Zadatak 261 (Tin, srednja škola)

Ako je $tg\,x + ctg\,x = 3$, koliko je $\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$?

Rješenje 261

Ponovimo!

$$tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} \quad , \quad ctg\,\alpha = \frac{\cos\alpha}{\sin\alpha} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \cos^2\alpha + \sin^2\alpha = 1.$$

$$(a \cdot b)^n = a^n \cdot b^n \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad \frac{a+b}{n} = \frac{a}{n} + \frac{b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

Transformiramo zadanu jednakost.

$$tg\,x + ctg\,x = 3 \Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = 3 \Rightarrow \frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x} = 3 \Rightarrow \frac{1}{\sin x \cdot \cos x} = 3.$$

Sada je

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} = \frac{1}{\sin^2 x \cdot \cos^2 x} = \left(\frac{1}{\sin x \cdot \cos x}\right)^2 = \left[\frac{1}{\sin x \cdot \cos x} = 3\right] = 3^2 = 9.$$

2.inačica

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} = \frac{1}{\sin^2 x \cdot \cos^2 x} = \left(\frac{1}{\sin x \cdot \cos x}\right)^2 = \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x}\right)^2 =$$

$$= \left(\frac{\sin^2 x}{\sin x \cdot \cos x} + \frac{\cos^2 x}{\sin x \cdot \cos x}\right)^2 = \left(\frac{\sin^2 x}{\sin x \cdot \cos x} + \frac{\cos^2 x}{\sin x \cdot \cos x}\right)^2 = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)^2 =$$

$$= (tg\,x + ctg\,x)^2 = [tg\,x + ctg\,x = 3] = 3^2 = 9.$$

Vježba 261

Ako je $tg\,x + ctg\,x = 5$, koliko je $\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$?

Rezultat: 25.

Zadatak 262 (Mirna, gimnazija)

Pojednostavnite izraz $A = \frac{\sin(2x+y) - \sin(2x-y) + \sin y}{\cos(2x+y) + \cos(2x-y) + \cos y}$,

pri čemu je $\cos y \neq 0$, $\cos 2 \cdot x \neq -\frac{1}{2}$.

Rješenje 262

Ponovimo!

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta \quad , \quad \sin(\alpha - \beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta.$$

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta \quad , \quad \cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta.$$

$$tg\,\alpha=\frac{\sin\alpha}{\cos\alpha} \quad , \quad ctg\,\alpha=\frac{\cos\alpha}{\sin\alpha}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$A=\frac{\sin(2\cdot x+y)-\sin(2\cdot x-y)+\sin y}{\cos(2\cdot x+y)+\cos(2\cdot x-y)+\cos y} \Rightarrow$$

$$\Rightarrow A=\frac{\sin 2x\cdot\cos y+\cos 2x\cdot\sin y-(\sin 2x\cdot\cos y-\cos 2x\cdot\sin y)+\sin y}{\cos 2x\cdot\cos y-\sin 2x\cdot\sin y+\cos 2x\cdot\cos y+\sin 2x\cdot\sin y+\cos y} \Rightarrow$$

$$\Rightarrow A=\frac{\sin 2x\cdot\cos y+\cos 2x\cdot\sin y-\sin 2x\cdot\cos y+\cos 2x\cdot\sin y+\sin y}{\cos 2x\cdot\cos y-\sin 2x\cdot\sin y+\cos 2x\cdot\cos y+\sin 2x\cdot\sin y+\cos y} \Rightarrow$$

$$\Rightarrow A=\frac{\sin 2x\cdot\cos y+\cos 2x\cdot\sin y-\sin 2x\cdot\cos y+\cos 2x\cdot\sin y+\sin y}{\cos 2x\cdot\cos y-\sin 2x\cdot\sin y+\cos 2x\cdot\cos y+\sin 2x\cdot\sin y+\cos y} \Rightarrow$$

$$\Rightarrow A=\frac{\cos 2x\cdot\sin y+\cos 2x\cdot\sin y+\sin y}{\cos 2x\cdot\cos y+\cos 2x\cdot\cos y+\cos y} \Rightarrow A=\frac{2\cdot\cos 2x\cdot\sin y+\sin y}{2\cdot\cos 2x\cdot\cos y+\cos y} \Rightarrow$$

$$\Rightarrow A=\frac{\sin y\cdot(2\cdot\cos 2x+1)}{\cos y\cdot(2\cdot\cos 2x+1)} \Rightarrow A=\frac{\sin y\cdot(2\cdot\cos 2x+1)}{\cos y\cdot(2\cdot\cos 2x+1)} \Rightarrow A=\frac{\sin y}{\cos y} \Rightarrow A=tg\,y.$$

