Zadatak 221 (Ana, srednja škola)

Izračunaj $\log\left(\cos 70^0\right)-\log\left(\sin 20^0\right)$.

Rješenje 221

Ponovimo!

$$\log_{10} a=\log a \quad , \quad \log\frac{a}{b}=\log a-\log b \quad , \quad \log 1=0 \quad , \quad \cos\left(90^0-\alpha\right)=\sin\alpha.$$

1.inačica

$$\log\left(\cos 70^0\right)-\log\left(\sin 20^0\right)=\log\left(\cos 70^0\right)-\log\left(\cos\left(90^0-20^0\right)\right)=$$

$$=\log\left(\cos 70^0\right)-\log\left(\cos 70^0\right)=\log\left(\cos 70^0\right)-\log\left(\cos 70^0\right)=0.$$

2.inačica

$$\log\left(\cos 70^0\right)-\log\left(\sin 20^0\right)=\log\frac{\cos 70^0}{\sin 20^0}=\log\frac{\cos 70^0}{\cos\left(90^0-20^0\right)}=\log\frac{\cos 70^0}{\cos 70^0}=$$

$$=\log\frac{\cos 70^0}{\cos 70^0}=\log 1=0.$$

Vježba 221

Izračunaj $\log\left(\cos 80^0\right)-\log\left(\sin 10^0\right)$.

Rezultat: 0.

Zadatak 222 (Franjo, srednja škola)

Pojednostavni: $\frac{1-\cos x}{\sin x}$.

Rješenje 222

Ponovimo!

$$\cos^2 x+\sin^2 x=1.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Trigonometrijske funkcije za dvostruki argument

$$\sin(2\cdot x)=2\cdot\sin x\cdot\cos x \quad , \quad \cos(2\cdot x)=\cos^2 x-\sin^2 x.$$

Trigonometrijske funkcije polovičnog argumenta

$$\sin^2\frac{x}{2}=\frac{1-\cos x}{2} \Rightarrow 1-\cos x=2\cdot\sin^2\frac{x}{2}.$$

$$\cos^2\frac{x}{2}=\frac{1+\cos x}{2} \Rightarrow 1+\cos x=2\cdot\cos^2\frac{x}{2}.$$

1.inačica

$$\frac{1-\cos x}{\sin x}=\frac{1-\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{1-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{\left(1-\cos^2\frac{x}{2}\right)+\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=$$

$$=\frac{\sin^2\frac{x}{2}+\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{2\cdot\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{2\cdot\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=tg\frac{x}{2}.$$

2.inačica

$$\frac{1-\cos x}{\sin x}=\frac{2\cdot\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{2\cdot\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=tg\frac{x}{2}.$$

3.inačica

$$\frac{1-\cos x}{\sin x}=\frac{\left(\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\right)-\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=$$

$$=\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-\cos^2\frac{x}{2}+\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{2\cdot\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{2\cdot\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=tg\frac{x}{2}.$$

Vježba 222

Pojednostavni: $\frac{1+\cos x}{\sin x}$.

Rezultat: $ctg\frac{x}{2}.$

Zadatak 223 (Ante, srednja škola)

Ako je $t\in\left\langle\frac{\pi}{2},\pi\right\rangle$ i $\sin t=0.6,$ koliko je $\cos t$?

$A.\ -0.8 \qquad B.\ -0.4 \qquad C.\ 0.4 \qquad D.\ 0.8$

Rješenje 223

Ponovimo!

$$\cos^2 x+\sin^2 x=1.$$

	I. kvadrant	II. kvadrant	III. kvadrant	IV. kvadrant
sin	+	+	—	—
cos	+	—	—	+
tg	+	—	+	—
ctg	+	—	+	—

$$\cos^2 t+\sin^2 t=1\ \Rightarrow\ \cos^2 t=1-\sin^2 t\ \Rightarrow\ \cos^2 t=1-\sin^2 t\ /\sqrt{\ }\ \Rightarrow\ \cos t=\pm\sqrt{1-\sin^2 t}\ \Rightarrow$$

$$\Rightarrow\ \left[\begin{matrix} t\in\left\langle\frac{\pi}{2},\pi\right\rangle \\ \text{drugi kvadrant}\end{matrix}\right]\ \Rightarrow\ \cos t=-\sqrt{1-\sin^2 t}\ \Rightarrow\ \cos t=-\sqrt{1-0.6^2}\ \Rightarrow\ \cos t=-\sqrt{1-0.36}\ \Rightarrow$$

$$\Rightarrow\ \cos t=-\sqrt{0.64}\ \Rightarrow\ \cos t=-0.8.$$

Odgovor je pod A.

Vježba 223

Ako je $t \in \left\langle \frac{3\cdot\pi}{2}, 2\cdot\pi \right\rangle$ i $\sin t = -0.6$, koliko je $\cos t$?

$A.\ -0.8 \qquad B.\ -0.4 \qquad C.\ 0.4 \qquad D.\ 0.8$

Rezultat: D.

Zadatak 224 (Sunčica + Sunčan = ♥, srednja škola)

Jednoga ljetnoga dana temperatura u pustinji mijenjala se prema formuli $T(t) = 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) + 32$, gdje je t vrijeme od 0 do 24 sata, a T temperatura u °C. Kolika je temperatura bila u 7 sati ujutro?

Rješenje 224

Ponovimo!

