Zadatak 081 (Tea, Mirela, Miroslava, Jure, hotelijerska škola)

Iz sustava jednakosti eliminirajte t: $\begin{cases} x = a \cdot \cos t, \\ y = a \cdot \sin t. \end{cases}$

Rješenje 081

Ponovimo!

$$\cos^2 x + \sin^2 x = 1$$

$$\left.\begin{array}{l} x = a \cdot \cos t \\ y = a \cdot \sin t \end{array}\right\} \Rightarrow \left.\begin{array}{l} \cos t = \frac{x}{a} \\ \sin t = \frac{y}{a} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \cos t = \frac{x}{a} \,/\, ^2 \\ \sin t = \frac{y}{a} \,/\, ^2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \cos^2 t = \frac{x^2}{a^2} \\ \sin^2 t = \frac{y^2}{a^2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \cos^2 t + \sin^2 t = 1 \\ \frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \,/ \cdot a^2 \Rightarrow x^2 + y^2 = a^2.$$

Vježba 081

Iz sustava jednakosti eliminirajte t: $\begin{cases} x = 3 \cdot \cos t, \\ y = 3 \cdot \sin t. \end{cases}$

Rezultat: $x^2 + y^2 = 9.$

Zadatak 082 (Tea, Mirela, Miroslava, Jure, hotelijerska škola)

Pojednostavnite: $\frac{\cos^3 x - \sin^3 x}{(\cos x - \sin x) \cdot \cos^2 x} - \frac{1}{\cos^2 x}.$

Rješenje 082

Ponovimo!

$$a^3 - b^3 = (a-b) \cdot \left(a^2 + a \cdot b + b^2\right) \quad , \quad \cos^2 x + \sin^2 x = 1 \quad , \quad \frac{a+b}{n} = \frac{a}{n} + \frac{b}{n}.$$

1.inačica

$$\frac{\cos^3 x - \sin^3 x}{(\cos x - \sin x) \cdot \cos^2 x} - \frac{1}{\cos^2 x} = \frac{(\cos x - \sin x) \cdot \left(\cos^2 x + \cos x \cdot \sin x + \sin^2 x\right)}{(\cos x - \sin x) \cdot \cos^2 x} - \frac{1}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \cos x \cdot \sin x + \sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{1 + \cos x \cdot \sin x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{1}{\cos^2 x} + \frac{\cos x \cdot \sin x}{\cos^2 x} - \frac{1}{\cos^2 x} =$$

$$= \frac{\cos x \cdot \sin x}{\cos^2 x} = \frac{\sin x}{\cos x} = tg\, x.$$

2.inačica

$$\frac{\cos^3 x - \sin^3 x}{(\cos x - \sin x) \cdot \cos^2 x} - \frac{1}{\cos^2 x} = \frac{1}{\cos^2 x} \cdot \left[\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} - 1\right] =$$

$$= \frac{1}{\cos^2 x} \cdot \left[\frac{(\cos x - \sin x) \cdot \left(\cos^2 x + \cos x \cdot \sin x + \sin^2 x\right)}{\cos x - \sin x} - 1\right] =$$

$$= \frac{1}{\cos^2 x} \cdot \left[\cos^2 x + \cos x \cdot \sin x + \sin^2 x - 1\right] = \frac{1}{\cos^2 x} \cdot [1 + \cos x \cdot \sin x - 1] =$$

$$=\frac{1}{\cos^2 x}\cdot\cos x\cdot\sin x=\frac{\sin x}{\cos x}=tg\ x.$$

Vježba 082

Pojednostavnite: $\frac{\cos^2 x-\sin^2 x}{\cos x-\sin x}-\sin x.$

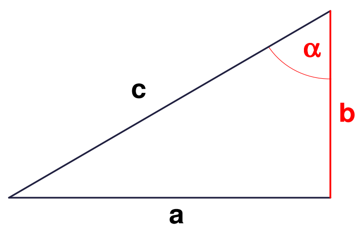
Rezultat: cos x.

Zadatak 083 (2A, hotelijerska škola)

U pravokutnom trokutu je b = 10 cm, a za kut α vrijedi $\sin\alpha=\frac{24}{25}, \cos\alpha=\frac{7}{25}, tg\,\alpha=\frac{24}{7}.$ Koliko iznosi duljina katete a?

Rješenje 083

Ponovimo!



$$\sin\alpha=\frac{a}{c},\ \cos\alpha=\frac{b}{c},\ tg\,\alpha=\frac{a}{b},\ tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}$$

Provjerimo da su za kut α točne dane jednakosti:

$$\left.\begin{array}{l}\sin\alpha=\frac{24}{25}\\ \cos\alpha=\frac{7}{25}\end{array}\right\}\Rightarrow tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}\Rightarrow tg\,\alpha=\frac{\frac{24}{25}}{\frac{7}{25}}\Rightarrow tg\,\alpha=\frac{24}{7}.$$

Budući da je b = 10 cm, slijedi:

$$\left.\begin{array}{l}tg\,\alpha=\frac{24}{7}\\ tg\,\alpha=\frac{a}{b}\end{array}\right\}\Rightarrow \frac{a}{b}=\frac{24}{7}\Rightarrow\frac{a}{10}=\frac{24}{7}\ /\cdot 10\Rightarrow a=\frac{240}{7}\ cm.$$

Vježba 083

U pravokutnom trokutu je b = 10 cm, a za kut α vrijedi $\sin\alpha=\frac{24}{25}, \cos\alpha=\frac{7}{25}, ctg\,\alpha=\frac{7}{24}.$ Koliko iznosi duljina katete a?

Rezultat: $a=\frac{240}{7}\ cm.$

Zadatak 084 (Anamarija, gimnazija)

Dokažite da vrijedi: $\frac{1-4\cdot\sin^4 x}{\cos 2x}-2\cdot\sin^2 x=1.$

Rješenje 084

Ponovimo!

Razlika kvadrata: $a^2-b^2=(a-b)\cdot(a+b).$

Kosinus dvostrukog kuta: $\cos 2x=\cos^2 x-\sin^2 x=1-2\cdot\sin^2 x.$

$$\frac{1-4\cdot\sin^4 x}{\cos 2x}-2\cdot\sin^2 x=\frac{\left(1-2\cdot\sin^2 x\right)\cdot\left(1+2\cdot\sin^2 x\right)}{\cos 2x}-2\cdot\sin^2 x=\frac{\cos 2x\cdot\left(1+2\cdot\sin^2 x\right)}{\cos 2x}-2\cdot\sin^2 x=$$

$$=1+2\cdot\sin^2 x-2\cdot\sin^2 x=1.$$

Vježba 084

Dokažite da vrijedi: $\frac{1-\sin^4 x}{\cos 2x} - 2 \cdot \sin^2 x - 1 = 0.$

Rezultat: Dokaz analogan.