Vježba 262

Pojednostavnite izraz $A=\frac{\cos(2\cdot x+y)+\cos(2\cdot x-y)+\cos y}{\sin(2\cdot x+y)-\sin(2\cdot x-y)+\sin y}$,

pri čemu je $\sin y\neq 0$, $\cos 2\cdot x\neq -\frac{1}{2}$.

Rezultat: $A=ctg\,y.$

Zadatak 263 (Mirna, gimnazija)

Ako je $m=tg\,x+\sin x$ i $n=tg\,x-\sin x$, koliko je $A=\frac{m-n}{m+n}$?

Rješenje 263

Ponovimo!

$$tg\,\alpha\cdot ctg\,\alpha=1 \quad , \quad ctg\,\alpha=\frac{1}{tg\,\alpha} \quad , \quad tg\,\alpha=\frac{\sin\alpha}{\cos\alpha} \quad , \quad ctg\,\alpha=\frac{\cos\alpha}{\sin\alpha} \quad , \quad n=\frac{n}{1}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$A=\frac{m-n}{m+n}\Rightarrow A=\frac{tg\ x+\sin x-(tg\ x-\sin x)}{tg\ x+\sin x+tg\ x-\sin x}\Rightarrow A=\frac{tg\ x+\sin x-tg\ x+\sin x}{tg\ x+\sin x+tg\ x-\sin x}\Rightarrow$$

$$\Rightarrow A=\frac{tg\ x+\sin x-tg\ x+\sin x}{tg\ x+\sin x+tg\ x-\sin x}\Rightarrow A=\frac{\sin x+\sin x}{tg\ x+tg\ x}\Rightarrow A=\frac{2\cdot\sin x}{2\cdot tg\ x}\Rightarrow A=\frac{2\cdot\sin x}{2\cdot tg\ x}\Rightarrow$$

$$\Rightarrow A=\frac{\sin x}{tg\ x}\Rightarrow\begin{bmatrix}\text{može na}\\ \text{dva načina}\end{bmatrix}\Rightarrow$$

- $$\Rightarrow A=\sin x\cdot ctg\ x\Rightarrow A=\sin x\cdot\frac{\cos x}{\sin x}\Rightarrow A=\frac{\sin x}{1}\cdot\frac{\cos x}{\sin x}\Rightarrow$$

$$\Rightarrow A=\frac{\sin x}{1}\cdot\frac{\cos x}{\sin x}\Rightarrow A=\cos x.$$

- $$\Rightarrow A=\frac{\frac{\sin x}{1}}{\frac{\sin x}{\cos x}}\Rightarrow A=\frac{\sin x\cdot\cos x}{\sin x}\Rightarrow A=\frac{\sin x\cdot\cos x}{\sin x}\Rightarrow A=\cos x.$$

Vježba 263

Ako je $m=tg\ x+\sin x$ i $n=tg\ x-\sin x$, koliko je $A=\frac{m+n}{m-n}$?

Rezultat: $A=\frac{1}{\cos x}.$

Zadatak 264 (Matija, gimnazija)

Bez uporabe računala izračunaj vrijednost izraza $\frac{\sin x+\sin 3x+\sin 5x}{\cos x+\cos 3x+\cos 5x}$ za $x=15^0$.

Rješenje 264

Ponovimo!

$$tg\ x=\frac{\sin x}{\cos x}\quad,\quad tg\ 45^0=1\quad,\quad cg\ x=\frac{\cos x}{\sin x}\quad,\quad ctg\ 45^0=1.$$

Za zbrajanje realnih brojeva vrijede ova svojstva:

- komutativnost (zamjena)

$$a+b=b+a.$$

Zbroj se ne mijenja ako pribrojnici zamijene svoja mjesta.

- asocijativnost (udruživanje)

$$(a+b)+c=a+(b+c).$$

Zbroj se ne mijenja ako pribrojnike udružimo na bilo koji način.

