrow \alpha+\beta=45^0.$$

Računamo tg β:

$$tg\,(\alpha+\beta) = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha\cdot tg\,\beta} \Rightarrow tg\,45^0 = \frac{\frac{2}{3}+tg\,\beta}{1-\frac{2}{3}\cdot tg\,\beta} \Rightarrow 1 = \frac{\frac{2}{3}+tg\,\beta}{1-\frac{2}{3}\cdot tg\,\beta} \Rightarrow 1-\frac{2}{3}\cdot tg\,\beta = \frac{2}{3}+tg\,\beta \Rightarrow$$

$$\Rightarrow -\frac{2}{3}\cdot tg\,\beta - tg\,\beta = \frac{2}{3}-1 \Rightarrow -\frac{5}{3}\cdot tg\,\beta = -\frac{1}{3} \,/\cdot\left(-\frac{3}{5}\right) \Rightarrow tg\,\beta = \frac{1}{5}.$$

Vježba 065

Ako je u trokutu $tg\,\alpha = \frac{1}{2}$ i $\gamma = 135^0$, koliko iznosi $tg\,\beta$?

Rezultat: $\frac{1}{3}.$

Zadatak 066 (Maja, gimnazija)

Koliki je broj rješenja trigonometrijske jednadžbe sin x + sin 2x + sin 3x + sin 4x = 0 koja se nalaze na segmentu [0, π]?

Rješenje 066

Ponovimo!

$$\sin x + \sin y = 2\cdot\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2} \quad , \quad \cos x + \cos y = 2\cdot\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2} \quad , \quad \cos(-x) = \cos x$$

$$a\cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0$$

Rješavamo jednadžbu:

$$\sin x+\sin 2x+\sin 3x+\sin 4x=0 \Rightarrow (\sin x+\sin 4x)+(\sin 2x+\sin 3x)=0 \Rightarrow$$

$$\Rightarrow 2\cdot\sin\frac{x+4x}{2}\cdot\cos\frac{x-4x}{2}+2\cdot\sin\frac{2x+3x}{2}\cdot\cos\frac{2x-3x}{2}=0 \Rightarrow$$

$$\Rightarrow 2\cdot\sin\frac{5x}{2}\cdot\cos\left(-\frac{3x}{2}\right)+2\cdot\sin\frac{5x}{2}\cdot\cos\left(-\frac{x}{2}\right)=0 \Rightarrow 2\cdot\sin\frac{5x}{2}\cdot\cos\frac{3x}{2}+2\cdot\sin\frac{5x}{2}\cdot\cos\frac{x}{2}=0 \;/:2 \Rightarrow$$

$$\Rightarrow \sin\frac{5x}{2}\cdot\cos\frac{3x}{2}+\sin\frac{5x}{2}\cdot\cos\frac{x}{2}=0 \Rightarrow \sin\frac{5x}{2}\cdot\left[\cos\frac{3x}{2}+\cos\frac{x}{2}\right]=0 \Rightarrow$$

$$\Rightarrow \sin\frac{5x}{2}\cdot 2\cdot\cos\frac{\frac{3x}{2}+\frac{x}{2}}{2}\cdot\cos\frac{\frac{3x}{2}-\frac{x}{2}}{2}=0 \Rightarrow 2\cdot\sin\frac{5x}{2}\cdot\cos\frac{2x}{2}\cdot\cos\frac{x}{2}=0 \Rightarrow$$

$$\Rightarrow 2\cdot\sin\frac{5x}{2}\cdot\cos x\cdot\cos\frac{x}{2}=0 \;/:2 \Rightarrow \sin\frac{5x}{2}\cdot\cos x\cdot\cos\frac{x}{2}=0 \Rightarrow \left.\begin{array}{l}\sin\frac{5x}{2}=0\\ \cos x=0\\ \cos\frac{x}{2}=0\end{array}\right\}.$$

Tražimo broj rješenja svake pojedine jednadžbe na segmentu [0, π]:

- $$\sin\frac{5x}{2}=0 \Rightarrow \frac{5x}{2}=k\cdot\pi,\ k\in Z \Rightarrow \frac{5x}{2}=k\cdot\pi \;/\cdot\frac{2}{5} \Rightarrow x=k\cdot\frac{2\cdot\pi}{5}.$$

  Budući da rješenja moraju biti na segmentu [0, π], slijedi:

  $$0\le x\le\pi \Rightarrow 0\le k\cdot\frac{2\cdot\pi}{5}\le\pi \Rightarrow 0\le k\cdot\frac{2\cdot\pi}{5}\le\pi \;/\cdot\frac{5}{2\cdot\pi} \Rightarrow 0\le k\le\frac{5}{2} \Rightarrow k=0,1,2 \Rightarrow \text{tri rješenja}.$$

- $$\cos x=0 \Rightarrow x=(2\cdot k+1)\cdot\frac{\pi}{2},\ k\in Z \Rightarrow x=(2\cdot k+1)\cdot\frac{\pi}{2}.$$

  Budući da rješenja moraju biti na segmentu [0, π], slijedi:

  $$0\le x\le\pi \Rightarrow 0\le(2\cdot k+1)\cdot\frac{\pi}{2}\le\pi \Rightarrow 0\le(2\cdot k+1)\cdot\frac{\pi}{2}\le\pi \;/\cdot\frac{2}{\pi} \Rightarrow 0\le 2\cdot k+1\le 2 \;/-1 \Rightarrow$$

  $$\Rightarrow 0-1\le 2\cdot k+1-1\le 2-1 \Rightarrow -1\le 2\cdot k\le 1 \;/:2 \Rightarrow -\frac{1}{2}\le k\le\frac{1}{2} \Rightarrow k=0 \Rightarrow \text{jedno rješenje}.$$