Zadatak 085 (Anamarija, gimnazija)

Dokažite da vrijedi: $\left(1-\sin^2 \alpha\right) \cdot \left(1+tg^2 \alpha\right) = 1.$

Rješenje 085

Ponovimo!

$$\cos^2 x + \sin^2 x = 1 \quad , \quad tg\, x = \frac{\sin x}{\cos x}.$$

Računamo:

$$\left(1-\sin^2 \alpha\right) \cdot \left(1+tg^2 \alpha\right) = \cos^2 \alpha \cdot \left(1 + \frac{\sin^2 \alpha}{\cos^2 \alpha}\right) = \cos^2 \alpha + \cos^2 \alpha \cdot \frac{\sin^2 \alpha}{\cos^2 \alpha} = \cos^2 \alpha + \sin^2 \alpha = 1.$$

Vježba 085

Dokažite da vrijedi: $tg\, x \cdot \cos x + \sin x = 2 \cdot \sin x.$

Rezultat: Identitet je točan.

Zadatak 086 (Anamarija, gimnazija)

Dokažite da vrijedi: $\cos^2 \alpha + 2 \cdot \sin^2 \alpha + \sin^2 \alpha \cdot tg^2 \alpha = \frac{1}{\cos^2 \alpha}.$

Rješenje 086

Ponovimo!

$$\cos^2 x + \sin^2 x = 1 \quad , \quad tg\, x = \frac{\sin x}{\cos x}.$$

Računamo:

$$\cos^2 \alpha + 2 \cdot \sin^2 \alpha + \sin^2 \alpha \cdot tg^2 \alpha = \cos^2 \alpha + \sin^2 \alpha + \sin^2 \alpha + \sin^2 \alpha \cdot \frac{\sin^2 \alpha}{\cos^2 \alpha} =$$

$$= 1 + \sin^2 \alpha + \frac{\sin^4 \alpha}{\cos^2 \alpha} = \frac{\cos^2 \alpha + \cos^2 \alpha \cdot \sin^2 \alpha + \sin^4 \alpha}{\cos^2 \alpha} = \frac{\cos^2 \alpha + \sin^2 \alpha \cdot \left(\cos^2 \alpha + \sin^2 \alpha\right)}{\cos^2 \alpha} =$$

$$= \frac{\cos^2 \alpha + \sin^2 \alpha \cdot 1}{\cos^2 \alpha} = \frac{\cos^2 \alpha + \sin^2 \alpha}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha}.$$

Vježba 086

Dokažite da vrijedi: $\frac{1}{\cos \alpha} - \frac{\sin^2 \alpha}{\cos \alpha} = \cos \alpha.$

Rezultat: Identitet je točan.

Zadatak 087 (Anamarija, gimnazija)

Koliko rješenja ima jednadžba sin 4x = sin 2x u intervalu $\left[0, 2 \cdot \pi\right\rangle$?

Rješenje 087

$$\sin 4x = \sin 2x \Rightarrow \left.\begin{matrix} 4x = 2x + k \cdot 2\pi, \ k \in Z \\ 4x = \pi - 2x + k \cdot 2\pi, \ k \in Z \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 4x - 2x = k \cdot 2\pi \\ 4x + 2x = \pi + k \cdot 2\pi \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2x = k \cdot 2\pi \\ 6x = \pi + k \cdot 2\pi \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 2x = k\cdot 2\pi \;/:2 \\ 6x = \pi + k\cdot 2\pi \;/:6 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = k\cdot\pi \\ x_2 = \frac{\pi}{6} + k\cdot\frac{\pi}{3} \end{array}\right\}.$$

Računamo broj rješenja za x_1 na intervalu $\left[0, 2\cdot\pi\right\rangle$:

$$x_1 \in \left[0, 2\cdot\pi\right\rangle \Rightarrow 0 \le x_1 < 2\cdot\pi \Rightarrow 0 \le k\cdot\pi < 2\cdot\pi \Rightarrow 0 \le k\cdot\pi < 2\cdot\pi \;/:\pi \Rightarrow 0 \le k < 2 \Rightarrow k = 0, 1.$$

Dva su rješenja.

Računamo broj rješenja za x_2 na intervalu $\left[0, 2\cdot\pi\right\rangle$:

$$x_2 \in \left[0, 2\cdot\pi\right\rangle \Rightarrow 0 \le x_2 < 2\cdot\pi \Rightarrow 0 \le \frac{\pi}{6} + k\cdot\frac{\pi}{3} < 2\cdot\pi \Rightarrow 0 \le \frac{\pi}{6} + k\cdot\frac{\pi}{3} < 2\cdot\pi \;/\cdot\frac{6}{\pi} \Rightarrow$$

$$\Rightarrow 0 \le 1 + 2\cdot k < 12 \Rightarrow 0 \le 1 + 2\cdot k < 12 \;/-1 \Rightarrow 0 - 1 \le 1 + 2\cdot k - 1 < 12 - 1 \Rightarrow -1 \le 2\cdot k < 11 \Rightarrow$$

$$\Rightarrow -1 \le 2\cdot k < 11 \;/\cdot\frac{1}{2} \Rightarrow -\frac{1}{2} \le k < \frac{11}{2} \Rightarrow k = 0, 1, 2, 3, 4, 5.$$

Šest je rješenja. Ukupno ima 8 rješenja.

Vježba 087

Koliko rješenja ima jednadžba sin 4x = sin 2x u intervalu $\left[0, \pi\right\rangle$?

Rezultat: Ukupno je 4 rješenja.

Zadatak 088 (Anamarija, gimnazija)

Riješite jednadžbu: $\left|\cos x\right| = \cos x + 1$, $0 < x < 2\pi$.

Rješenje 088

Na zadanom intervalu $\left\langle 0, 2\pi\right\rangle$ jednadžba cos x = 0 ima rješenja: $x_1 = \frac{\pi}{2}$ i $x_2 = \frac{3\cdot\pi}{2}$.

Polaznu jednadžbu promatramo u podintervalima: $\left\langle 0, \frac{\pi}{2}\right\rangle$, $\left\langle \frac{\pi}{2}, \frac{3\cdot\pi}{2}\right\rangle$ i $\left\langle \frac{3\cdot\pi}{2}, 2\cdot\pi\right\rangle$.

- Za $x \in \left\langle 0, \frac{\pi}{2}\right\rangle$ je $\cos x > 0$ pa je $\left.\begin{array}{l} \left|\cos x\right| = \cos x + 1 \\ \cos x > 0 \end{array}\right\} \Rightarrow \cos x = \cos x + 1 \Rightarrow 0 = 1$ (nema smisla)

Jednadžba u promatranom intervalu nema rješenja.