Formule pretvorbe zbroja u umnožak

$$\sin x+\sin y=2\cdot\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2}\quad,\quad\cos x+\cos y=2\cdot\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}.$$

Parnost funkcije

Za funkciju $f : D_f \to R$ kažemo da je parna ako za svaki $x \in D_f$ vrijedi

$$f(-x) = f(x).$$

Parnost kosinusa

Za svaki realni broj x vrijedi

$$\cos(-x) = \cos x,$$

tj. kosinus je parna funkcija.

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\frac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \frac{(\sin x + \sin 5x) + \sin 3x}{(\cos x + \cos 5x) + \cos 3x} = \frac{2 \cdot \sin \frac{x + 5 \cdot x}{2} \cdot \cos \frac{x - 5 \cdot x}{2} + \sin 3x}{2 \cdot \cos \frac{x + 5 \cdot x}{2} \cdot \cos \frac{x - 5 \cdot x}{2} + \cos 3x} =$$

$$= \frac{2 \cdot \sin \frac{6 \cdot x}{2} \cdot \cos \frac{-4 \cdot x}{2} + \sin 3x}{2 \cdot \cos \frac{6 \cdot x}{2} \cdot \cos \frac{-4 \cdot x}{2} + \cos 3x} = \frac{2 \cdot \sin \frac{6 \cdot x}{2} \cdot \cos \frac{-4 \cdot x}{2} + \sin 3x}{2 \cdot \cos \frac{6 \cdot x}{2} \cdot \cos \frac{-4 \cdot x}{2} + \cos 3x} = \frac{2 \cdot \sin 3x \cdot \cos(-2x) + \sin 3x}{2 \cdot \cos 3x \cdot \cos(-2x) + \cos 3x} =$$

$$= \frac{2 \cdot \sin 3x \cdot \cos 2x + \sin 3x}{2 \cdot \cos 3x \cdot \cos 2x + \cos 3x} = \frac{\sin 3x \cdot (2 \cdot \cos 2x + 1)}{\cos 3x \cdot (2 \cdot \cos 2x + 1)} = \frac{\sin 3x \cdot (2 \cdot \cos 2x + 1)}{\cos 3x \cdot (2 \cdot \cos 2x + 1)} = \frac{\sin 3x}{\cos 3x} =$$

$$= tg\, 3x = \left[x = 15^0 \right] = tg\left(3 \cdot 15^0\right) = tg\, 45^0 = 1.$$

Vježba 264

Bez uporabe računala izračunaj vrijednost izraza $\frac{\cos x + \cos 3x + \cos 5x}{\sin x + \sin 3x + \sin 5x}$ za $x = 15^0$.

Rezultat: 1.

Zadatak 265 (Ana, gimnazija)

Izračunaj: $\frac{\cos x \cdot \sin 2x}{\sin x - \sin x \cdot \cos 2x}$.

$A.\ tg^2 x \qquad B.\ \cos^2 x \qquad C.\ ctg^2 x \qquad D.\ \sin^2 x$

Rješenje 265

Ponovimo!

$$\sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha \quad , \quad \cos^2 \alpha + \sin^2 \alpha = 1 \quad , \quad \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \quad , \quad ctg\, \alpha = \frac{\cos \alpha}{\sin \alpha}.$$

$$tg\, \alpha = \frac{\sin \alpha}{\cos \alpha} \quad , \quad 1 - \cos 2\alpha = 2 \cdot \sin^2 \alpha \quad , \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

1.inačica

$$\frac{\cos x \cdot \sin 2x}{\sin x - \sin x \cdot \cos 2x} = \frac{\cos x \cdot 2 \cdot \sin x \cdot \cos x}{\sin x \cdot (1 - \cos 2x)} = \frac{\cos x \cdot 2 \cdot \sin x \cdot \cos x}{\sin x \cdot (1 - \cos 2x)} = \frac{\cos x \cdot 2 \cdot \cos x}{1 - \cos 2x} =$$

$$= \frac{2 \cdot \cos^2 x}{2 \cdot \sin^2 x} = \frac{2 \cdot \cos^2 x}{2 \cdot \sin^2 x} = \frac{\cos^2 x}{\sin^2 x} = \left(\frac{\cos x}{\sin x}\right)^2 = ctg^2 x.$$

Odgovor je pod C.