- $$\cos\frac{x}{2}=0 \Rightarrow \frac{x}{2}=(2\cdot k+1)\cdot\frac{\pi}{2},\ k\in Z \Rightarrow \frac{x}{2}=(2\cdot k+1)\cdot\frac{\pi}{2} \;/\cdot 2 \Rightarrow x=(2\cdot k+1)\cdot\pi.$$

  Budući da rješenja moraju biti na segmentu [0, π], slijedi:

  $$0\le x\le\pi \Rightarrow 0\le(2\cdot k+1)\cdot\pi\le\pi \Rightarrow 0\le(2\cdot k+1)\cdot\pi\le\pi \;/:\pi \Rightarrow 0\le 2\cdot k+1\le 1 \Rightarrow$$

  $$\Rightarrow 0\le 2\cdot k+1\le 1 \;/-1 \Rightarrow 0-1\le 2\cdot k+1-1\le 1-1 \Rightarrow -1\le 2\cdot k\le 0 \;/:2 \Rightarrow$$

  $$\Rightarrow -\frac{1}{2}\le k\le 0 \Rightarrow k=0 \Rightarrow \text{jedno rješenje}.$$

Broj rješenja je 5.

Vježba 066

Koliki je zbroj rješenja trigonometrijske jednadžbe sin x + sin 2x + sin 3x + sin 4x = 0 koja se nalaze na segmentu [0, π]?

Rezultat: $\frac{27}{10}\cdot\pi.$

Zadatak 067 (Maja, gimnazija)

Pojednostavnite: $\sin^2\alpha+\cos\left(\frac{\pi}{3}-\alpha\right)\cdot\cos\left(\frac{\pi}{3}+\alpha\right).$

Rješenje 067

Ponovimo!

$$\cos x\cdot\cos y=\frac{1}{2}\cdot\left[\cos(x+y)+\cos(x-y)\right]\quad,\quad\cos(\pi-x)=-\cos x\quad,\quad\cos(-x)=\cos x$$

$$\cos 2x=\cos^2 x-\sin^2 x\quad,\quad\cos^2 x+\sin^2 x=1.$$

$$\sin^2\alpha+\cos\left(\frac{\pi}{3}-\alpha\right)\cdot\cos\left(\frac{\pi}{3}+\alpha\right)=\sin^2\alpha+\frac{1}{2}\cdot\left[\cos\left(\left(\frac{\pi}{3}-\alpha\right)+\left(\frac{\pi}{3}+\alpha\right)\right)+\cos\left(\left(\frac{\pi}{3}-\alpha\right)-\left(\frac{\pi}{3}+\alpha\right)\right)\right]=$$

$$=\sin^2\alpha+\frac{1}{2}\cdot\left[\cos\left(\frac{\pi}{3}-\alpha+\frac{\pi}{3}+\alpha\right)+\cos\left(\frac{\pi}{3}-\alpha-\frac{\pi}{3}-\alpha\right)\right]=\sin^2\alpha+\frac{1}{2}\cdot\left[\cos\frac{2\cdot\pi}{3}+\cos(-2\cdot\alpha)\right]=$$

$$=\sin^2\alpha+\frac{1}{2}\cdot\left[\cos\left(\pi-\frac{\pi}{3}\right)+\cos 2\cdot\alpha\right]=\sin^2\alpha+\frac{1}{2}\cdot\left[-\cos\frac{\pi}{3}+\cos^2\alpha-\sin^2\alpha\right]=$$

$$=\sin^2\alpha+\frac{1}{2}\cdot\left[-\frac{1}{2}+1-\sin^2\alpha-\sin^2\alpha\right]=\sin^2\alpha+\frac{1}{2}\cdot\left[\frac{1}{2}-2\cdot\sin^2\alpha\right]=\sin^2\alpha+\frac{1}{4}-\sin^2\alpha=\frac{1}{4}.$$

Vježba 067

Pojednostavnite: $\sin^2\alpha+\cos\left(\frac{\pi}{2}-\alpha\right)\cdot\cos\left(\frac{\pi}{2}+\alpha\right).$

Rezultat: 0.

Zadatak 068 (Maja, gimnazija)

Nađite zbroj rješenja jednadžbe $\log_3(tg\,3x)=\frac{1}{2}$ na segmentu [0, π].

Rješenje 068

Ponovimo!

$$\log_b a=c\ \Leftrightarrow\ b^c=a\quad,\quad x^{\frac{1}{2}}=\sqrt{x}.$$

Rješavamo jednadžbu:

$$\log_3(tg\,3x)=\frac{1}{2}\ \Rightarrow\ 3^{\frac{1}{2}}=tg\,3x\ \Rightarrow\ tg\,3x=\sqrt{3}\ \Rightarrow\ \left[\begin{array}{l}\text{supstitucija}\\ t=3\cdot x\end{array}\right]\ \Rightarrow\ tg\,t=\sqrt{3}\ \Rightarrow\ t=\frac{\pi}{3}+k\cdot\pi,\ k\in Z\ \Rightarrow$$

$$\Rightarrow\ 3\cdot x=\frac{\pi}{3}+k\cdot\pi\ /:3\ \Rightarrow\ x=\frac{\pi}{9}+k\cdot\frac{\pi}{3}.$$