- Za $x \in \left\langle \frac{\pi}{2}, \frac{3\cdot\pi}{2}\right\rangle$ je $\cos x < 0$ pa je $\left.\begin{array}{l} \left|\cos x\right| = \cos x + 1 \\ \cos x < 0 \end{array}\right\} \Rightarrow -\cos x = \cos x + 1 \Rightarrow -\cos x - \cos x = 1 \Rightarrow$

$$\Rightarrow -2\cdot\cos x = 1 \;/:(-2) \Rightarrow \cos x = -\frac{1}{2} \Rightarrow \left.\begin{array}{l} x_1 = \frac{2\cdot\pi}{3} \in \left\langle \frac{\pi}{2}, \frac{3\cdot\pi}{2}\right\rangle \\ x_2 = \frac{4\cdot\pi}{3} \in \left\langle \frac{\pi}{2}, \frac{3\cdot\pi}{2}\right\rangle \end{array}\right\}.$$

- Za $x \in \left\langle \frac{3\cdot\pi}{2}, 2\cdot\pi\right\rangle$ je $\cos x > 0$ pa je $\left.\begin{array}{l} \left|\cos x\right| = \cos x + 1 \\ \cos x > 0 \end{array}\right\} \Rightarrow \cos x = \cos x + 1 \Rightarrow 0 = 1$ (nema smisla)

Jednadžba u promatranom intervalu nema rješenja.

Dakle, rješenja zadane jednadžbe su: $x_1 = \frac{2\cdot\pi}{3}$, $x_2 = \frac{4\cdot\pi}{3}$.

Vježba 088

Nađite zbroj rješenja jednadžbe: $\left|\cos x\right| = \cos x + 1$, $0 < x < 2\pi$.

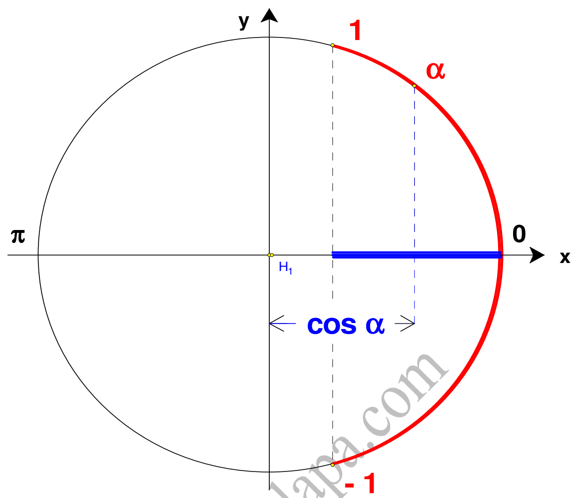
Rezultat: $2\cdot\pi$.

Zadatak 089 (Max, gimnazija)

Ako je y_M maksimalna, a y_m minimalna vrijednost funkcije f(x) = cos (sin x), nađite razliku $y_M - y_m$.

Rješenje 089

Kodomena (antidomena, područje vrijednosti funkcije, izlazni skup) funkcije sin x je segment $[-1, 1]$, tj. funkcija sin x poprima vrijednosti iz segmenta $[-1, 1]$. To znači da je za funkciju cos α = cos (sin x) argument α iz segmenta $[-1, 1]$. Taj je segment označen debljom crvenom crtom na trigonometrijskoj kružnici. Na kosinusovoj osi (apscisi) je označen segment u kojem leži y = cos (sin x). Iz slike vidi se:



$$\left.\begin{array}{l} y_M = \cos 0 \\ y_m = \cos 1 \end{array}\right\} \Rightarrow y_M - y_m = \cos 0 - \cos 1 \Rightarrow y_M - y_m = 1 - 0.5403 \Rightarrow y_M - y_m = 0.4597.$$

Vježba 089

Ako je y_M maksimalna, a y_m minimalna vrijednost funkcije f(x) = cos (sin x), nađite zbroj $y_M + y_m$.

Rezultat: 1.5403.

Zadatak 090 (Maturant, gimnazija)

Koliko nultočaka u intervalu $\left[-\frac{\pi}{2}, \pi\right]$ ima funkcija $y = 2 \cdot \sin(3 \cdot x + 7 \cdot \pi)$?

Rješenje 090

Tražimo nultočke:

$$2 \cdot \sin(3 \cdot x + 7 \cdot \pi) = 0 \Rightarrow 2 \cdot \sin(3 \cdot x + 7 \cdot \pi) = 0 \;/:2 \Rightarrow \sin(3 \cdot x + 7 \cdot \pi) = 0 \Rightarrow 3 \cdot x + 7 \cdot \pi = k \cdot \pi, k \in Z \Rightarrow$$

$$\Rightarrow 3 \cdot x = k \cdot \pi - 7 \cdot \pi \;/:3 \Rightarrow x = \frac{k}{3} \cdot \pi - \frac{7}{3} \cdot \pi.$$

Budući da se nultočke moraju nalaziti u segmentu $\left[-\frac{\pi}{2}, \pi\right]$, slijedi:

$$-\frac{\pi}{2} \le \frac{k}{3} \cdot \pi - \frac{7}{3} \cdot \pi \le \pi \Rightarrow -\frac{\pi}{2} \le \frac{k}{3} \cdot \pi - \frac{7}{3} \cdot \pi \le \pi \;/\cdot\frac{6}{\pi} \Rightarrow -3 \le 2 \cdot k - 14 \le 6 \Rightarrow -3 \le 2 \cdot k - 14 \le 6 \;/+14 \Rightarrow$$

$$\Rightarrow -3 + 14 \le 2 \cdot k - 14 + 14 \le 6 + 14 \Rightarrow 11 \le 2 \cdot k \le 20 \Rightarrow 11 \le 2 \cdot k \le 20 \;/:2 \Rightarrow \frac{11}{2} \le k \le 10 \Rightarrow k \in \{6, 7, 8, 9, 10\}.$$

Funkcija ima 5 nultočaka na zadanom intervalu.

Vježba 090

Koliko nultočaka u intervalu $[0, \pi]$ ima funkcija $y = 2 \cdot \sin(3 \cdot x + 7 \cdot \pi)$?

Rezultat: Funkcija ima 4 nultočke na zadanom intervalu.

Zadatak 091 (Maturant, gimnazija)

Ako je $a = \cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5}$, nađite a^{-1}.