2.inačica

$$\frac{\cos x \cdot \sin 2x}{\sin x - \sin x \cdot \cos 2x} = \frac{\cos x \cdot 2 \cdot \sin x \cdot \cos x}{\sin x \cdot (1 - \cos 2x)} = \frac{\cos x \cdot 2 \cdot \sin x \cdot \cos x}{\sin x \cdot (1 - \cos 2x)} = \frac{\cos x \cdot 2 \cdot \cos x}{1 - \cos 2x} =$$

$$= \frac{2 \cdot \cos^2 x}{\cos^2 x + \sin^2 x - \left(\cos^2 x - \sin^2 x\right)} = \frac{2 \cdot \cos^2 x}{\cos^2 x + \sin^2 x - \cos^2 x + \sin^2 x} =$$

$$= \frac{2 \cdot \cos^2 x}{\cos^2 x + \sin^2 x - \cos^2 x + \sin^2 x} = \frac{2 \cdot \cos^2 x}{\sin^2 x + \sin^2 x} = \frac{2 \cdot \cos^2 x}{2 \cdot \sin^2 x} = \frac{2 \cdot \cos^2 x}{2 \cdot \sin^2 x} = \frac{\cos^2 x}{\sin^2 x} = \left(\frac{\cos x}{\sin x}\right)^2 = ctg^2 x.$$

Odgovor je pod C.

Vježba 265

Izračunaj: $\dfrac{\sin x - \sin x \cdot \cos 2x}{\cos x - \sin 2x}$.

$A.\ tg^2 x \qquad B.\ \cos^2 x \qquad C.\ ctg^2 x \qquad D.\ \sin^2 x$

Rezultat: A.

Zadatak 266 (Goga, gimnazija)

Ako je $f(x - \pi) = 2 \cdot \cos \frac{x}{2}$, onda je $f(2 \cdot x)$ jednako:

$A.\ 2 \cdot \cos(\pi - x) \qquad B.\ 2 \cdot \sin x \qquad C.\ -2 \cdot \sin x \qquad D.\ 2 \cdot \cos(x + \pi)$

Rješenje 266

Ponovimo!

$$\frac{a+b}{n} = \frac{a}{n} + \frac{b}{n} \quad , \quad \cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y \quad , \quad \cos^2 \alpha + \sin^2 \alpha = 1.$$

$$\cos \frac{\pi}{2} = 0 \quad , \quad \sin \frac{\pi}{2} = 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

Najprije odredimo funkciju f.

$$f(x - \pi) = 2 \cdot \cos \frac{x}{2} \Rightarrow \left[\begin{array}{c} \text{supstitucija} \\ x - \pi = y \\ x = y + \pi \end{array}\right] \Rightarrow f(y) = 2 \cdot \cos \frac{y + \pi}{2} \Rightarrow f(y) = 2 \cdot \cos\left(\frac{y}{2} + \frac{\pi}{2}\right) \Rightarrow$$

$$\Rightarrow f(y)=2\cdot\left(\cos\frac{y}{2}\cdot\cos\frac{\pi}{2}-\sin\frac{y}{2}\cdot\sin\frac{\pi}{2}\right) \Rightarrow f(y)=2\cdot\left(\cos\frac{y}{2}\cdot 0-\sin\frac{y}{2}\cdot 1\right) \Rightarrow$$

$$\Rightarrow f(y)=2\cdot\left(0-\sin\frac{y}{2}\right) \Rightarrow f(y)=-2\cdot\sin\frac{y}{2}.$$

Funkcija f zadana je izrazom

$$f(x)=-2\cdot\sin\frac{x}{2}$$

pa je tada

$$f(2\cdot x)=-2\cdot\sin\frac{2\cdot x}{2} \Rightarrow f(2\cdot x)=-2\cdot\sin\frac{2\cdot x}{2} \Rightarrow f(2\cdot x)=-2\cdot\sin x.$$

Odgovor je pod C.

Vježba 266

Ako je $f(x+\pi)=2\cdot\cos\frac{x}{2}$, onda je $f(2\cdot x)$ jednako:

$A.\ 2\cdot\cos(\pi-x)$ $\quad B.\ 2\cdot\sin x$ $\quad C.\ -2\cdot\sin x$ $\quad D.\ 2\cdot\cos(x+\pi)$

Rezultat: B.