Tražimo broj rješenja jednadžbe na segmentu [0, π]:

$$0\le x\le\pi\ \Rightarrow\ 0\le\frac{\pi}{9}+k\cdot\frac{\pi}{3}\le\pi\ \Rightarrow\ 0\le\frac{\pi}{9}+k\cdot\frac{\pi}{3}\le\pi\ /:\pi\ \Rightarrow\ 0\le\frac{1}{9}+\frac{k}{3}\le 1\ \Rightarrow\ 0\le\frac{1}{9}+\frac{k}{3}\le 1\ /-\frac{1}{9}\ \Rightarrow$$

$$\Rightarrow\ 0-\frac{1}{9}\le\frac{1}{9}+\frac{k}{3}-\frac{1}{9}\le 1-\frac{1}{9}\ \Rightarrow\ -\frac{1}{9}\le\frac{k}{3}\le\frac{8}{9}\ \Rightarrow\ -\frac{1}{9}\le\frac{k}{3}\le\frac{8}{9}\ /\cdot 3\ \Rightarrow\ -\frac{1}{3}\le k\le\frac{8}{3}\ \Rightarrow\ \left[\begin{array}{l}k\text{ je cijeli}\\ \text{broj}\end{array}\right]\ \Rightarrow$$

$$\Rightarrow\ k=0,1,2\ \Rightarrow\ \text{tri rješenja}.$$

Prvo rješenje iznosi:

$$\left.\begin{array}{l}k=0\\ x=\frac{\pi}{9}+k\cdot\frac{\pi}{3}\end{array}\right\}\ \Rightarrow\ x_1=\frac{\pi}{9}+0\cdot\frac{\pi}{3}\ \Rightarrow\ x_1=\frac{\pi}{9}.$$

Drugo rješenje iznosi:

$$\left.\begin{array}{l} k=1 \\ x=\frac{\pi}{9}+k\cdot\frac{\pi}{3} \end{array}\right\} \Rightarrow x_2=\frac{\pi}{9}+1\cdot\frac{\pi}{3} \Rightarrow x_2=\frac{\pi}{9}+\frac{\pi}{3}=\frac{\pi+3\cdot\pi}{9}=\frac{4\cdot\pi}{9}.$$

Treće rješenje iznosi:

$$\left.\begin{array}{l} k=2 \\ x=\frac{\pi}{9}+k\cdot\frac{\pi}{3} \end{array}\right\} \Rightarrow x_3=\frac{\pi}{9}+2\cdot\frac{\pi}{3} \Rightarrow x_3=\frac{\pi}{9}+\frac{2\cdot\pi}{3}=\frac{\pi+6\cdot\pi}{9}=\frac{7\cdot\pi}{9}.$$

Zbroj rješenja je:

$$x_1+x_2+x_3=\frac{\pi}{9}+\frac{4\cdot\pi}{9}+\frac{7\cdot\pi}{9}=\frac{12\cdot\pi}{9}=\frac{4\cdot\pi}{3}.$$

Vježba 068

Nađite zbroj rješenja jednadžbe $\log_9(tg\,3x)=\frac{1}{4}$ na segmentu [0, π].

Rezultat: $\frac{4}{3}\cdot\pi.$

Zadatak 069 (Iva, gimnazija)

Neka su α i β kutovi pravokutnog trokuta (α ≠ 90°, β ≠ 90°). Ako je $tg\,\alpha=\frac{7}{24}$, koliko je sin β?

Rješenje 069

Ponovimo!

$$tg\,x=ctg\left(90^0-x\right) \quad , \quad \cos^2 x+\sin^2 x=1 \quad , \quad ctg\,x=\frac{\cos x}{\sin x}.$$

Budući da su α i β kutovi pravokutnog trokuta, slijedi:

$$\alpha+\beta=90^0 \Rightarrow \beta=90^0-\alpha \Rightarrow ctg\,\beta=ctg\left(90^0-\alpha\right) \Rightarrow ctg\,\beta=tg\,\alpha \Rightarrow ctg\,\beta=\frac{7}{24} \Rightarrow \frac{\cos\beta}{\sin\beta}=\frac{7}{24} \;/^2 \Rightarrow$$

$$\Rightarrow \frac{\cos^2\beta}{\sin^2\beta}=\frac{49}{576} \Rightarrow \frac{1-\sin^2\beta}{\sin^2\beta}=\frac{49}{576} \Rightarrow \frac{1}{\sin^2\beta}-\frac{\sin^2\beta}{\sin^2\beta}=\frac{49}{576} \Rightarrow \frac{1}{\sin^2\beta}-1=\frac{49}{576} \Rightarrow$$

$$\Rightarrow \frac{1}{\sin^2\beta}=1+\frac{49}{576} \Rightarrow \frac{1}{\sin^2\beta}=\frac{625}{576} \Rightarrow \sin^2\beta=\frac{576}{625} \;/\sqrt{\;} \Rightarrow \sin\beta=\sqrt{\frac{576}{625}} \Rightarrow \sin\beta=\frac{24}{25}.$$

Vježba 069

Neka su α i β kutovi pravokutnog trokuta (α ≠ 90°, β ≠ 90°). Ako je $tg\,\alpha=\frac{7}{24}$, koliko je cos β?

Rezultat: $\frac{7}{25}.$

Zadatak 070 (Beny, strojarska škola)

Nađi rješenja trigonometrijske jednadžbe $\cos^4 x-\sin^4 x=\cos\frac{5\cdot\pi}{6}$ na segmentu $\left[\frac{\pi}{2},\ \pi\right]$.