Rješenje 091

Ponovimo!

$$\sin 2x = 2 \cdot \sin x \cdot \cos x \quad , \quad \sin(\pi - \alpha) = \sin \alpha.$$

$$a = \cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5} \Rightarrow a^{-1} = \frac{1}{\cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5}} \Rightarrow a^{-1} = \frac{1}{\cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5}} \cdot \frac{2 \cdot \sin\frac{\pi}{5}}{2 \cdot \sin\frac{\pi}{5}} \Rightarrow$$

$$\Rightarrow a^{-1} = \frac{2 \cdot \sin\frac{\pi}{5}}{2 \cdot \sin\frac{\pi}{5} \cdot \cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5}} \Rightarrow a^{-1} = \frac{2 \cdot \sin\frac{\pi}{5}}{\sin\frac{2\pi}{5} \cdot \cos\frac{2\pi}{5}} \Rightarrow a^{-1} = \frac{2 \cdot \sin\frac{\pi}{5}}{\sin\frac{2\pi}{5} \cdot \cos\frac{2\pi}{5}} \cdot \frac{2}{2} \Rightarrow$$

$$\Rightarrow a^{-1} = \frac{4 \cdot \sin\frac{\pi}{5}}{2 \cdot \sin\frac{2\pi}{5} \cdot \cos\frac{2\pi}{5}} \Rightarrow a^{-1} = \frac{4 \cdot \sin\frac{\pi}{5}}{\sin\frac{4\pi}{5}} \Rightarrow a^{-1} = \frac{4 \cdot \sin\frac{\pi}{5}}{\sin\left(\pi - \frac{\pi}{5}\right)} \Rightarrow a^{-1} = \frac{4 \cdot \sin\frac{\pi}{5}}{\sin\frac{\pi}{5}} \Rightarrow a^{-1} = 4.$$

Vježba 091

Ako je $a = \cos\frac{\pi}{5} \cdot \cos\frac{2\pi}{5}$, nađite $\frac{1}{4} \cdot a^{-1}$.

Rezultat: 1.

Zadatak 092 (Ana Marija, gimnazija)

Riješite trigonometrijsku jednadžbu sin x + sin 4x = 0.

Rješenje 092

Ponovimo!

Rješenje osnovne jednadžbe:

$$\sin x = a \;,\; |a| \le 1 \Rightarrow x_1 = \alpha + k \cdot 2 \cdot \pi \;,\; x_2 = \pi - \alpha + k \cdot 2 \cdot \pi \;,\; k \in Z \quad , \quad \sin(-x) = -\sin x$$

$$\sin(\pi + x) = -\sin x \quad , \quad \cos(-x) = \cos x.$$

Transformaciona formula:

$$\sin x + \sin y = 2 \cdot \sin\frac{x+y}{2} \cdot \cos\frac{x-y}{2}.$$

1.inačica

$$\sin x + \sin 4x = 0 \Rightarrow \sin 4x = -\sin x \Rightarrow \left[\sin(\pi + x) = -\sin x\right] \Rightarrow \sin 4x = \sin(\pi + x).$$

$\sin 4x = \sin(\pi + x)$	$\sin 4x = \sin(\pi + x)$
$4 \cdot x = \pi + x + k \cdot 2 \cdot \pi$	$4 \cdot x = \pi - (\pi + x) + k \cdot 2 \cdot \pi$
$4 \cdot x - x = \pi + k \cdot 2 \cdot \pi$	$4 \cdot x = \pi - \pi - x + k \cdot 2 \cdot \pi$
$3 \cdot x = (2 \cdot k + 1) \cdot \pi \;/:3$	$4 \cdot x + x = k \cdot 2 \cdot \pi$
$x_1 = (2 \cdot k + 1) \cdot \frac{\pi}{3} \;,\; k \in Z.$	$5 \cdot x = k \cdot 2 \cdot \pi \;/:5$
	$x_2 = 2 \cdot k \cdot \frac{\pi}{5} \;,\; k \in Z.$

2.inačica

$$\sin x+\sin 4x=0 \Rightarrow \sin 4x=-\sin x \Rightarrow \left[\sin(-x)=-\sin x\right] \Rightarrow \sin 4x=\sin(-x).$$

$\sin 4x=\sin(-x)$	$\sin 4x=\sin(-x)$
$4\cdot x=-x+k\cdot 2\cdot\pi$ $4\cdot x+x=k\cdot 2\cdot\pi$ $5\cdot x=k\cdot 2\cdot\pi \;/:5$ $x_1=2\cdot k\cdot\frac{\pi}{5}\;,\;k\in Z.$	$4\cdot x=\pi-(-x)+k\cdot 2\cdot\pi$ $4\cdot x=\pi+x+k\cdot 2\cdot\pi$ $4\cdot x-x=\pi+k\cdot 2\cdot\pi$ $3\cdot x=(2\cdot k+1)\cdot\pi \;/:3$ $x_2=(2\cdot k+1)\cdot\frac{\pi}{3}\;,\;k\in Z.$

3.inačica

$$\sin x+\sin 4x=0 \Rightarrow \left[\sin x+\sin y=2\cdot\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2}\right] \Rightarrow 2\cdot\sin\frac{x+4x}{2}\cdot\cos\frac{x-4x}{2}=0 \Rightarrow$$

$$\Rightarrow 2\cdot\sin\frac{5x}{2}\cdot\cos\frac{-3x}{2}=0 \Rightarrow 2\cdot\sin\frac{5x}{2}\cdot\cos\frac{3x}{2}=0 \;/:2 \Rightarrow \sin\frac{5x}{2}\cdot\cos\frac{3x}{2}=0.$$

$\sin\frac{5x}{2}=0$	$\cos\frac{3x}{2}=0$
$\frac{5\cdot x}{2}=k\cdot\pi \;/\cdot\frac{2}{5}$ $x_1=2\cdot k\cdot\frac{\pi}{5}\;,\;k\in Z.$	$\frac{3\cdot x}{2}=\frac{\pi}{2}+k\cdot\pi$ $\frac{3\cdot x}{2}=\left(k+\frac{1}{2}\right)\cdot\pi \;/\cdot\frac{2}{3}$ $x_2=(2\cdot k+1)\cdot\frac{\pi}{3}\;,\;k\in Z.$

Vježba 092

Riješite trigonometrijsku jednadžbu sin 2x = cos x.

Rezultat: $x_1=\frac{\pi}{6}+k\cdot 2\cdot\pi \;,\; x_2=\frac{5\cdot\pi}{6}+k\cdot 2\cdot\pi \;,\; x_3=\frac{\pi}{2}+k\cdot\pi \;,\; k\in Z.$

Zadatak 093 (Ana Marija, gimnazija)

Riješite trigonometrijsku jednadžbu $\sqrt{2}\cdot\cos\left(2\cdot x-\frac{\pi}{5}\right)-1=0.$

Rješenje 093

Ponovimo!