Zadatak 267 (Goga, gimnazija)

Izraz $\dfrac{\sin^2 x\cdot\cos x+\cos^3 x-\sin x\cdot\cos x}{\cos^2 x}$ jednak je:

$A.\ \sin x$ $\quad B.\ \dfrac{1-\sin x}{\cos x}$ $\quad C.\ tg\ x$ $\quad D.\ 1-\cos x$

Rješenje 267

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{\sin^2 x\cdot\cos x+\cos^3 x-\sin x\cdot\cos x}{\cos^2 x}=\frac{\cos x\cdot\left(\sin^2 x+\cos^2 x-\sin x\right)}{\cos^2 x}=$$

$$=\frac{\cos x\cdot\left(\left(\sin^2 x+\cos^2 x\right)-\sin x\right)}{\cos^2 x}=\frac{\cos x\cdot(1-\sin x)}{\cos^2 x}=\frac{\cos x\cdot(1-\sin x)}{\cos^2 x}=\frac{1-\sin x}{\cos x}.$$

Odgovor je pod B.

Vježba 267

Izraz $\dfrac{\cos^2 x\cdot\sin x+\sin^3 x-\sin x\cdot\cos x}{\sin x}$ jednak je:

$$A.\ \sin x \qquad B.\ \frac{1-\sin x}{\cos x} \qquad C.\ tg\ x \qquad D.\ 1-\cos x$$

Rezultat: D.

Zadatak 268 (4A, 4B, TUPŠ)

Mjera kuta je 162°. Koliko je to radijana?

$$A.\ \frac{9\cdot\pi}{10} \qquad B.\ \frac{10\cdot\pi}{9} \qquad C.\ \frac{9\cdot\pi}{20} \qquad D.\ \frac{20\cdot\pi}{9}$$

Rješenje 268

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Kut s mjerom od α° stupnjeva ima

$$\alpha_r=\frac{\pi}{180^0}\cdot\alpha^0$$

radijana.

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left.\begin{array}{l}\alpha^0=162^0\\ \alpha_r=\dfrac{\pi}{180^0}\cdot\alpha^0\end{array}\right\}\Rightarrow \alpha_r=\frac{\pi}{180^0}\cdot 162^0 \Rightarrow \alpha_r=\frac{\pi}{180^0}\cdot\frac{162^0}{1} \Rightarrow \alpha_r=\frac{\pi}{180^0}\cdot\frac{162^0}{1} \Rightarrow$$

$$\Rightarrow \alpha_r=\frac{\pi}{10}\cdot\frac{9}{1} \Rightarrow \alpha_r=\frac{9\cdot\pi}{10}.$$

Odgovor je pod A.

Vježba 268

Mjera kuta je 81°. Koliko je to radijana?

$$A.\ \frac{9\cdot\pi}{10} \qquad B.\ \frac{10\cdot\pi}{9} \qquad C.\ \frac{9\cdot\pi}{20} \qquad D.\ \frac{20\cdot\pi}{9}$$

Rezultat: C.

Zadatak 269 (4A, 4B, TUPŠ)

Mjera kuta je $\frac{7\cdot\pi}{10}$ radijana. Koliko je to stupnjeva?

$$A.\ 21^0 \qquad B.\ 63^0 \qquad C.\ 94^0 \qquad D.\ 126^0$$

Rješenje 269

Ponovimo!

$$n=\frac{n}{1}.$$

Kut s mjerom od α_r radijana ima

$$\alpha^0=\frac{180^0}{\pi}\cdot\alpha_r$$

stupnjeva.

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left.\begin{array}{l}\alpha_r=\dfrac{7\cdot\pi}{10}\\[2ex] \alpha^0=\dfrac{180^0}{\pi}\cdot\alpha_r\end{array}\right\}\Rightarrow \alpha^0=\frac{180^0}{\pi}\cdot\frac{7\cdot\pi}{10}\Rightarrow \alpha^0=\frac{180^0}{\pi}\cdot\frac{7\cdot\pi}{10}\Rightarrow \alpha^0=\frac{180^0}{1}\cdot\frac{7}{10}\Rightarrow$$

$$\Rightarrow \alpha^0=\frac{180^0}{1}\cdot\frac{7}{10}\Rightarrow \alpha^0=\frac{18^0}{1}\cdot\frac{7}{1}\Rightarrow \alpha^0=18^0\cdot 7\Rightarrow \alpha^0=126^0.$$

Odgovor je pod D.