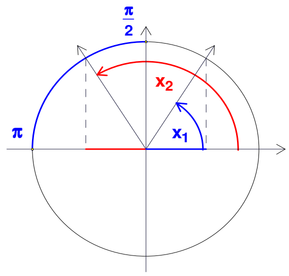
Rješenje 070

Ponovimo!

$$x^2-y^2=(x+y)\cdot(x-y) \quad , \quad \cos^2 x+\sin^2 x=1 \quad , \quad \cos^2 x-\sin^2 x=\cos 2x.$$

Rješavamo jednadžbu:

$$\cos^4 x-\sin^4 x=\cos\frac{5\cdot\pi}{6} \Rightarrow \left(\cos^2 x+\sin^2 x\right)\cdot\left(\cos^2 x-\sin^2 x\right)=\cos\frac{5\cdot\pi}{6} \Rightarrow$$



$$\Rightarrow \underbrace{\left(\cos^2 x+\sin^2 x\right)}_{1}\cdot\left(\cos^2 x-\sin^2 x\right)=\cos\frac{5\cdot\pi}{6} \Rightarrow$$

$$\cos^2 x-\sin^2 x=\cos\frac{5\cdot\pi}{6} \Rightarrow \cos 2x=\cos\frac{5\cdot\pi}{6} \Rightarrow 2x=\frac{5\cdot\pi}{6} \Rightarrow$$

$$\Rightarrow 2x=\frac{5\cdot\pi}{6}\ /\cdot\frac{1}{2} \Rightarrow x_1=\frac{5\cdot\pi}{12}$$ (nije rješenje jer je u I. kvadrantu).

Rezultat je:

$$x_2=\pi-x_1 \Rightarrow x_2=\pi-\frac{5\cdot\pi}{12} \Rightarrow x_2=\frac{7\cdot\pi}{12}$$ (to je u II. kvadrantu).

Vježba 070

Nađi rješenja trigonometrijske jednadžbe $\cos^4 x-\sin^4 x=\cos\frac{5\cdot\pi}{6}$ na segmentu $\left[0,\ \frac{\pi}{2}\right]$.

Rezultat: $\frac{5\cdot\pi}{12}$.

Zadatak 071 (Goga, gimnazija)

Ako je $\cos^2\alpha+\cos^2\beta=a$, koliko je $\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)$?

Rješenje 071

Ponovimo!

$$\cos(x+y)=\cos x\cdot\cos y-\sin x\cdot\sin y\ ,\quad \cos(x-y)=\cos x\cdot\cos y+\sin x\cdot\sin y\ ,\quad (a-b)\cdot(a+b)=a^2-b^2$$

$$(a\cdot b)^n=a^n\cdot b^n\ ,\quad \cos^2 x+\sin^2 x=1.$$

$$\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)=(\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta)\cdot(\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta)=$$

$$=(\cos\alpha\cdot\cos\beta)^2-(\sin\alpha\cdot\sin\beta)^2=\cos^2\alpha\cdot\cos^2\beta-\sin^2\alpha\cdot\sin^2\beta=$$

$$=\cos^2\alpha\cdot\cos^2\beta-\left(1-\cos^2\alpha\right)\cdot\left(1-\cos^2\beta\right)=\cos^2\alpha\cdot\cos^2\beta-\left(1-\cos^2\beta-\cos^2\alpha+\cos^2\alpha\cdot\cos^2\beta\right)=$$

$$=\cos^2\alpha\cdot\cos^2\beta-1+\cos^2\beta+\cos^2\alpha-\cos^2\alpha\cdot\cos^2\beta=\cos^2\alpha+\cos^2\beta-1=a-1.$$

Vježba 071

Ako je $\cos^2\alpha+\cos^2\beta=a+1$, koliko je $\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)$?

Rezultat: a.

Zadatak 072 (Anamarija, hotelijerska škola)

Nađite skup rješenja jednadžbe: $\sin x\cdot\cos 3x=\cos x\cdot\sin 3x$.

Rješenje 072

$$\sin x\cdot\cos 3x=\cos x\cdot\sin 3x \Rightarrow \sin x\cdot\cos 3x=\cos x\cdot\sin 3x\ /\cdot\frac{1}{\cos x\cdot\cos 3x} \Rightarrow \frac{\sin x\cdot\cos 3x}{\cos x\cdot\cos 3x}=\frac{\cos x\cdot\sin 3x}{\cos x\cdot\cos 3x} \Rightarrow$$

$$\Rightarrow \frac{\sin x}{\cos x}=\frac{\sin 3x}{\cos 3x} \Rightarrow tg\,x=tg\,3x \Rightarrow x=3\cdot x+k\cdot\pi\ ,\ k\in Z \Rightarrow x-3\cdot x=k\cdot\pi\ ,\ k\in Z \Rightarrow$$

$$\Rightarrow -2\cdot x=k\cdot\pi\ /:(-2)\ ,\ k\in Z \Rightarrow x=k\cdot\frac{\pi}{2}\ ,\ k\in Z.$$

Vježba 072

Nađite skup rješenja jednadžbe: $\sin x\cdot\cos 2x=\cos x\cdot\sin 2x$.

Rezultat: $x=k\cdot\pi\ ,\ k\in Z.$

Zadatak 073 (Nena, gimnazija)

Koliki je ukupan broj rješenja jednadžbe $\sin^2 x + \sin^2 2x = 1$ na intervalu $\langle 0, 2\cdot\pi \rangle$?