Parnost funkcije kosinus

$$\cos(-x) = \cos x.$$

$$\cos\frac{2\cdot\pi}{3} = \cos 120^0 = -\frac{1}{2} \quad , \quad n = \frac{n}{1}.$$

Računamo temperaturu pustinje u 7 sati ujutro.

$$\left.\begin{array}{l} T(t) = 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) + 32 \\ t = 7 \end{array}\right\} \Rightarrow T(7) = 16\cdot\cos\left(\frac{7\cdot\pi - 15\cdot\pi}{12}\right) + 32 \Rightarrow$$

$$\Rightarrow T(7) = 16\cdot\cos\left(\frac{-8\cdot\pi}{12}\right) + 32 \Rightarrow \left[\begin{array}{c}\text{parnost} \\ \text{kosinusa}\end{array}\right] \Rightarrow T(7) = 16\cdot\cos\left(\frac{8\cdot\pi}{12}\right) + 32 \Rightarrow$$

$$\Rightarrow T(7) = 16\cdot\cos\left(\frac{8\cdot\pi}{12}\right) + 32 \Rightarrow T(7) = 16\cdot\cos\left(\frac{2\cdot\pi}{3}\right) + 32 \Rightarrow T(7) = 16\cdot\left(-\frac{1}{2}\right) + 32 \Rightarrow$$

$$\Rightarrow T(7) = \frac{16}{1}\cdot\left(-\frac{1}{2}\right) + 32 \Rightarrow T(7) = -8 + 32 \Rightarrow T(7) = 24\ {}^0C.$$

t  T = ?

Vježba 224

Jednoga ljetnoga dana temperatura u pustinji mijenjala se prema formuli $T(t) = 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) + 32$, gdje je t vrijeme od 0 do 24 sata, a T temperatura u °C. Kolika je temperatura bila u 15 sati?

Rezultat: 48 °C.

Zadatak 225 (Sunčica + Sunčan = ♥, srednja škola)

Jednoga ljetnoga dana temperatura u pustinji mijenjala se prema formuli $T(t) = 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) + 32$, gdje je t vrijeme od 0 do 24 sata, a T temperatura u °C. U koje je vrijeme poslijepodne temperatura bila 41 °C?

Rješenje 225

Ponovimo!

$$1\ h = 60\ \min.$$

Skup rješenja trigonometrijske jednadžbe

$$\cos x = a\ ,\ \left|a\right| \le 1$$

je

$$\left\{\pm x_0 + k\cdot 2\cdot\pi\ ,\ k\in Z\right\},$$

gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

Računamo u koje je vrijeme poslijepodne temperatura bila 41 °C?

$$\left.\begin{array}{l} T(t) = 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) + 32 \\ T(t) = 41 \end{array}\right\} \Rightarrow 41 = 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) + 32 \Rightarrow$$

$$\Rightarrow 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) + 32 = 41 \Rightarrow 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) = 41 - 32 \Rightarrow 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) = 9 \Rightarrow$$

$$\Rightarrow 16\cdot\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) = 9\ /:16 \Rightarrow \cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) = \frac{9}{16}.$$

Rješavamo trigonometrijsku jednadžbu

$$\cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) = \frac{9}{16}.$$

Uvodimo zamjenu (supstituciju)

$$x = \frac{t\cdot\pi - 15\cdot\pi}{12}.$$

Računalo stavimo u stanje (mod): **RAD**.

$$\left.\begin{array}{l} \cos\left(\frac{t\cdot\pi - 15\cdot\pi}{12}\right) = \frac{9}{16} \\ x = \frac{t\cdot\pi - 15\cdot\pi}{12} \end{array}\right\} \Rightarrow \cos x = \frac{9}{16} \Rightarrow x = \cos^{-1}\left(\frac{9}{16}\right) \Rightarrow x = 0.97338991.$$

Vraćamo se supstituciji.

$$\left.\begin{array}{l} x = \frac{t\cdot\pi - 15\cdot\pi}{12} \\ x = 0.97338991 \end{array}\right\} \Rightarrow \frac{t\cdot\pi - 15\cdot\pi}{12} = 0.97338991 \Rightarrow \frac{t\cdot\pi - 15\cdot\pi}{12} = 0.97338991\ /\cdot 12 \Rightarrow$$

$$\Rightarrow t\cdot\pi - 15\cdot\pi = 0.97338991\cdot 12 \Rightarrow t\cdot\pi = 0.97338991\cdot 12 + 15\cdot\pi \Rightarrow$$

$$\Rightarrow t\cdot\pi = 0.97338991\cdot 12 + 15\cdot\pi\ /:\pi \Rightarrow t = \frac{0.97338991\cdot 12 + 15\cdot\pi}{\pi} \Rightarrow \left[\begin{array}{l}\text{na računalu} \\ \text{pritisnemo} \\ \text{tipku } \pi\end{array}\right] \Rightarrow$$

$$\Rightarrow t = 18.71807558\ h \Rightarrow t = 18.71807558\ h \Rightarrow t = 18\ h + 0.71807558\ h \Rightarrow$$

$$\Rightarrow t = 18\ h + 0.71807558\cdot 60\ \min \Rightarrow t = 18\ h + 43.08453469\ \min \Rightarrow$$

$$\Rightarrow t = 18\ h + 43.08453469\ \min \Rightarrow t \approx 18\ h\ 43\ \min.$$

Vježba 225

Jednoga ljetnoga dana temperatura u pustinji mijenjala se prema formuli

$T(t)=16\cdot\cos\left(\frac{t\cdot\pi-15\cdot\pi}{12}\right)+32,$ gdje je t vrijeme od 0 do 24 sata, a T temperatura u °C. U koje je vrijeme poslijepodne temperatura bila 48 °C?

Rezultat: 15 h.

Zadatak 226 (Sunčica + Sunčan = ♥, srednja škola)

Jednoga ljetnoga dana temperatura u pustinji mijenjala se prema formuli

$T(t)=16\cdot\cos\left(\frac{t\cdot\pi-15\cdot\pi}{12}\right)+32,$ gdje je t vrijeme od 0 do 24 sata, a T temperatura u °C. Kolika je bila najviša temperatura toga dana?

Rješenje 226

Ponovimo!

Za funkciju kosinus vrijedi:

$$\cos : R\rightarrow\left[-1,\ 1\right].$$

Maksimum funkcije kosinus iznosi 1. Minimum funkcije kosinus iznosi – 1.