Vježba 269

Mjera kuta je $\frac{7\cdot\pi}{20}$ radijana. Koliko je to stupnjeva?

$$A.\ 21^0 \qquad B.\ 63^0 \qquad C.\ 94^0 \qquad D.\ 126^0$$

Rezultat: B.

Zadatak 270 (Ivana, gimnazija)

Pojednostavni $\dfrac{ctg^2 x\cdot\cos^2 x}{ctg^2 x-\cos^2 x}$.

$$A.\ ctg\, x \qquad B.\ 1 \qquad C.\ -1 \qquad D.\ \sin x$$

Rješenje 270

Ponovimo!

$$ctg\, x=\frac{\cos x}{\sin x}\ ,\ \left(\frac{a}{b}\right)^2=\frac{a^2}{b^2}\ ,\ n=\frac{n}{1}\ ,\ \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\ ,\ \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\cos^2 x+\sin^2 x=1\ ,\ a^n\cdot a^m=a^{n+m}\ ,\ \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\ ,\ a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{ctg^2 x\cdot\cos^2 x}{ctg^2 x-\cos^2 x}=\frac{\frac{\cos^2 x}{\sin^2 x}\cdot\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}-\cos^2 x}=\frac{\frac{\cos^2 x}{\sin^2 x}\cdot\frac{\cos^2 x}{1}}{\frac{\cos^2 x}{\sin^2 x}-\frac{\cos^2 x}{1}}=\frac{\frac{\cos^4 x}{\sin^2 x}}{\frac{\cos^2 x-\cos^2 x\cdot\sin^2 x}{\sin^2 x}}=$$

$$=\frac{\frac{\cos^4 x}{\sin^2 x}}{\frac{\cos^2 x-\cos^2 x\cdot\sin^2 x}{\sin^2 x}}=\frac{\frac{\cos^4 x}{1}}{\frac{\cos^2 x-\cos^2 x\cdot\sin^2 x}{1}}=\frac{\cos^4 x}{\cos^2 x-\cos^2 x\cdot\sin^2 x}=\frac{\cos^4 x}{\cos^2 x\cdot\left(1-\sin^2 x\right)}=$$

$$=\frac{\cos^4 x}{\cos^2 x\cdot\left(1-\sin^2 x\right)}=\frac{\cos^2 x}{1-\sin^2 x}=\frac{\cos^2 x}{\cos^2 x}=\frac{\cos^2 x}{\cos^2 x}=1.$$

Odgovor je pod B.

Vježba 270

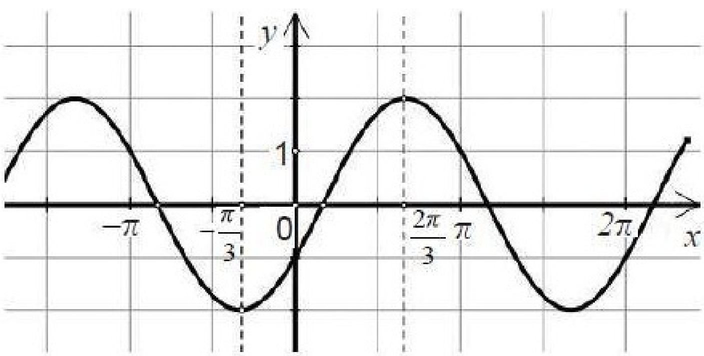
Pojednostavni $\frac{ctg^2 x-\cos^2 x}{ctg^2 x\cdot\cos^2 x}$.

$A.\ ctg\,x \qquad B.\ 1 \qquad C.\ -1 \qquad D.\ \sin x$

Rezultat: B.

Zadatak 271 (Mihaela, gimnazija)

Grafom je zadana funkcija $f(x)=A\cdot\sin(x+C)$. Odredite A i C.



Rješenje 271

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \sin\frac{\pi}{2}=1.$$

Harmonijska funkcija je funkcija oblika

$$f(x)=A\cdot\sin(B\cdot x+C).$$

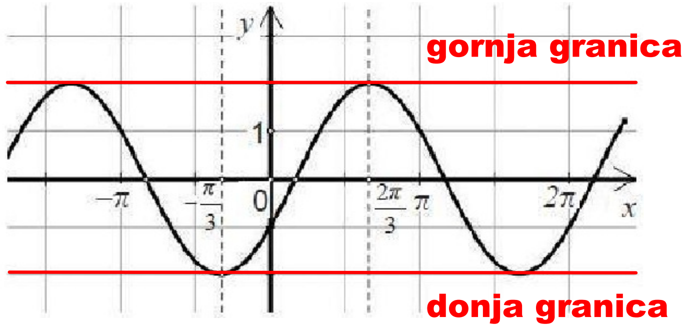
Koeficijent A zove se amplituda, B se naziva kružna frekvencija, a C fazni pomak.