Rješenje 073

Ponovimo!

$$\sin 2x = 2\cdot\sin x\cdot\cos x \quad , \quad \cos^2 x + \sin^2 x = 1 \quad , \quad (a\cdot b)^n = a^n\cdot b^n$$

$$a\cdot b = 0 \;\Rightarrow\; a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

1.inačica

$$\sin^2 x + \sin^2 2x = 1 \Rightarrow \sin^2 x + \sin^2 2x = \cos^2 x + \sin^2 x \Rightarrow \sin^2 2x = \cos^2 x \Rightarrow \sin^2 2x - \cos^2 x = 0 \Rightarrow$$

$$\Rightarrow (2\cdot\sin x\cdot\cos x)^2 - \cos^2 x = 0 \Rightarrow 4\cdot\sin^2 x\cdot\cos^2 x - \cos^2 x = 0 \Rightarrow \cos^2 x\cdot\left(4\cdot\sin^2 x - 1\right) = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix}\cos^2 x = 0 \\ 4\cdot\sin^2 x - 1 = 0\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\cos x = 0 \\ 4\cdot\sin^2 x = 1 \;/:4\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\cos x = 0 \\ \sin^2 x = \frac{1}{4}\end{matrix}\right\}.$$

Svaku jednadžbu posebno rješavamo:

$$\cos x = 0 \Rightarrow x_1 = \frac{\pi}{2} + k\cdot 2\cdot\pi \;,\; x_2 = \frac{3\cdot\pi}{2} + k\cdot 2\cdot\pi \;,\; k\in Z,$$

$$\sin^2 x = \frac{1}{4} \;/\sqrt{\;} \Rightarrow \sin x = \pm\sqrt{\frac{1}{4}} \Rightarrow \left.\begin{matrix}\sin x = \frac{1}{2} \\ \sin x = -\frac{1}{2}\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x_3 = \frac{\pi}{6} + k\cdot 2\cdot\pi \;,\; x_4 = \frac{5\cdot\pi}{6} + k\cdot 2\cdot\pi \;,\; k\in Z \\ x_5 = \frac{7\cdot\pi}{6} + k\cdot 2\cdot\pi \;,\; x_6 = \frac{11\cdot\pi}{6} + k\cdot 2\cdot\pi \;,\; k\in Z\end{matrix}\right\}.$$

Budući da rezultati moraju biti na intervalu $\langle 0, 2\cdot\pi \rangle$, slijedi:

$$x_1 = \frac{\pi}{2} \quad , \quad x_2 = \frac{3\cdot\pi}{2} \quad , \quad x_3 = \frac{\pi}{6} \quad , \quad x_4 = \frac{5\cdot\pi}{6} \quad , \quad x_5 = \frac{7\cdot\pi}{6} \quad , \quad x_2 = \frac{11\cdot\pi}{6}.$$

Ukupno ima 6 rješenja.

2.inačica

$$\sin^2 x + \sin^2 2x = 1 \Rightarrow \sin^2 x + (2\cdot\sin x\cdot\cos x)^2 = 1 \Rightarrow \sin^2 x + 4\cdot\sin^2 x\cdot\cos^2 x = 1 \Rightarrow$$

$$\Rightarrow \sin^2 x + 4\cdot\sin^2 x\cdot\left(1 - \sin^2 x\right) = 1 \Rightarrow \begin{bmatrix}\text{supstitucija} \\ t = \sin^2 x\end{bmatrix} \Rightarrow t + 4\cdot t\cdot(1 - t) = 1 \Rightarrow t + 4\cdot t - 4\cdot t^2 = 1 \Rightarrow$$

$$\Rightarrow -4\cdot t^2 + 5\cdot t - 1 = 0 \;/\cdot(-1) \Rightarrow 4\cdot t^2 - 5\cdot t + 1 = 0 \Rightarrow t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{5 \pm \sqrt{25 - 4\cdot 4\cdot 1}}{2\cdot 4} \Rightarrow t_{1,2} = \frac{5 \pm \sqrt{9}}{8} \Rightarrow t_{1,2} = \frac{5 \pm 3}{8} \Rightarrow \left.\begin{matrix}t_1 = \frac{5+3}{8} = 1 \\ t_2 = \frac{5-2}{8} = \frac{1}{4}\end{matrix}\right\}.$$

Tražimo rješenja zadane jednadžbe:

$$\left.\begin{matrix}\sin^2 x = 1 \\ \sin^2 x = \frac{1}{4}\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\sin^2 x = 1 \;/\sqrt{\;} \\ \sin^2 x = \frac{1}{4} \;/\sqrt{\;}\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\sin x = \pm 1 \\ \sin x = \pm\frac{1}{2}\end{matrix}\right\}.$$

Svaku jednadžbu posebno rješavamo:

- $\sin x = 1 \Rightarrow x_1 = \frac{\pi}{2} + k\cdot 2\cdot\pi \;,\; k\in Z,$
- $\sin x = -1 \Rightarrow x_2 = \frac{3\cdot\pi}{2} + k\cdot 2\cdot\pi \;,\; k\in Z,$

- $\sin x = \frac{1}{2} \Rightarrow x_3 = \frac{\pi}{6} + k \cdot 2 \cdot \pi \quad , \quad x_4 = \frac{5 \cdot \pi}{6} + k \cdot 2 \cdot \pi \ , \ k \in Z,$
- $\sin x = -\frac{1}{2} \Rightarrow x_5 = \frac{7 \cdot \pi}{6} + k \cdot 2 \cdot \pi \quad , \quad x_6 = \frac{11 \cdot \pi}{6} + k \cdot 2 \cdot \pi \ , \ k \in Z.$

Budući da rezultati moraju biti na intervalu $\langle 0, 2 \cdot \pi \rangle$, slijedi:

$$x_1 = \frac{\pi}{2} \quad , \quad x_2 = \frac{3 \cdot \pi}{2} \quad , \quad x_3 = \frac{\pi}{6} \quad , \quad x_4 = \frac{5 \cdot \pi}{6} \quad , \quad x_5 = \frac{7 \cdot \pi}{6} \quad , \quad x_2 = \frac{11 \cdot \pi}{6}.$$

Ukupno ima 6 rješenja.