Budući da je maksimum funkcije kosinus 1, najviša temperatura toga dana u pustinji bila je

$$\left.\begin{array}{l} T(t)=16\cdot\cos\left(\frac{t\cdot\pi-15\cdot\pi}{12}\right)+32 \\ \cos\left(\frac{t\cdot\pi-15\cdot\pi}{12}\right)=1 \quad \text{maksimum} \end{array}\right\} \Rightarrow T(t)=16\cdot 1+32 \Rightarrow T(t)=16+32 \Rightarrow T(t)=48\ {}^{0}C.$$

Vježba 226

Jednoga ljetnoga dana temperatura u pustinji mijenjala se prema formuli

$T(t)=16\cdot\cos\left(\frac{t\cdot\pi-15\cdot\pi}{12}\right)+32,$ gdje je t vrijeme od 0 do 24 sata, a T temperatura u °C. Kolika je bila najniža temperatura toga dana?

Rezultat: 16 °C.

Zadatak 227 (Mirjana, srednja škola)

Ako je $tg\ x=a$, izračunajte $\frac{\sin x+\cos x}{\sin x-\cos x}$.

Rješenje 227

Ponovimo!

$$tg\ x=\frac{\sin x}{\cos x}\quad,\quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}\quad,\quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$\frac{\sin x+\cos x}{\sin x-\cos x}=\left[\begin{array}{l}\text{brojnik i nazivnik} \\ \text{dijelimo sa } \cos x\end{array}\right]=\frac{\frac{\sin x+\cos x}{\cos x}}{\frac{\sin x-\cos x}{\cos x}}=\frac{\frac{\sin x}{\cos x}+\frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}-\frac{\cos x}{\cos x}}=\frac{\frac{\sin x}{\cos x}+\frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}-\frac{\cos x}{\cos x}}=\frac{tg\ x+1}{tg\ x-1}=\frac{a+1}{a-1}.$$

Vježba 227

Ako je $tg\ x=a$, izračunajte $\frac{\sin x-\cos x}{\sin x+\cos x}$.

Rezultat: $\frac{a-1}{a+1}.$

Zadatak 228 (Ante, srednja škola)

Odredite sva rješenja jednadžbe $2\cdot\cos^2 x=\sin 2x.$

Rješenje 228

Ponovimo!

$$\sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha \quad , \quad ctg\, x=\frac{\cos x}{\sin x}.$$

Trigonometrijska jednadžba

$$ctg\, x=ctg\,\alpha \Rightarrow x=\alpha+k\cdot\pi \; , \; k\in Z.$$

Trigonometrijska jednadžba cos x = a, $|a|\le 1$

Skup rješenja jednadžbe cos x = a, $|a|\le 1$, je $\{\pm x_0+k\cdot 2\cdot\pi : k\in Z\}$ gdje je $x_0\in R$ jedno rješenje te jednadžbe.

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a\cdot b=0 \Leftrightarrow a=0 \text{ ili } b=0 \text{ ili } a=b=0.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$2\cdot\cos^2 x=\sin 2x \Rightarrow 2\cdot\cos^2 x=2\cdot\sin x\cdot\cos x \Rightarrow 2\cdot\cos^2 x=2\cdot\sin x\cdot\cos x \;/:2 \Rightarrow$$

$$\Rightarrow \cos^2 x=\sin x\cdot\cos x \Rightarrow \cos^2 x-\sin x\cdot\cos x=0 \Rightarrow \cos x\cdot(\cos x-\sin x)=0 \Rightarrow \left.\begin{array}{l}\cos x=0\\ \cos x-\sin x=0\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\cos x=0\\ \cos x=\sin x\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos x=0\\ \cos x=\sin x \;/\cdot\frac{1}{\sin x}\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos x=0\\ \frac{\cos x}{\sin x}=1\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos x=0\\ ctg\, x=1\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}x_{1,2}=\pm\frac{\pi}{2}+k\cdot\pi \; , \; k\in Z\\ x_3=\frac{\pi}{4}+k\cdot\pi \; , \; k\in Z\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{na drugi način}\\ \text{zapisano}\end{array}\right] \Rightarrow \left.\begin{array}{l}x_{1,2}=(2\cdot k+1)\cdot\frac{\pi}{2} \; , \; k\in Z\\ x_3=(4\cdot k+1)\cdot\frac{\pi}{4} \; , \; k\in Z\end{array}\right\}.$$

Vježba 228

Odredite sva rješenja jednadžbe $\cos^2 x=\frac{1}{2}\cdot\sin 2x.$

Rezultat: $x_{1,2}=\pm\frac{\pi}{2}+k\cdot\pi \; , \; x_3=\frac{\pi}{4}+k\cdot\pi \; , \; k\in Z.$

Zadatak 229 (Irena, srednja škola)

Izračunajmo $\frac{\cos 35^0+\cos 85^0}{\cos 65^0}.$

$A.\; tg\, 35^0 \qquad B.\; \cos 24^0 \qquad C.\; ctg\, 25^0 \qquad D.\; tg\, 60^0$

Rješenje 229

Ponovimo!

$$\cos 60^0=\frac{1}{2} \quad , \quad ctg\, x=\frac{\cos x}{\sin x}.$$

Formula pretvorbe zbroja u umnožak

$$\cos x+\cos y=2\cdot\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}.$$

Kosinus je parna funkcija

$$\cos(-x)=\cos x,\ \forall x\in R.$$

Formula redukcije

$$\sin\left(90^{0}-\alpha\right)=\cos\alpha.$$

$$\frac{\cos 35^{0}+\cos 85^{0}}{\cos 65^{0}}=\frac{2\cdot\cos\frac{35^{0}+85^{0}}{2}\cdot\cos\frac{35^{0}-85^{0}}{2}}{\cos 65^{0}}=\frac{2\cdot\cos\frac{120^{0}}{2}\cdot\cos\left(-\frac{50^{0}}{2}\right)}{\cos 65^{0}}=$$

$$=\frac{2\cdot\cos 60^{0}\cdot\cos\left(-25^{0}\right)}{\cos 65^{0}}=\left[\cos(-x)=\cos x\right]=\frac{2\cdot\frac{1}{2}\cdot\cos 25^{0}}{\cos 65^{0}}=\frac{2\cdot\frac{1}{2}\cdot\cos 25^{0}}{\cos 65^{0}}=\frac{\cos 25^{0}}{\cos 65^{0}}=$$

$$=\left[\sin\left(90^{0}-x\right)=\cos x\right]=\frac{\cos 25^{0}}{\sin\left(90^{0}-65^{0}\right)}=\frac{\cos 25^{0}}{\sin 25^{0}}=ctg\, 25^{0}.$$

Odgovor je pod C.