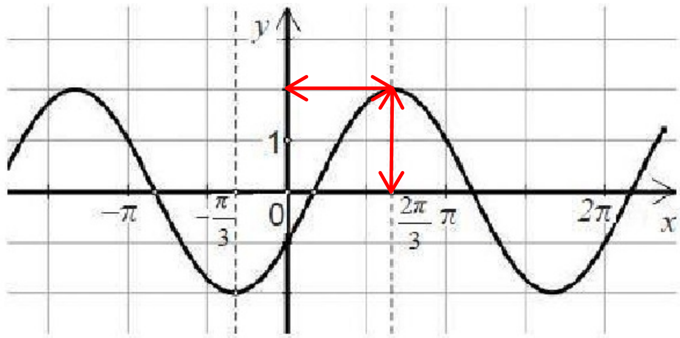
Trigonometrijska jednadžba $\sin x=a \ , \ |a|\le 1$

Skup rješenja jednadžbe $\sin x=a,\ |a|\le 1,$ je $\left\{x_0+k\cdot 2\cdot\pi : k\in Z\right\}\cup\left\{\pi-x_0+k\cdot 2\cdot\pi : k\in Z\right\}$

gdje je $x_0\in R$ jedno rješenje te jednadžbe.

$$\sin x=\sin x_0 \ \Rightarrow \ x=x_0.$$





Sa slike vidi se da je funkcija omeđena. Po y osi njezine vrijednosti variraju između – 2 i 2 pa je amplituda (maksimalna udaljenost od položaja ravnoteže) A = 2. Budući da u točki $\frac{2\cdot\pi}{3}$ na x osi zadana funkcija postiže maksimalni iznos 2, slijedi:

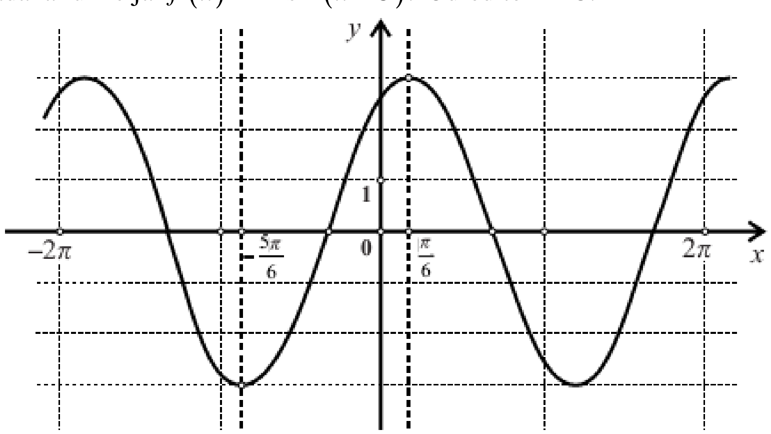
$$\left.\begin{array}{l} A=2 \;\;,\;\; f\left(\frac{2\cdot\pi}{3}\right)=2 \\ f(x)=A\cdot\sin(x+C) \end{array}\right\} \Rightarrow \left.\begin{array}{l} f\left(\frac{2\cdot\pi}{3}\right)=2 \\ f(x)=2\cdot\sin(x+C) \end{array}\right\} \Rightarrow \left.\begin{array}{l} f\left(\frac{2\cdot\pi}{3}\right)=2 \\ f\left(\frac{2\cdot\pi}{3}\right)=2\cdot\sin\left(\frac{2\cdot\pi}{3}+C\right) \end{array}\right\} \Rightarrow$$

$$\Rightarrow 2\cdot\sin\left(\frac{2\cdot\pi}{3}+C\right)=2 \Rightarrow 2\cdot\sin\left(\frac{2\cdot\pi}{3}+C\right)=2 \;/:2 \Rightarrow \sin\left(\frac{2\cdot\pi}{3}+C\right)=1 \Rightarrow$$

$$\Rightarrow \sin\left(\frac{2\cdot\pi}{3}+C\right)=\sin\frac{\pi}{2} \Rightarrow \frac{2\cdot\pi}{3}+C=\frac{\pi}{2} \Rightarrow C=\frac{\pi}{2}-\frac{2\cdot\pi}{3} \Rightarrow C=\frac{3\cdot\pi-4\cdot\pi}{6} \Rightarrow C=-\frac{\pi}{6}.$$

Vježba 271

Grafom je zadana funkcija $f(x)=A\cdot\sin(x+C)$. Odredite A i C.