Rješenje osnovne jednadžbe:

$$\cos x=a\;,\;|a|\le 1 \Rightarrow x_{1,2}=\pm\alpha+k\cdot 2\cdot\pi\;,\;k\in Z.$$

$$\sqrt{2}\cdot\cos\left(2\cdot x-\frac{\pi}{5}\right)-1=0 \Rightarrow \sqrt{2}\cdot\cos\left(2\cdot x-\frac{\pi}{5}\right)=1 \;/:\sqrt{2} \Rightarrow \cos\left(2\cdot x-\frac{\pi}{5}\right)=\frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \cos\left(2\cdot x-\frac{\pi}{5}\right)=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \cos\left(2\cdot x-\frac{\pi}{5}\right)=\frac{\sqrt{2}}{2} \Rightarrow \left[\begin{array}{l}\text{supstitucija}\\ t=2\cdot x-\frac{\pi}{5}\end{array}\right] \Rightarrow \cos t=\frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\pm\frac{\pi}{4}+k\cdot 2\cdot\pi\;,\;k\in Z.$$

Sada se vraćamo na supstituciju:

$t=2\cdot x-\frac{\pi}{5}$, $t_1=\frac{\pi}{4}+k\cdot 2\cdot\pi$	$t=2\cdot x-\frac{\pi}{5}$, $t_2=-\frac{\pi}{4}+k\cdot 2\cdot\pi$
$2\cdot x-\frac{\pi}{5}=\frac{\pi}{4}+k\cdot 2\cdot\pi$ $2\cdot x=\frac{\pi}{4}+\frac{\pi}{5}+k\cdot 2\cdot\pi$ $2\cdot x=\frac{5\cdot\pi+4\cdot\pi}{20}+k\cdot 2\cdot\pi$ $2\cdot x=\frac{9\cdot\pi}{20}+k\cdot 2\cdot\pi\ /:2$ $x_1=\frac{9\cdot\pi}{40}+k\cdot\pi\ ,\ k\in Z.$	$2\cdot x-\frac{\pi}{5}=-\frac{\pi}{4}+k\cdot 2\cdot\pi$ $2\cdot x=-\frac{\pi}{4}+\frac{\pi}{5}+k\cdot 2\cdot\pi$ $2\cdot x=\frac{-5\cdot\pi+4\cdot\pi}{20}+k\cdot 2\cdot\pi$ $2\cdot x=-\frac{\pi}{20}+k\cdot 2\cdot\pi\ /:2$ $x_2=-\frac{\pi}{40}+k\cdot\pi\ ,\ k\in Z.$

Vježba 093

Riješite trigonometrijsku jednadžbu $2\cdot\sin\left(x-\frac{\pi}{3}\right)=\sqrt{3}.$

Rezultat: $x_1=\frac{2\cdot\pi}{3}+k\cdot 2\cdot\pi\quad ,\quad x_2=(2\cdot k+1)\cdot\pi\ ,\ k\in Z.$

Zadatak 094 (Los-Habalos, gimnazija)

Skup svih rješenja nejednadžbe $\left(\cos x+\frac{1}{2}\right)\cdot(3+2\cdot\sin x)\le 0$ iz intervala $[\,0,\ 2\cdot\pi\,]$ je:

A) $\left[\frac{\pi}{2},\frac{2\cdot\pi}{3}\right]$ B) $\left[\frac{2\cdot\pi}{3},\frac{4\cdot\pi}{3}\right]$ C) $\left[\frac{4\cdot\pi}{3},\frac{5\cdot\pi}{3}\right]$ D) $\left[\frac{\pi}{3},\frac{4\cdot\pi}{3}\right]$ E) $\left[\frac{\pi}{6},\frac{5\cdot\pi}{6}\right]$

Rješenje 094

Ponovimo!

$$a\cdot b\le 0\ \Rightarrow\ \left.\begin{matrix}a\ge 0\\ b\le 0\end{matrix}\right\}\quad \text{i}\quad \left.\begin{matrix}a\le 0\\ b\ge 0\end{matrix}\right\}.$$

$$\left(\cos x+\frac{1}{2}\right)\cdot(3+2\cdot\sin x)\le 0\ \Rightarrow\ \left.\begin{matrix}\text{1. slučaj}\\ \cos x+\frac{1}{2}\ge 0\\ 3+2\cdot\sin x\le 0\end{matrix}\right\}\quad \text{i}\quad \left.\begin{matrix}\text{2. slučaj}\\ \cos x+\frac{1}{2}\le 0\\ 3+2\cdot\sin x\ge 0\end{matrix}\right\}.$$

1.slučaj

$$\left.\begin{matrix}\cos x+\frac{1}{2}\ge 0\\ 3+2\cdot\sin x\le 0\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix}\cos x\ge -\frac{1}{2}\\ 2\cdot\sin x\le -3\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix}\cos x\ge -\frac{1}{2}\\ 2\cdot\sin x\le -3\ /:2\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix}\cos x\ge -\frac{1}{2}\\ \sin x\le -\frac{3}{2}\end{matrix}\right\}.$$

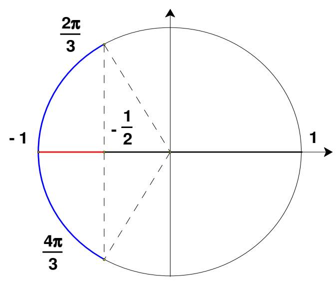
Uočimo drugu nejednadžbu sustava:

$$\sin x\le -\frac{3}{2}.$$

Budući da za funkciju sinus vrijedi

$$|\sin x|\le 1\ \Rightarrow\ -1\le\sin x\le 1,$$

Zaključujemo da u ovom slučaju nema rješenja, tj. rješenje je prazan skup, $\varnothing$.



2.slučaj

$$\left.\begin{array}{l}\cos x+\frac{1}{2}\le 0\\ 3+2\cdot\sin x\ge 0\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos x\le-\frac{1}{2}\\ 2\cdot\sin x\ge-3\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos x\le-\frac{1}{2}\\ 2\cdot\sin x\ge-3\ /:2\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos x\le-\frac{1}{2}\\ \sin x\ge-\frac{3}{2}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}x\in\left[\frac{2\cdot\pi}{3},\frac{4\cdot\pi}{3}\right]\\ x\in R\ \text{jer je}\ \sin x\ge-1\ \text{za svaki realan broj}\end{array}\right\}.$$

Konačno rješenje je: $x\in\left[\frac{2\cdot\pi}{3},\frac{4\cdot\pi}{3}\right]$. Odgovor je pod B.