Vježba 229

Izračunajmo $\dfrac{\cos 65^{0}}{\cos 35^{0}+\cos 85^{0}}$.

$A.\ tg\,25^{0}$ $\quad B.\ \cos 24^{0}$ $\quad C.\ ctg\,25^{0}$ $\quad D.\ tg\,60^{0}$

Rezultat: A.

Zadatak 230 (Ante, srednja škola)

Prosječna dnevna temperatura T (u °C) u nekom gradu može se procijeniti prema formuli $T(d)=a\cdot\sin\left(\frac{2\cdot\pi}{365}\cdot(d-123)\right)+12$, gdje je d redni broj dana u godini (primjerice, 1. veljače d = 32).
Razlika u temperaturi 22. veljače i 2. veljače je 1.3 °C. Kolika je vrijednost parametra a?

$A.\ 18.6$ $\quad B.\ 19.7$ $\quad C.\ 20.3$ $\quad D.\ 21.4$

Rješenje 230

Ponovimo!

$$siječanj=31\ dan\quad,\quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c}.$$

Sinus je neparna funkcija

$$\sin(-x)=-\sin x\ ,\ \forall x\in R.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Budući da je d redni broj dana u godini, slijedi:

- 22. *veljače* $\Rightarrow d_1 = 31 + 22 = 53$
- 2. *veljače* $\Rightarrow d_2 = 31 + 2 = 33.$

Razlika u temperaturi 22. veljače i 2. veljače je 1.3 °C pa vrijedi:

$$T(d_1) - T(d_2) = 1.3 \Rightarrow \left(a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d_1 - 123)\right) + 12\right) - \left(a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d_2 - 123)\right) + 12\right) = 1.3 \Rightarrow$$

$$\Rightarrow a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d_1 - 123)\right) + 12 - a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d_2 - 123)\right) - 12 = 1.3 \Rightarrow$$

$$\Rightarrow a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d_1 - 123)\right) + 12 - a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d_2 - 123)\right) - 12 = 1.3 \Rightarrow$$

$$\Rightarrow a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d_1 - 123)\right) - a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d_2 - 123)\right) = 1.3 \Rightarrow$$

$$\Rightarrow a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (53 - 123)\right) - a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (33 - 123)\right) = 1.3 \Rightarrow$$

$$\Rightarrow a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (-70)\right) - a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (-90)\right) = 1.3 \Rightarrow \left[\sin(-x) = -\sin x\right] \Rightarrow$$

$$\Rightarrow -a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot 70\right) + a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot 90\right) = 1.3 \Rightarrow -a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot 70\right) + a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot 90\right) = 1.3 \Rightarrow$$

$$\Rightarrow -a \cdot \sin\left(\frac{2 \cdot \pi}{73} \cdot 14\right) + a \cdot \sin\left(\frac{2 \cdot \pi}{73} \cdot 18\right) = 1.3 \Rightarrow -a \cdot \sin\left(\frac{28 \cdot \pi}{73}\right) + a \cdot \sin\left(\frac{36 \cdot \pi}{73}\right) = 1.3 \Rightarrow$$

$$\Rightarrow a \cdot \left(\sin\left(\frac{36 \cdot \pi}{73}\right) - \sin\left(\frac{28 \cdot \pi}{73}\right)\right) = 1.3 \Rightarrow$$

$$\Rightarrow a \cdot \left(\sin\left(\frac{36 \cdot \pi}{73}\right) - \sin\left(\frac{28 \cdot \pi}{73}\right)\right) = 1.3 \; / \cdot \frac{1}{\sin\left(\frac{36 \cdot \pi}{73}\right) - \sin\left(\frac{28 \cdot \pi}{73}\right)} \Rightarrow a = \frac{1.3}{\sin\left(\frac{36 \cdot \pi}{73}\right) - \sin\left(\frac{28 \cdot \pi}{73}\right)} \Rightarrow$$

$$\Rightarrow \left[\text{mod : rad}\right] \Rightarrow a = 19.7.$$

Odgovor je pod B.

Vježba 230

Prosječna dnevna temperatura T (u °C) u nekom gradu može se procijeniti prema formuli $T(d) = a \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (d - 123)\right) + 10$, gdje je d redni broj dana u godini (primjerice, 1. veljače d = 32). Razlika u temperaturi 22. veljače i 2. veljače je 1.3 °C. Kolika je vrijednost parametra a?

A. 18.6 *B.* 19.7 *C.* 20.3 *D.* 21.4

Rezultat: B.

Zadatak 231 (MaturantTT, gimnazija)

Odredite $x \in \left\langle 0, \frac{\pi}{2} \right\rangle$ za koji je $\cos^2 x - \sin 2x = 0.$ Rješenje zapišite zaokruženo na četiri decimale.

Rješenje 231

Ponovimo!

$$\sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha \quad , \quad tg\, x = \frac{\sin x}{\cos x} \quad , \quad \pi \approx 3.1416 \quad , \quad \frac{\pi}{2} \approx 1.5708.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a \cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Trigonometrijska jednadžba

$$\cos x = a \quad , \quad |a| \le 1.$$

Postupak rješavanja:

$$\cos x = a$$

$$x_0 = \cos^{-1} a \;,\; x_0 \in R \text{ je jedno rješenje jednadžbe}$$

$$\cos x = \cos x_0$$

$$x = \pm x_0 + k \cdot 2 \cdot \pi \quad , \quad k \in Z.$$

Trigonometrijska jednadžba

$$tg\, x = a \quad , \quad a \in R.$$

Postupak rješavanja:

$$tg\, x = a$$

$$x_0 = tg^{-1} a \;,\; x_0 \in R \text{ je jedno rješenje jednadžbe}$$

$$tg\, x = tg\, x_0$$

$$x = x_0 + k \cdot \pi \quad , \quad k \in Z.$$

$$\cos^2 x - \sin 2x = 0 \Rightarrow \cos^2 x - 2 \cdot \sin x \cdot \cos x = 0 \Rightarrow \cos x \cdot (\cos x - 2 \cdot \sin x) = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} \cos x = 0 \\ \cos x - 2 \cdot \sin x = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x = \cos^{-1} 0 \\ \cos x = 2 \cdot \sin x \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = \frac{\pi}{2} + k \cdot \pi, \in Z, \text{ nije rješenje zbog } x \in \left\langle 0, \frac{\pi}{2} \right\rangle \\ 2 \cdot \sin x = \cos x \end{array}\right\} \Rightarrow$$

$$\Rightarrow 2 \cdot \sin x = \cos x \;/ \cdot \frac{1}{2 \cdot \cos x} \Rightarrow \frac{\sin x}{\cos x} = \frac{1}{2} \Rightarrow tg\, x = \frac{1}{2} \Rightarrow x = tg^{-1}\left(\frac{1}{2}\right) \Rightarrow \left[\begin{array}{l} \text{mod :} \\ \text{rad} \end{array}\right] \Rightarrow x = 0.4636.$$

Vježba 231

Odredite $x \in \left\langle 0, \frac{\pi}{2} \right\rangle$ za koji je $\sin 2x - \cos^2 x = 0.$ Rješenje zapišite zaokruženo na tri decimale.

Rezultat: 0.464.

Zadatak 232 (Max, gimnazija)

Dokažite jednakost: $\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}=4.$

Rješenje 232

Ponovimo!

$$tg\,60^0=\sqrt{3}\quad,\quad \cos 60^0=\frac{1}{2}\quad,\quad tg\,x=\frac{\sin x}{\cos x}\quad,\quad \sin 2x=2\cdot\sin x\cdot\cos x\quad,\quad n=\frac{n}{1}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad \cos(x+y)=\cos x\cdot\cos y-\sin x\cdot\sin y.$$

$$\cos x=\sin\left(90^0-x\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{1}{\sin 10^0}-\frac{\sqrt{3}}{\cos 10^0}=\frac{1}{\sin 10^0}-\frac{tg\,60^0}{\cos 10^0}=\frac{1}{\sin 10^0}-\frac{\frac{\sin 60^0}{\cos 60^0}}{\cos 10^0}=\frac{1}{\sin 10^0}-\frac{\frac{\sin 60^0}{\cos 60^0}}{\frac{\cos 10^0}{1}}=$$

$$=\frac{1}{\sin 10^0}-\frac{\sin 60^0}{\cos 10^0\cdot\cos 60^0}=\frac{\cos 60^0\cdot\cos 10^0-\sin 60^0\cdot\sin 10^0}{\sin 10^0\cdot\cos 10^0\cdot\cos 60^0}=\frac{\cos\left(60^0+10^0\right)}{\sin 10^0\cdot\cos 10^0\cdot\cos 60^0}=$$

$$=\frac{\cos 70^0}{\sin 10^0\cdot\cos 10^0\cdot\cos 60^0}=\frac{\cos\left(90^0-20^0\right)}{\sin 10^0\cdot\cos 10^0\cdot\cos 60^0}=\frac{\sin 20^0}{\sin 10^0\cdot\cos 10^0\cdot\cos 60^0}=$$

$$=\frac{\sin 20^0}{\sin 10^0\cdot\cos 10^0\cdot\frac{1}{2}}=\frac{2\cdot\sin 20^0}{\sin 10^0\cdot\cos 10^0}=\left[\begin{array}{l}\text{razlomak}\\ \text{proširujemo s 2}\end{array}\right]=\frac{2\cdot 2\cdot\sin 20^0}{2\cdot\sin 10^0\cdot\cos 10^0}=$$

$$=\frac{4\cdot\sin 20^0}{\sin 20^0}=\frac{4\cdot\sin 20^0}{\sin 20^0}=4.$$

Vježba 232

Dokažite jednakost: $\frac{1}{\sin 10^0}-\frac{3}{\sqrt{3}\cdot\cos 10^0}=4.$

Rezultat: Dokaz analogan.

Zadatak 233 (Nina, gimnazija)

Dokažite jednakost: $(\sin\alpha+\sin\beta)^2+\cos^2\alpha\cdot\cos^2\beta=(\sin\alpha\cdot\sin\beta+1)^2$.

Rješenje 233

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \cos^2 x+\sin^2 x=1 \quad , \quad (a\cdot b)^n=a^n\cdot b^n.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$(\sin\alpha+\sin\beta)^2+\cos^2\alpha\cdot\cos^2\beta=\sin^2\alpha+2\cdot\sin\alpha\cdot\sin\beta+\sin^2\beta+\cos^2\alpha\cdot\cos^2\beta=$$

$$=\sin^2\alpha+2\cdot\sin\alpha\cdot\sin\beta+\sin^2\beta+\left(1-\sin^2\alpha\right)\cdot\left(1-\sin^2\beta\right)=$$

$$=\sin^2\alpha+2\cdot\sin\alpha\cdot\sin\beta+\sin^2\beta+1-\sin^2\beta-\sin^2\alpha+\sin^2\alpha\cdot\sin^2\beta=$$

$$=\sin^2\alpha+2\cdot\sin\alpha\cdot\sin\beta+\sin^2\beta+1-\sin^2\beta-\sin^2\alpha+\sin^2\alpha\cdot\sin^2\beta=2\cdot\sin\alpha\cdot\sin\beta+1+\sin^2\alpha\cdot\sin^2\beta=$$

$$=\sin^2\alpha\cdot\sin^2\beta+2\cdot\sin\alpha\cdot\sin\beta+1=(\sin\alpha\cdot\sin\beta)^2+2\cdot\sin\alpha\cdot\sin\beta+1=(\sin\alpha\cdot\sin\beta+1)^2.$$

Vježba 233

Dokažite jednakost: $(\cos\alpha+\cos\beta)^2+\sin^2\alpha\cdot\sin^2\beta=(\cos\alpha\cdot\cos\beta+1)^2$.