Vježba 073

Koliki je ukupan broj rješenja jednadžbe $\sin^2 x + \sin^2 2x = 1$ na intervalu $\langle 0, \pi \rangle$?

Rezultat: 3.

Zadatak 074 (Mira, Vedrana, gimnazija)

Odredi broj korijena (rješenja) jednadžbe $\sin \left| x + \frac{\pi}{2} \right| = \frac{1}{2}$ u intervalu $\langle -\pi, \pi \rangle$.

Rješenje 074

Ponovimo!

$$\sin x = a \ , \ |a| \le 1 \Rightarrow x_1 = \alpha + k \cdot 2 \cdot \pi \ , \ x_2 = \pi - \alpha + k \cdot 2 \cdot \pi \ , \ k \in Z$$

$$|x| = x \text{ za } x \ge 0 \quad , \quad |x| = -x \text{ za } x < 0.$$

Uvođenjem supstitucije

$$t = \left| x + \frac{\pi}{2} \right|$$

najprije riješimo elementarnu trigonometrijsku jednadžbu:

$$\sin t = \frac{1}{2} \Rightarrow \left. \begin{array}{l} t_1 = \frac{\pi}{6} + k \cdot 2 \cdot \pi \ , \ k \in Z \\ t_2 = \frac{5 \cdot \pi}{6} + k \cdot 2 \cdot \pi \ , \ k \in Z \end{array} \right\}.$$

Sada rješavamo jednadžbe koje sadrže apsolutne vrijednosti:

$$\left| x + \frac{\pi}{2} \right| = \frac{\pi}{6} + k \cdot 2 \cdot \pi \qquad \text{i} \qquad \left| x + \frac{\pi}{2} \right| = \frac{5 \cdot \pi}{6} + k \cdot 2 \cdot \pi.$$

● $\left| x + \frac{\pi}{2} \right| = \frac{\pi}{6} + k \cdot 2 \cdot \pi$

Razlikujemo dva slučaja

- $x + \frac{\pi}{2} \ge 0 \Rightarrow x \ge -\frac{\pi}{2}$

Tada je:

$$\left| x + \frac{\pi}{2} \right| = x + \frac{\pi}{2} \Rightarrow x + \frac{\pi}{2} = \frac{\pi}{6} + k \cdot 2 \cdot \pi \Rightarrow x_1 = \frac{\pi}{6} - \frac{\pi}{2} + k \cdot 2 \cdot \pi \Rightarrow x_1 = -\frac{\pi}{3} + k \cdot 2 \cdot \pi.$$

Na intervalu $\langle -\pi, \pi \rangle$ rješenje je $x_1 = -\frac{\pi}{3}$. Primijetimo da je zadovoljena nejednadžba $x \ge -\frac{\pi}{2}$.

- $x + \frac{\pi}{2} < 0 \Rightarrow x < -\frac{\pi}{2}$

Tada je:

$$\left| x + \frac{\pi}{2} \right| = -x - \frac{\pi}{2} \Rightarrow -x - \frac{\pi}{2} = \frac{\pi}{6} + k \cdot 2 \cdot \pi \Rightarrow -x = \frac{\pi}{6} + \frac{\pi}{2} + k \cdot 2 \cdot \pi \Rightarrow -x = \frac{2 \cdot \pi}{3} + k \cdot 2 \cdot \pi \ / \cdot (-1) \Rightarrow$$

$$\Rightarrow x_2 = -\frac{2\cdot\pi}{3} + k\cdot 2\cdot\pi.$$

Na intervalu $\langle -\pi, \pi \rangle$ rješenje je $x_2 = -\frac{2\cdot\pi}{3}$. Primijetimo da je zadovoljena nejednadžba $x < -\frac{\pi}{2}$.

- $\left| x + \frac{\pi}{2} \right| = \frac{5\cdot\pi}{6} + k\cdot 2\cdot\pi$

  Razlikujemo dva slučaja

  - $x + \frac{\pi}{2} \geq 0 \Rightarrow x \geq -\frac{\pi}{2}$

  Tada je:

  $$\left| x + \frac{\pi}{2} \right| = x + \frac{\pi}{2} \Rightarrow x + \frac{\pi}{2} = \frac{5\cdot\pi}{6} + k\cdot 2\cdot\pi \Rightarrow x_3 = \frac{5\cdot\pi}{6} - \frac{\pi}{2} + k\cdot 2\cdot\pi \Rightarrow x_3 = \frac{\pi}{3} + k\cdot 2\cdot\pi.$$

Na intervalu $\langle -\pi, \pi \rangle$ rješenje je $x_3 = \frac{\pi}{3}$. Primijetimo da je zadovoljena nejednadžba $x \geq -\frac{\pi}{2}$.

  - $x + \frac{\pi}{2} < 0 \Rightarrow x < -\frac{\pi}{2}$

  Tada je:

$$\left| x + \frac{\pi}{2} \right| = -x - \frac{\pi}{2} \Rightarrow -x - \frac{\pi}{2} = \frac{5\cdot\pi}{6} + k\cdot 2\cdot\pi \Rightarrow -x = \frac{5\cdot\pi}{6} + \frac{\pi}{2} + k\cdot 2\cdot\pi \Rightarrow -x = \frac{4\cdot\pi}{3} + k\cdot 2\cdot\pi \;/\cdot(-1) \Rightarrow$$

$$\Rightarrow x_4 = -\frac{4\cdot\pi}{3} + k\cdot 2\cdot\pi.$$

Na intervalu $\langle -\pi, \pi \rangle$ to nije rješenje zadane jednadžbe.

Zadana jednadžba ima 3 korijena u intervalu $\langle 0, \pi \rangle$.