Vježba 094

Skup svih rješenja nejednadžbe $\left(-\cos x-\frac{1}{2}\right)\cdot(3+2\cdot\sin x)\ge 0$ iz intervala $[0, 2\cdot\pi]$ je:

$A)\ \left[\frac{\pi}{2},\frac{2\cdot\pi}{3}\right]$ $B)\ \left[\frac{2\cdot\pi}{3},\frac{4\cdot\pi}{3}\right]$ $C)\ \left[\frac{4\cdot\pi}{3},\frac{5\cdot\pi}{3}\right]$ $D)\ \left[\frac{\pi}{3},\frac{4\cdot\pi}{3}\right]$ $E)\ \left[\frac{\pi}{6},\frac{5\cdot\pi}{6}\right]$

Rezultat: Odgovor je pod B.

Zadatak 095 (Los-Habalos, gimnazija)

U trokutu je a = 1, $c=\frac{\sqrt{6}}{2}$, γ = 120°. Nađite kut β.

Rješenje 095

Ponovimo!

Sinusov poučak: $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$, $\sin\left(180^0-\alpha\right)=\sin\alpha$, $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$.

Uporabom sinusovog poučka dobije se kut α:

$$\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}\Rightarrow c\cdot\sin\alpha=a\cdot\sin\gamma\Rightarrow\sin\alpha=\frac{a\cdot\sin\gamma}{c}\Rightarrow\sin\alpha=\frac{1\cdot\sin 120^0}{\frac{\sqrt{6}}{2}}\Rightarrow\sin\alpha=\frac{\sin\left(180^0-60^0\right)}{\frac{\sqrt{6}}{2}}\Rightarrow$$

$$\Rightarrow\sin\alpha=\frac{\sin 60^0}{\frac{\sqrt{6}}{2}}\Rightarrow\sin\alpha=\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{6}}{2}}\Rightarrow\sin\alpha=\frac{\sqrt{3}}{\sqrt{6}}\Rightarrow\sin\alpha=\sqrt{\frac{3}{6}}\Rightarrow\sin\alpha=\sqrt{\frac{1}{2}}\Rightarrow\sin\alpha=\frac{1}{\sqrt{2}}\Rightarrow$$

$$\Rightarrow\sin\alpha=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\Rightarrow\sin\alpha=\frac{\sqrt{2}}{2}\Rightarrow\alpha=45^0.$$

Kut β iznosi:

$$\alpha+\beta+\gamma=180^0\Rightarrow\beta=180^0-(\alpha+\gamma)\Rightarrow\beta=180^0-\left(45^0+120^0\right)\Rightarrow\beta=180^0-165^0\Rightarrow\beta=15^0.$$

Vježba 095

U trokutu je a = 2, $c=\sqrt{6}$, γ = 120°. Nađite kut β.

Rezultat: $\beta=15^0$.

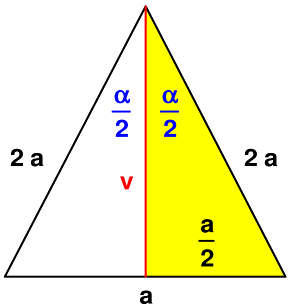
Zadatak 096 (Los-Habalos, gimnazija)

U jednakokračnom trokutu krak je dva puta dulji od osnovice. Neka je α kut nasuprot osnovici. Nađite sin α.

Rješenje 096

Ponovimo!

$$\sin 2x = 2 \cdot \sin x \cdot \cos x.$$



Iz osjenčanog pravokutnog trokuta pomoću Pitagorina poučka dobije se visina v:

$$v^2 = (2 \cdot a)^2 - \left(\frac{a}{2}\right)^2 \Rightarrow v^2 = 4 \cdot a^2 - \frac{a^2}{4} \Rightarrow v^2 = \frac{15 \cdot a^2}{4} \,/\sqrt{\ } \Rightarrow v = \frac{a}{2} \cdot \sqrt{15}.$$

Računamo sin α:

$$\sin\alpha = 2 \cdot \sin\frac{\alpha}{2} \cdot \cos\frac{\alpha}{2} = 2 \cdot \frac{\frac{a}{2}}{2 \cdot a} \cdot \frac{v}{2 \cdot a} = \frac{v}{4 \cdot a} = \frac{\frac{a}{2} \cdot \sqrt{15}}{4 \cdot a} = \frac{\sqrt{15}}{8}.$$

Vježba 096

U jednakokračnom trokutu krak je dva puta dulji od osnovice. Neka je β kut na osnovici. Nađite sin β.

Rezultat: $\frac{\sqrt{15}}{4}.$

Zadatak 097 (Ivan, pomorska škola)

Odredi vrijednosti ostalih trigonometrijskih funkcija ako je zadano $\cos\alpha = \frac{7}{25}.$

Rješenje 097

Ponovimo!

$$\cos^2 x + \sin^2 x = 1 \quad , \quad tg\, x = \frac{\sin x}{\cos x} \quad , \quad ctg\, x = \frac{\cos x}{\sin x} \quad , \quad tg\, x \cdot ctg\, x = 1.$$

Računamo vrijednost trigonometrijske funkcije sin α:

$$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \sin^2\alpha = 1 - \cos^2\alpha \,/\sqrt{\ } \Rightarrow \sin\alpha = \pm\sqrt{1 - \cos^2\alpha} \Rightarrow \sin\alpha = \pm\sqrt{1 - \left(\frac{7}{25}\right)^2} \Rightarrow$$

$$\Rightarrow \sin\alpha = \pm\sqrt{1 - \frac{49}{625}} \Rightarrow \sin\alpha = \pm\sqrt{\frac{625 - 49}{625}} \Rightarrow \sin\alpha = \pm\sqrt{\frac{576}{625}} \Rightarrow \sin\alpha = \pm\frac{24}{25}.$$

Računamo vrijednost trigonometrijske funkcije tg α:

$$tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} \Rightarrow tg\,\alpha = \frac{\pm\frac{24}{25}}{\frac{7}{25}} \Rightarrow tg\,\alpha = \pm\frac{24}{7}.$$

Računamo vrijednost trigonometrijske funkcije ctg α:

1.inačica

$$ctg\,\alpha = \frac{\cos\alpha}{\sin\alpha} \Rightarrow ctg\,\alpha = \frac{\frac{7}{25}}{\pm\frac{24}{25}} \Rightarrow ctg\,\alpha = \pm\frac{7}{24}.$$

2.inačica

$$tg\,\alpha \cdot ctg\,\alpha = 1 \Rightarrow ctg\,\alpha = \frac{1}{tg\,\alpha} \Rightarrow ctg\,\alpha = \frac{1}{\pm\frac{24}{7}} \Rightarrow ctg\,\alpha = \pm\frac{7}{24}.$$

Vježba 097

Odredi vrijednosti ostalih trigonometrijskih funkcija ako je zadano $\cos\alpha=\frac{3}{5}.$