Rezultat: Dokaz analogan.

Zadatak 234 (Tomo, srednja škola)

Ako je $\sin\frac{x}{2}+\cos\frac{x}{2}=1.4$, izračunati $\sin x$.

$A.\ 0.96 \qquad B.\ 1.96 \qquad C.\ 2.8 \qquad D.\ 0$

Rješenje 234

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \cos^2\alpha+\sin^2\alpha=1 \quad , \quad b\cdot\frac{a}{b}=a \quad , \quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha.$$

$$\sin\frac{x}{2}+\cos\frac{x}{2}=1.4 \;\Rightarrow\; \sin\frac{x}{2}+\cos\frac{x}{2}=1.4 \;/^2 \;\Rightarrow\; \left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2=1.4^2 \;\Rightarrow$$

$$\Rightarrow\; \sin^2\frac{x}{2}+2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}+\cos^2\frac{x}{2}=1.96 \;\Rightarrow\; \left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)+2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}=1.96 \;\Rightarrow$$

$$\Rightarrow\; 1+\sin\left(2\cdot\frac{x}{2}\right)=1.96 \;\Rightarrow\; 1+\sin\left(2\cdot\frac{x}{2}\right)=1.96 \;\Rightarrow\; 1+\sin x=1.96 \;\Rightarrow\; \sin x=1.96-1 \;\Rightarrow\; \sin x=0.96.$$

Odgovor je pod A.

Vježba 234

Ako je $\sin\frac{x}{2}+\cos\frac{x}{2}=1.2$, izračunati $\sin x$.

$A.\ 1.44 \qquad B.\ 0.44 \qquad C.\ 2.4 \qquad D.\ -0.44$

Rezultat: B.

Zadatak 235 (Vedran, srednja škola)

Ako je $(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)=\cos\alpha\cdot\cos\beta\cdot\cos\gamma$, pojednostavnite izraz: $(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)$.

Rješenje 235

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \cos^2 x+\sin^2 x=1.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \; , \; n\neq 0 \; , \; n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

$$(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)=\left[\begin{array}{c}\text{proširimo izraz s}\\(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)\end{array}\right]=$$

$$=(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)\cdot\frac{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}=$$

$$=\frac{(1-\sin\alpha)\cdot(1+\sin\alpha)\cdot(1-\sin\beta)\cdot(1+\sin\beta)\cdot(1-\sin\gamma)\cdot(1+\sin\gamma)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}=$$

$$=\frac{(1-\sin\alpha)\cdot(1+\sin\alpha)\cdot(1-\sin\beta)\cdot(1+\sin\beta)\cdot(1-\sin\gamma)\cdot(1+\sin\gamma)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}=$$

$$=\frac{\left(1-\sin^2\alpha\right)\cdot\left(1-\sin^2\beta\right)\cdot\left(1-\sin^2\gamma\right)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}=$$

$$=\left[\begin{array}{c}\text{uvjet}\\(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)=\cos\alpha\cdot\cos\beta\cdot\cos\gamma\end{array}\right]=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{\cos\alpha\cdot\cos\beta\cdot\cos\gamma}=$$

$$=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{\cos\alpha\cdot\cos\beta\cdot\cos\gamma}=\cos\alpha\cdot\cos\beta\cdot\cos\gamma.$$

Vježba 235

Ako je $(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)=\cos\alpha\cdot\cos\beta\cdot\cos\gamma,$ pojednostavnite izraz: $(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma).$

Rezultat: $\cos\alpha\cdot\cos\beta\cdot\cos\gamma.$

Zadatak 236 (Matea, gimnazija)

Prikažite izraz $\frac{\sin x+\cos x}{\cos^3 x}$ kao racionalnu funkciju od tg x.

Rješenje 236

Ponovimo!

$$\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n} \quad , \quad \cos^2 x+\sin^2 x=1 \quad , \quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad tg\, x=\frac{\sin x}{\cos x}.$$

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \neq 0 \; , \; n \neq 1.$$

$$\frac{\sin x + \cos x}{\cos^3 x} = \frac{\sin x}{\cos^3 x} + \frac{\cos x}{\cos^3 x} = \frac{\sin x \cdot 1}{\cos^3 x} + \frac{\cos x}{\cos^3 x} = \frac{\sin x \cdot \left(\cos^2 x + \sin^2 x\right)}{\cos^3 x} + \frac{1}{\cos^2 x} =$$

$$= \frac{\sin x \cdot \cos^2 x + \sin^3 x}{\cos^3 x} + \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{\sin x \cdot \cos^2 x}{\cos^3 x} + \frac{\sin^3 x}{\cos^3 x} + \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} =$$

$$= \frac{\sin x \cdot \cos^2 x}{\cos^3 x} + \frac{\sin^3 x}{\cos^3 x} + \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{\sin x}{\cos x} + \frac{\sin^3 x}{\cos^3 x} + 1 + \frac{\sin^2 x}{\cos^2 x} =$$

$$= \frac{\sin x}{\cos x} + \left(\frac{\sin x}{\cos x}\right)^3 + 1 + \left(\frac{\sin x}{\cos x}\right)^2 = tg\, x + tg^3 x + 1 + tg^2 x = tg^3 x + tg^2 x + tg\, x + 1.$$

Vježba 236

Prikažite izraz $\frac{\sin x + \cos x}{\sin^3 x}$ kao racionalnu funkciju od ctg x.

Rezultat: $ctg^3 x + ctg^2 x + ctg\, x + 1.$

Zadatak 237 (Dubravka, srednja škola)

Ako je $\sin\alpha + \cos\beta = m$ i $\cos\alpha - \cos\beta = n$ koliko je $\sin 2\alpha$?