Vježba 074

Odredi broj korijena (rješenja) jednadžbe $\sin\left| x + \frac{\pi}{2} \right| = \frac{1}{2}$ u intervalu $\langle 0, \pi \rangle$.

Rezultat: Zadana jednadžba ima 2 rješenja u intervalu $\langle 0, \pi \rangle$.

Zadatak 075 (Mira, gimnazija)

Koja je od sljedećih nejednakosti točna za svaki $x \in R$?

$A.\ \cos(2\cdot\sin x) > 0 \quad B.\ \sin(\sin x) > 0 \quad C.\ \sin(\cos x) > 0 \quad D.\ \cos(2\cdot\cos x) > 0 \quad E.\ \cos(\cos x) > 0$

Rješenje 075

Ponovimo!

$$-1 \leq \sin x \leq 1 \text{ za svaki } x \in R \quad , \quad -1 \leq \cos x \leq 1 \text{ za svaki } x \in R.$$

$$\text{Sinus je neparna funkcija} : \sin(-x) = -\sin x. \ \text{Kosinus je parna funkcija} : \cos(-x) = \cos x.$$

$A.\ \cos(2\cdot\sin x) > 0$

Za 2 · sin x vrijedi: $-2 \leq 2\cdot\sin x \leq 2$.

Budući da je

$$-1 \leq \cos x \leq 1,$$

zadana nejednakost nije točna.

$B.\ \sin(\sin x) > 0$

Za sin x vrijedi: $-1 \leq \sin x \leq 1$.

Budući da je sin x neparna funkcija, nije uvijek ispunjen uvjet:

$$\sin(\sin x) > 0.$$

Zadana nejednakost nije točna.

$C.\ \sin(\cos x) > 0$

Za cos x vrijedi: $-1 \le \cos x \le 1.$

Budući da je sin x neparna funkcija, nije uvijek ispunjen uvjet:

$$\sin(\cos x) > 0.$$

Zadana nejednakost nije točna.

$D.\ \cos(2 \cdot \cos x) > 0$

Za 2 · cos x vrijedi: $-2 \le 2 \cdot \cos x \le 2.$

Budući da je

$$-1 \le \cos x \le 1,$$

zadana nejednakost nije točna.

$E.\ \cos(\cos x) > 0$

Za cos x vrijedi: $-1 \le \cos x \le 1.$

Budući da je cos x parna funkcija, zadani uvjet je uvijek ispunjen:

$$\cos(\cos x) > 0.$$

Odgovor je pod E.

Vježba 075

Koja je od sljedećih nejednakosti točna za svaki $x \in R$?

$A.\ \cos(2 \cdot \sin x) > 0 \quad B.\ \sin(\sin x) < 0 \quad C.\ \sin(\cos x) < 0 \quad D.\ \cos(2 \cdot \cos x) > 0 \quad E.\ \cos(\cos x) > 0$

Rezultat: Odgovor je pod E.

Zadatak 076 (Tea, Mirela, Miroslava, Jure, hotelijerska škola)

Ako je $tg\, x = 3$, izračunajte $\dfrac{\sin^3 x - \cos^3 x}{(\sin x - \cos x)^3}$.

Rješenje 076

Ponovimo!

$$a^3 - b^3 = (a-b)\cdot\left(a^2 + a \cdot b + b^2\right) \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \quad , \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad , \quad \frac{a-b}{n} = \frac{a}{n} - \frac{b}{n}.$$

1.inačica

$$\frac{\sin^3 x - \cos^3 x}{(\sin x - \cos x)^3} = \frac{(\sin x - \cos x)\cdot\left(\sin^2 x + \sin x \cdot \cos x + \cos^2 x\right)}{(\sin x - \cos x)^3} = \frac{\overbrace{(\sin x - \cos x)}^{1}\cdot\left(\sin^2 x + \sin x \cdot \cos x + \cos^2 x\right)}{\underbrace{(\sin x - \cos x)^3}_{(\sin x - \cos x)^2}} =$$

$$= \frac{\sin^2 x + \sin x \cdot \cos x + \cos^2 x}{(\sin x - \cos x)^2} = \frac{\sin^2 x + \sin x \cdot \cos x + \cos^2 x}{\sin^2 x - 2 \cdot \sin x \cdot \cos x + \cos^2 x} = \frac{\sin^2 x + \sin x \cdot \cos x + \cos^2 x \ /{:}\cos^2 x}{\sin^2 x - 2 \cdot \sin x \cdot \cos x + \cos^2 x \ /{:}\cos^2 x} =$$

$$= \frac{\dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\sin x \cdot \cos x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x}}{\dfrac{\sin^2 x}{\cos^2 x} - 2 \cdot \dfrac{\sin x \cdot \cos x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x}} = \frac{tg^2 x + \dfrac{\sin x}{\cos x} + 1}{tg^2 x - 2 \cdot \dfrac{\sin x}{\cos x} + 1} = \frac{tg^2 x + tg\, x + 1}{tg^2 x - 2 \cdot tg\, x + 1} = \frac{3^2 + 3 + 1}{3^2 - 2 \cdot 3 + 1} = \frac{13}{4}.$$

2.inačica

$$\frac{\sin^3 x-\cos^3 x}{(\sin x-\cos x)^3}=\frac{\sin^3 x-\cos^3 x\ /:\cos^3 x}{(\sin x-\cos x)^3\ /:\cos^3 x}=\frac{\frac{\sin^3 x}{\cos^3 x}-\frac{\cos^3 x}{\cos^3 x}}{\frac{(\sin x-\cos x)^3}{\cos^3 x}}=\frac{tg^3 x-1}{\left(\frac{\sin x-\cos x}{\cos x}\right)^3}=\frac{tg^3 x-1}{\left(\frac{\sin x}{\cos x}-\frac{\cos x}{\cos x}\right)^3}=$$

$$=\frac{tg^3 x-1}{(tg\, x-1)^3}=\frac{3^3-1}{(3-1)^3}=\frac{26}{8}=\frac{13}{4}.$$

Vježba 076

Ako je $tg\, x=2$, izračunajte $\frac{\sin^3 x-\cos^3 x}{(\sin x-\cos x)^3}$.