Rezultat: $\sin\alpha=\pm\frac{4}{5}\ ,\ tg\,\alpha=\pm\frac{4}{3}\ ,\ ctg\,\alpha=\pm\frac{3}{4}.$

Zadatak 098 (Ivan, pomorska škola)

Odredi vrijednosti ostalih trigonometrijskih funkcija ako je zadano $tg\,\alpha=\frac{1}{3}.$

Rješenje 098

Ponovimo!

$$\cos^2 x+\sin^2 x=1\quad,\quad tg\,x=\frac{\sin x}{\cos x}\quad,\quad ctg\,x=\frac{\cos x}{\sin x}\quad,\quad tg\,x\cdot ctg\,x=1\quad,\quad\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}.$$

Računamo vrijednost trigonometrijske funkcije sin α:

1.inačica

$$\cos^2\alpha+\sin^2\alpha=1\Rightarrow\cos^2\alpha+\sin^2\alpha=1\ /:\sin^2\alpha\Rightarrow\frac{\cos^2\alpha}{\sin^2\alpha}+\frac{\sin^2\alpha}{\sin^2\alpha}=\frac{1}{\sin^2\alpha}\Rightarrow ctg^2\alpha+1=\frac{1}{\sin^2\alpha}\Rightarrow$$

$$\Rightarrow\frac{1}{tg^2\alpha}+1=\frac{1}{\sin^2\alpha}\Rightarrow\frac{1+tg^2\alpha}{tg^2\alpha}=\frac{1}{\sin^2\alpha}\Rightarrow\left[\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{b}{a}=\frac{d}{c}\right]\Rightarrow\frac{tg^2\alpha}{1+tg^2\alpha}=\frac{\sin^2\alpha}{1}\Rightarrow$$

$$\Rightarrow\sin^2\alpha=\frac{tg^2\alpha}{1+tg^2\alpha}\ /\sqrt{\ }\Rightarrow\sin\alpha=\pm\sqrt{\frac{tg^2\alpha}{1+tg^2\alpha}}\Rightarrow\sin\alpha=\pm\frac{tg\,\alpha}{\sqrt{1+tg^2\alpha}}\Rightarrow\sin\alpha=\pm\frac{\frac{1}{3}}{\sqrt{1+\left(\frac{1}{3}\right)^2}}\Rightarrow$$

$$\Rightarrow\sin\alpha=\pm\frac{\frac{1}{3}}{\sqrt{1+\frac{1}{9}}}\Rightarrow\sin\alpha=\pm\frac{\frac{1}{3}}{\sqrt{\frac{9+1}{9}}}\Rightarrow\sin\alpha=\pm\frac{\frac{1}{3}}{\sqrt{\frac{10}{9}}}\Rightarrow\sin\alpha=\pm\frac{\frac{1}{3}}{\frac{\sqrt{10}}{3}}\Rightarrow\sin\alpha=\pm\frac{1}{\sqrt{10}}\Rightarrow$$

$$\Rightarrow\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]\Rightarrow\sin\alpha=\pm\frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}\Rightarrow\sin\alpha=\pm\frac{\sqrt{10}}{10}.$$

2.inačica

$$tg\,\alpha=\frac{1}{3}\Rightarrow\frac{\sin\alpha}{\cos\alpha}=\frac{1}{3}\Rightarrow\frac{\sin\alpha}{\cos\alpha}=\frac{1}{3}\ /^2\Rightarrow\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{1}{9}\Rightarrow\frac{\sin^2\alpha}{1-\sin^2\alpha}=\frac{1}{9}\Rightarrow$$

$$\Rightarrow 9\cdot\sin^2\alpha=1-\sin^2\alpha\Rightarrow 9\cdot\sin^2\alpha+\sin^2\alpha=1\Rightarrow 10\cdot\sin^2\alpha=1\Rightarrow\sin^2\alpha=\frac{1}{10}\ /\sqrt{\ }\Rightarrow$$

$$\Rightarrow\sin\alpha=\pm\sqrt{\frac{1}{10}}\Rightarrow\sin\alpha=\pm\frac{1}{\sqrt{10}}\Rightarrow\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]\Rightarrow\sin\alpha=\pm\frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}\Rightarrow\sin\alpha=\pm\frac{\sqrt{10}}{10}.$$

3.inačica

$$tg\,\alpha=\frac{1}{3}\Rightarrow\frac{\sin\alpha}{\cos\alpha}=\frac{1}{3}\Rightarrow\frac{\sin\alpha}{\pm\sqrt{1-\sin^2\alpha}}=\frac{1}{3}\ /^2\Rightarrow\frac{\sin^2\alpha}{1-\sin^2\alpha}=\frac{1}{9}\Rightarrow$$

$$\Rightarrow 9\cdot\sin^2\alpha=1-\sin^2\alpha\Rightarrow 9\cdot\sin^2\alpha+\sin^2\alpha=1\Rightarrow 10\cdot\sin^2\alpha=1\Rightarrow\sin^2\alpha=\frac{1}{10}\ /\sqrt{\ }\Rightarrow$$

$$\Rightarrow \sin\alpha = \pm\sqrt{\frac{1}{10}} \Rightarrow \sin\alpha = \pm\frac{1}{\sqrt{10}} \Rightarrow \begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix} \Rightarrow \sin\alpha = \pm\frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} \Rightarrow \sin\alpha = \pm\frac{\sqrt{10}}{10}.$$

Računamo vrijednost trigonometrijske funkcije cos α:

1.inačica

$$\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow \cos^2\alpha + \sin^2\alpha = 1 \;/:\cos^2\alpha \Rightarrow \frac{\cos^2\alpha}{\cos^2\alpha} + \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha} \Rightarrow 1 + tg^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow$$

$$\Rightarrow \left[\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}\right] \Rightarrow \frac{1}{1+tg^2\alpha} = \frac{\cos^2\alpha}{1} \Rightarrow \cos^2\alpha = \frac{1}{1+tg^2\alpha} \;/\sqrt{} \Rightarrow \cos\alpha = \pm\sqrt{\frac{1}{1+tg^2\alpha}} \Rightarrow$$

$$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{1+tg^2\alpha}} \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{1+\left(\frac{1}{3}\right)^2}} \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{1+\frac{1}{9}}} \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{\frac{9+1}{9}}} \Rightarrow$$

$$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{\frac{10}{9}}} \Rightarrow \cos\alpha = \pm\frac{1}{\frac{\sqrt{10}}{3}} \Rightarrow \cos\alpha = \pm\frac{3}{\sqrt{10}} \Rightarrow \begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix} \Rightarrow$$

$$\Rightarrow \cos\alpha = \pm\frac{3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} \Rightarrow \cos\alpha = \pm\frac{3\cdot\sqrt{10}}{10}.$$