A. $(m+n)^2 - 2$ B. $(m+n)^2 + 2$ C. $(m+n)^2 + 1$ D. $(m+n)^2 - 1$

Rješenje 237

Ponovimo!

$$\left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow a + c = b + d \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

$$\cos^2 x + \sin^2 x = 1 \quad , \quad \sin 2x = 2 \cdot \sin x \cdot \cos x.$$

$$\left.\begin{array}{l} \sin\alpha + \cos\beta = m \\ \cos\alpha - \cos\beta = n \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow \sin\alpha + \cos\beta + \cos\alpha - \cos\beta = m + n \Rightarrow$$

$$\Rightarrow \sin\alpha + \cos\beta + \cos\alpha - \cos\beta = m + n \Rightarrow \sin\alpha + \cos\alpha = m + n \Rightarrow \left[\begin{array}{l} \text{kvadriramo} \\ \text{jednadžbu} \end{array}\right] \Rightarrow$$

$$\Rightarrow (\sin\alpha + \cos\alpha) = m + n \; /^2 \Rightarrow (\sin\alpha + \cos\alpha)^2 = (m+n)^2 \Rightarrow$$

$$\Rightarrow \sin^2\alpha + 2 \cdot \sin\alpha \cdot \cos\alpha + \cos^2\alpha = (m+n)^2 \Rightarrow$$

$$\Rightarrow \left(\sin^2\alpha + \cos^2\alpha\right) + 2 \cdot \sin\alpha \cdot \cos\alpha = (m+n)^2 \Rightarrow 1 + \sin 2\alpha = (m+n)^2 \Rightarrow \sin 2\alpha = (m+n)^2 - 1.$$

Odgovor je pod D.

Vježba 237

Ako je $\sin\alpha - \cos\beta = m$ i $\cos\alpha + \cos\beta = n$ koliko je $\sin 2\alpha$?

$A.\ (m+n)^2 - 2 \qquad B.\ (m+n)^2 + 2 \qquad C.\ (m+n)^2 + 1 \qquad D.\ (m+n)^2 - 1$

Rezultat: D.

Zadatak 238 (Dubravka, srednja škola)

Nađi maksimum funkcije $f(x) = 2 + 2\cdot\cos x - \sin^2 x$.

$A.\ 2 \qquad B.\ 3 \qquad C.\ 4 \qquad D.\ 0$

Rješenje 238

Ponovimo!

$\cos^2\alpha + \sin^2\alpha = 1 \quad , \quad (a+b)^2 = a^2 + 2\cdot a\cdot b + b^2 \quad , \quad \sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha.$

Zakon distribucije množenja prema zbrajanju.

$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$

Maksimum funkcije f(x) = cos x je 1, a poprima se za $x = k\cdot 2\cdot\pi,\ k\in Z.$

Minimum funkcije f(x) = cos x je – 1, a poprima se za $x = (2\cdot k+1)\cdot\pi,\ k\in Z.$

$$f(x) = 2 + 2\cdot\cos x - \sin^2 x \Rightarrow f(x) = 2 + 2\cdot\cos x - \left(1 - \cos^2 x\right) \Rightarrow f(x) = 2 + 2\cdot\cos x - 1 + \cos^2 x \Rightarrow$$

$$\Rightarrow f(x) = \cos^2 x + 2\cdot\cos x + 2 - 1 \Rightarrow f(x) = \cos^2 x + 2\cdot\cos x + 1 + 1 - 1 \Rightarrow$$

$$\Rightarrow f(x) = \left(\cos^2 x + 2\cdot\cos x + 1\right) + 1 - 1 \Rightarrow f(x) = (\cos x + 1)^2.$$

Budući da funkcija kosinus ima maksimum 1, vrijedi:

$$\left.\begin{array}{l} f(x) = (\cos x + 1)^2 \\ \cos x = 1 \end{array}\right\} \Rightarrow f_{maks}(x) = (1+1)^2 \Rightarrow f_{maks}(x) = 2^2 \Rightarrow f_{maks}(x) = 4.$$

Odgovor je pod C.

Vježba 238

Nađi minimum funkcije $f(x) = 2 + 2\cdot\cos x - \sin^2 x$.

$A.\ 2 \qquad B.\ 3 \qquad C.\ 4 \qquad D.\ 0$

Rezultat: D.

Zadatak 239 (Vesna, gimnazija)

Ako je $x = \dfrac{1}{\cos\alpha\cdot\cos\beta}$, $y = \dfrac{tg\,\alpha}{\cos\beta}$, $z = tg\,\beta$, izračunajte vrijednost izraza $A = x^2 - y^2 - z^2$.

$A.\ 1 \qquad B.\ 0 \qquad C.\ \sin\alpha \qquad D.\ \cos\beta$

Rješenje 239

Ponovimo!

$\cos^2 x + \sin^2 x = 1 \quad , \quad \dfrac{a}{b} - \dfrac{c}{d} = \dfrac{a\cdot d - b\cdot c}{b\cdot d} \quad , \quad tg\,x = \dfrac{\sin x}{\cos x} \quad , \quad \dfrac{1}{\cos^2 x} = 1 + tg^2 x.$

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n} \quad , \quad (a\cdot b)^n = a^n\cdot b^n \quad , \quad \dfrac{a}{b}\cdot\dfrac{c}{d} = \dfrac{a\cdot c}{b\cdot d} \quad , \quad \dfrac{a}{b} = a\cdot\dfrac{1}{b}.$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Iz uvjeta zadatka slijedi:

$$A=x^2-y^2-z^2 \Rightarrow A=\left(\frac{1}{\cos\alpha\cdot\cos\beta}\right)^2-\left(\frac{tg\,\alpha}{\cos\beta}\right)^2-(tg\,\beta)^2 \Rightarrow$$

$$\Rightarrow A=\frac{1^2}{(\cos\alpha\cdot\cos\beta)^2}-\frac{tg^2\,\alpha}{\cos^2\beta}-tg^2\,\beta \Rightarrow A=\frac{1}{\cos^2\alpha\cdot\cos^2\beta}-\frac{tg^2\,\alpha}{\cos^2\beta}-\left(\frac{\sin\beta}{\cos\beta}\right)^2 \Rightarrow$$

$$\Rightarrow A=\frac{1}{\cos^2\alpha\cdot\cos^2\beta}-\frac{tg^2\,\alpha}{\cos^2\beta}-\frac{\sin^2\beta}{\cos^2\beta} \Rightarrow A=\frac{1-\cos^2\alpha\cdot tg^2\,\alpha-\cos^2\alpha\cdot\sin^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow$$