Rezultat: 7.

Zadatak 077 (Tea, Mirela, Miroslava, Jure, hotelijerska škola)

Pojednostavnite: $\frac{1-\cos x-\sin^2 x}{1-\cos x}$.

Rješenje 077

Ponovimo!

$$\cos^2 x+\sin^2 x=1\quad,\quad a-b=-(b-a)\quad,\quad a^2-b^2=(a-b)\cdot(a+b).$$

1.inačica (Jurina ideja)

$$\frac{1-\cos x-\sin^2 x}{1-\cos x}=\frac{\cos^2 x+\sin^2 x-\cos x-\sin^2 x}{1-\cos x}=\frac{\cos^2 x-\cos x}{1-\cos x}=\frac{\cos x\cdot(\cos x-1)}{1-\cos x}=$$

$$=\frac{-\cos x\cdot(1-\cos x)}{1-\cos x}=-\cos x.$$

2.inačica

$$\frac{1-\cos x-\sin^2 x}{1-\cos x}=\frac{1-\cos x-\left(1-\cos^2 x\right)}{1-\cos x}=\frac{1-\cos x-1+\cos^2 x}{1-\cos x}=\frac{-\cos x+\cos^2 x}{1-\cos x}=\frac{-\cos x\cdot(1-\cos x)}{1-\cos x}=-\cos x.$$

3.inačica

$$\frac{1-\cos x-\sin^2 x}{1-\cos x}=\frac{1-\sin^2 x-\cos x}{1-\cos x}=\frac{\cos^2 x-\cos x}{1-\cos x}=\frac{\cos x\cdot(\cos x-1)}{1-\cos x}=\frac{-\cos x\cdot(1-\cos x)}{1-\cos x}=-\cos x.$$

4.inačica

$$\frac{1-\cos x-\sin^2 x}{1-\cos x}=\frac{1-\cos x-\left(1-\cos^2 x\right)}{1-\cos x}=\frac{1-\cos x-(1-\cos x)\cdot(1+\cos x)}{1-\cos x}=\frac{(1-\cos x)-(1-\cos x)\cdot(1+\cos x)}{1-\cos x}=$$

$$=\frac{(1-\cos x)\cdot(1-(1+\cos x))}{1-\cos x}=\frac{(1-\cos x)\cdot(1-1-\cos x)}{1-\cos x}=1-1-\cos x=-\cos x.$$

Vježba 077

Pojednostavnite: $\frac{1-\cos x-\sin^2 x}{\cos x-1}$.

Rezultat: cos x.

Zadatak 078 (Tea, Mirela, Miroslava, Jure, hotelijerska škola)

Ako je $tg\, x = 4$, izračunajte $\frac{1}{\cos^2 x}$.

Rješenje 078

Ponovimo!

$$\cos^2 x + \sin^2 x = 1 \quad , \quad \frac{a+b}{n} = \frac{a}{n} + \frac{b}{n}.$$

$$\frac{1}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = 1 + tg^2 x = 1 + 4^2 = 17.$$

Vježba 078

Ako je $tg\, x = 3$, izračunajte $\frac{1}{\cos^2 x}$.

Rezultat: 10.

Zadatak 079 (Tea, Mirela, Miroslava, Jure, hotelijerska škola)

Ako je $tg\, x = 4$, izračunajte $\cos^2 x$.

Rješenje 079

Ponovimo!

$$n = \frac{n}{1} \quad , \quad \cos^2 x + \sin^2 x = 1.$$

$$\cos^2 x = \frac{\cos^2 x}{1} = \frac{\cos^2 x}{\cos^2 x + \sin^2 x} = \frac{\cos^2 x \;/:\cos^2 x}{\cos^2 x + \sin^2 x \;/:\cos^2 x} = \frac{\frac{\cos^2 x}{\cos^2 x}}{\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}} = \frac{1}{1 + tg^2 x} = \frac{1}{1 + 4^2} = \frac{1}{17}.$$

Vježba 079

Ako je $tg\, x = 10$, izračunajte $\cos^2 x$.

Rezultat: $\frac{1}{101}$.

Zadatak 080 (Tea, Mirela, Miroslava, Jure, hotelijerska škola)

Pojednostavnite: $\sin x - \sin x \cdot \cos^2 x$.

Rješenje 080

Ponovimo!

$$a - a \cdot b = a \cdot (1 - b) \quad , \quad \cos^2 x + \sin^2 x = 1.$$

1.inačica (Teina ideja)

$$\sin x - \sin x \cdot \cos^2 x = \sin x \cdot \left(1 - \cos^2 x\right) = \sin x \cdot \underbrace{\left(1 - \cos^2 x\right)}_{\sin^2 x} = \sin x \cdot \sin^2 x = \sin^3 x.$$

2.inačica

$$\sin x - \sin x \cdot \cos^2 x = \sin x - \sin x \cdot \left(1 - \sin^2 x\right) = \sin x - \sin x + \sin^3 x = \sin^3 x.$$

Vježba 080

Pojednostavnite: $\cos x - \cos x \cdot \sin^2 x$.

Rezultat: $\cos^3 x$.