2.inačica

$$tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \cos\alpha = \frac{\sin\alpha}{tg\,\alpha} \Rightarrow \cos\alpha = \frac{\pm\frac{\sqrt{10}}{10}}{\frac{1}{3}} \Rightarrow \cos\alpha = \pm\frac{3\cdot\sqrt{10}}{10}.$$

Računamo vrijednost trigonometrijske funkcije ctg α:

1.inačica

$$ctg\,\alpha = \frac{\cos\alpha}{\sin\alpha} \Rightarrow ctg\,\alpha = \frac{\pm\frac{3\cdot\sqrt{10}}{10}}{\pm\frac{\sqrt{10}}{10}} \Rightarrow ctg\,\alpha = \pm 3.$$

2.inačica

$$tg\,\alpha\cdot ctg\,\alpha = 1 \Rightarrow ctg\,\alpha = \frac{1}{tg\,\alpha} \Rightarrow ctg\,\alpha = \frac{1}{\frac{1}{3}} \Rightarrow ctg\,\alpha = 3.$$

Vježba 098

Odredi vrijednosti ostalih trigonometrijskih funkcija ako je zadano $tg\,\alpha = 1$.

Rezultat: $\sin\alpha = \pm\frac{\sqrt{2}}{2}$, $\cos\alpha = \pm\frac{\sqrt{2}}{2}$, $ctg\,\alpha = 1$.

Zadatak 099 (Ines, kemijska škola)

Dokaži: $\frac{\cos x}{1-tg\,x} - \frac{\sin x}{ctg\,x - 1} = \sin x + \cos x.$

Rješenje 099

Ponovimo!

$$tg\,x = \frac{\sin x}{\cos x} \quad , \quad ctg\,x = \frac{\cos x}{\sin x} \quad , \quad a^2 - b^2 = (a-b)\cdot(a+b).$$

1.inačica

$$\frac{\cos x}{1-tg\,x}-\frac{\sin x}{ctg\,x-1}=\frac{\cos x}{1-\frac{\sin x}{\cos x}}-\frac{\sin x}{\frac{\cos x}{\sin x}-1}=\frac{\cos x}{\frac{\cos x-\sin x}{\cos x}}-\frac{\sin x}{\frac{\cos x-\cos x}{\sin x}}=\frac{\cos^2 x}{\cos x-\sin x}-\frac{\sin^2 x}{\cos x-\sin x}=$$

$$=\frac{\cos^2 x-\sin^2 x}{\cos x-\sin x}=\frac{(\cos x-\sin x)\cdot(\cos x+\sin x)}{\cos x-\sin x}=\cos x+\sin x=\sin x+\cos x.\quad Q.E.D.$$

2.inačica

$$\frac{\cos x}{1-tg\,x}-\frac{\sin x}{ctg\,x-1}=\sin x+\cos x\Rightarrow\frac{\cos x}{1-\frac{\sin x}{\cos x}}-\frac{\sin x}{\frac{\cos x}{\sin x}-1}=\sin x+\cos x\Rightarrow$$

$$\Rightarrow\frac{\cos x}{\frac{\cos x-\sin x}{\cos x}}-\frac{\sin x}{\frac{\cos x-\sin x}{\sin x}}=\sin x+\cos x\Rightarrow\frac{\cos^2 x}{\cos x-\sin x}-\frac{\sin^2 x}{\cos x-\sin x}=\sin x+\cos x\Rightarrow$$

$$\Rightarrow\frac{\cos^2 x-\sin^2 x}{\cos x-\sin x}=\sin x+\cos x\ /\cdot(\cos x-\sin x)\Rightarrow\cos^2 x-\sin^2 x=(\sin x+\cos x)\cdot(\cos x-\sin x)\Rightarrow$$

$$\Rightarrow\cos^2 x-\sin^2 x=\cos^2 x-\sin^2 x.\quad Q.E.D.$$

Vježba 099

Dokaži: $\frac{\cos x}{1-tg\,x}+\frac{\sin x}{1-ctg\,x}=\sin x+\cos x.$

Rezultat: Dokaz analogan.

Zadatak 100 (Vesna, kemijska škola)

Nađite b : c ako su kutovi α = 45° i β = 30°.

Rješenje 100

Ponovimo!

Sinusov poučak : $a:b:c=\sin\alpha:\sin\beta:\sin\gamma\Rightarrow b:c=\sin\beta:\sin\gamma,$

sinus zbroja : $\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta.$

Odredimo kut γ:

$$\alpha+\beta+\gamma=180^0\Rightarrow\gamma=180^0-(\alpha+\beta)\Rightarrow\gamma=180^0-\left(45^0+30^0\right)\Rightarrow\gamma=180^0-75^0\Rightarrow\gamma=105^0.$$

Računamo omjer:

$$\frac{b}{c}=\frac{\sin\beta}{\sin\gamma}\Rightarrow\frac{b}{c}=\frac{\sin 30^0}{\sin 105^0}\Rightarrow\frac{b}{c}=\frac{\sin 30^0}{\sin\left(60^0+45^0\right)}\Rightarrow\frac{b}{c}=\frac{\sin 30^0}{\sin 60^0\cdot\cos 45^0+\cos 60^0\cdot\sin 45^0}\Rightarrow$$

$$\Rightarrow\frac{b}{c}=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2}}\Rightarrow\frac{b}{c}=\frac{\frac{1}{2}}{\frac{\sqrt{2}}{4}\cdot\sqrt{3}+\frac{\sqrt{2}}{4}\cdot 1}\Rightarrow\frac{b}{c}=\frac{\frac{1}{2}}{\frac{\sqrt{2}}{4}\cdot\left(\sqrt{3}+1\right)}\Rightarrow\frac{b}{c}=\frac{2}{\sqrt{2}\cdot\left(\sqrt{3}+1\right)}\Rightarrow$$

$$\Rightarrow\begin{bmatrix}\text{racionalizacija}\\\text{nazivnika}\end{bmatrix}\Rightarrow\frac{b}{c}=\frac{2}{\sqrt{2}\cdot\left(\sqrt{3}+1\right)}\cdot\frac{\sqrt{2}}{\sqrt{2}}\Rightarrow\frac{b}{c}=\frac{2\cdot\sqrt{2}}{\left(\sqrt{2}\right)^2\cdot\left(\sqrt{3}+1\right)}\Rightarrow\frac{b}{c}=\frac{2\cdot\sqrt{2}}{2\cdot\left(\sqrt{3}+1\right)}\Rightarrow\frac{b}{c}=\frac{\sqrt{2}}{1+\sqrt{3}}.$$

Vježba 100

Nađite c : b ako su kutovi α = 45° i β = 30°.

Rezultat: $c:b=\left(1+\sqrt{3}\right):\sqrt{2}.$