$$\Rightarrow A=\frac{1-\cos^2\alpha\cdot\left(\frac{\sin\alpha}{\cos\alpha}\right)^2-\cos^2\alpha\cdot\sin^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow A=\frac{1-\cos^2\alpha\cdot\frac{\sin^2\alpha}{\cos^2\alpha}-\cos^2\alpha\cdot\sin^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow$$

$$\Rightarrow A=\frac{1-\cos^2\alpha\cdot\frac{\sin^2\alpha}{\cos^2\alpha}-\cos^2\alpha\cdot\sin^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow A=\frac{1-\sin^2\alpha-\cos^2\alpha\cdot\sin^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow$$

$$\Rightarrow A=\frac{\left(1-\sin^2\alpha\right)-\cos^2\alpha\cdot\sin^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow A=\frac{\cos^2\alpha-\cos^2\alpha\cdot\sin^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow A=\frac{\cos^2\alpha\cdot\left(1-\sin^2\beta\right)}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow$$

$$\Rightarrow A=\frac{\cos^2\alpha\cdot\cos^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow A=\frac{\cos^2\alpha\cdot\cos^2\beta}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow A=1.$$

Odgovor je pod A.

2.inačica

Izračunamo x^2, y^2 i z^2.

- $$x=\frac{1}{\cos\alpha\cdot\cos\beta} \Rightarrow x=\frac{1}{\cos\alpha\cdot\cos\beta}\ /^2 \Rightarrow x^2=\left(\frac{1}{\cos\alpha\cdot\cos\beta}\right)^2 \Rightarrow x^2=\frac{1^2}{(\cos\alpha\cdot\cos\beta)^2} \Rightarrow$$
$$\Rightarrow x^2=\frac{1}{\cos^2\alpha\cdot\cos^2\beta} \Rightarrow x^2=\frac{1}{\cos^2\alpha}\cdot\frac{1}{\cos^2\beta} \Rightarrow x^2=\left(1+tg^2\,\alpha\right)\cdot\left(1+tg^2\,\beta\right).$$
- $$y=\frac{tg\,\alpha}{\cos\beta} \Rightarrow y=\frac{tg\,\alpha}{\cos\beta}\ /^2 \Rightarrow y^2=\left(\frac{tg\,\alpha}{\cos\beta}\right)^2 \Rightarrow y^2=\frac{tg^2\,\alpha}{\cos^2\beta} \Rightarrow$$
$$\Rightarrow y^2=tg^2\,\alpha\cdot\frac{1}{\cos^2\beta} \Rightarrow y^2=tg^2\,\alpha\cdot\left(1+tg^2\,\beta\right).$$
- $$z=tg\,\beta \Rightarrow z=tg\,\beta\ /^2 \Rightarrow z^2=tg^2\,\beta.$$

Sada je:

$$A=x^2-y^2-z^2 \Rightarrow A=\left(1+tg^2\alpha\right)\cdot\left(1+tg^2\beta\right)-tg^2\alpha\cdot\left(1+tg^2\beta\right)-tg^2\beta \Rightarrow$$

$$\Rightarrow A=1+tg^2\beta+tg^2\alpha+tg^2\alpha\cdot tg^2\beta-tg^2\alpha-tg^2\alpha\cdot tg^2\beta-tg^2\beta \Rightarrow$$

$$\Rightarrow A=1+tg^2\beta+tg^2\alpha+tg^2\alpha\cdot tg^2\beta-tg^2\alpha-tg^2\alpha\cdot tg^2\beta-tg^2\beta \Rightarrow A=1.$$

Odgovor je pod A.

Vježba 239

Ako je $x=\frac{\cos\alpha}{\cos\beta},\ y=\cos\alpha\cdot tg\,\beta,\ z=\sin\alpha,$ izračunajte vrijednost izraza $A=x^2-y^2+z^2.$

A. 1 *B.* 0 *C.* $\sin\alpha$ *D.* $\cos\beta$

Rezultat: A.

Zadatak 240 (Đurđica, srednja škola)

Pojednostavnite: $\frac{\cos^4 x-2\cdot\sin x\cdot\cos x-\sin^4 x}{1-tg\,2x}.$

Rješenje 240

Ponovimo!

$$tg\,x=\frac{\sin x}{\cos x}\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^2-b^2=(a+b)\cdot(a-b)\quad,\quad\cos^2 x+\sin^2 x=1.$$

$$\cos 2x=\cos^2 x-\sin^2 x\quad,\quad n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad\sin 2x=2\cdot\sin x\cdot\cos x.$$

$$\frac{\cos^4 x-2\cdot\sin x\cdot\cos x-\sin^4 x}{1-tg\,2x}=\frac{\cos^4 x-\sin^4 x-2\cdot\sin x\cdot\cos x}{1-\frac{\sin 2x}{\cos 2x}}=$$

$$=\frac{\left(\cos^2 x\right)^2-\left(\sin^2 x\right)^2-\sin 2x}{\frac{1}{1}-\frac{\sin 2x}{\cos 2x}}=\frac{\left(\cos^2 x+\sin^2 x\right)\cdot\left(\cos^2 x-\sin^2 x\right)-\sin 2x}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=$$

$$=\frac{1\cdot\left(\cos^2 x-\sin^2 x\right)-\sin 2x}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=\frac{\cos^2 x-\sin^2 x-\sin 2x}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=\frac{\cos 2x-\sin 2x}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=\frac{\frac{\cos 2x-\sin 2x}{1}}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=$$

$$=\frac{\frac{\cos 2x-\sin 2x}{1}}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=\frac{\frac{1}{1}}{\frac{1}{\cos 2x}}=\cos 2x.$$

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Pojednostavnite: $\dfrac{1-tg\,2x}{\cos^4 x-2\cdot\sin x\cdot\cos x-\sin^4 x}$.

Rezultat: $\dfrac{1}{\cos 2x}$.