Rezultat: $A = 3\ ,\ C = \frac{\pi}{3}.$

Zadatak 272 (Valentina, gimnazija)

Izračunaj $\sin\left(x + \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right).$

Rješenje 272

Ponovimo!

$$\sin\frac{\pi}{6} = \frac{1}{2}\quad,\quad \sin\alpha - \sin\beta = 2\cdot\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}.$$

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta\quad,\quad \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c\quad,\quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b}\ ,\ n \neq 0\ ,\ n \neq 1.$$

1.inačica

$$\sin\left(x+\frac{\pi}{6}\right) - \sin\left(x-\frac{\pi}{6}\right) = \sin x\cdot\cos\frac{\pi}{6} + \cos x\cdot\sin\frac{\pi}{6} - \left(\sin x\cdot\cos\frac{\pi}{6} - \cos x\cdot\sin\frac{\pi}{6}\right) =$$

$$= \sin x\cdot\cos\frac{\pi}{6} + \cos x\cdot\sin\frac{\pi}{6} - \sin x\cdot\cos\frac{\pi}{6} + \cos x\cdot\sin\frac{\pi}{6} =$$

$$= \sin x\cdot\cos\frac{\pi}{6} + \cos x\cdot\sin\frac{\pi}{6} - \sin x\cdot\cos\frac{\pi}{6} + \cos x\cdot\sin\frac{\pi}{6} = \cos x\cdot\sin\frac{\pi}{6} + \cos x\cdot\sin\frac{\pi}{6} =$$

$$= 2\cdot\cos x\cdot\sin\frac{\pi}{6} = 2\cdot\cos x\cdot\frac{1}{2} = 2\cdot\cos x\cdot\frac{1}{2} = \cos x.$$

2.inačica

$$\sin\left(x+\frac{\pi}{6}\right) - \sin\left(x-\frac{\pi}{6}\right) = 2\cdot\cos\frac{\left(x+\frac{\pi}{6}\right)+\left(x-\frac{\pi}{6}\right)}{2}\cdot\sin\frac{\left(x+\frac{\pi}{6}\right)-\left(x-\frac{\pi}{6}\right)}{2} =$$

$$= 2\cdot\cos\frac{x+\frac{\pi}{6}+x-\frac{\pi}{6}}{2}\cdot\sin\frac{x+\frac{\pi}{6}-x+\frac{\pi}{6}}{2} = 2\cdot\cos\frac{x+\frac{\pi}{6}+x-\frac{\pi}{6}}{2}\cdot\sin\frac{x+\frac{\pi}{6}-x+\frac{\pi}{6}}{2} =$$

$$= 2\cdot\cos\frac{x+x}{2}\cdot\sin\frac{\frac{\pi}{6}+\frac{\pi}{6}}{2} = 2\cdot\cos\frac{2\cdot x}{2}\cdot\sin\frac{2\cdot\frac{\pi}{6}}{2} = 2\cdot\cos\frac{2\cdot x}{2}\cdot\sin\frac{2\cdot\frac{\pi}{6}}{2} =$$

$$= 2\cdot\cos x\cdot\sin\frac{\pi}{6} = 2\cdot\cos x\cdot\frac{1}{2} = 2\cdot\cos x\cdot\frac{1}{2} = \cos x.$$

Vježba 272

Izračunaj $\sin\left(x + \frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6} - x\right).$

Rezultat: $\cos x.$

Zadatak 273 (Matea, gimnazija)

Broj $\frac{\pi}{8}$ rješenje je jednadžbe $\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=a$ ako je:

$A.\ a=8 \qquad B.\ a=6 \qquad C.\ a=4 \qquad D.\ a=2$

Rješenje 273

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \cos^2\alpha+\sin^2\alpha=1 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha.$$

$$n=\frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad \sin\frac{\pi}{4}=\frac{\sqrt{2}}{2} \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad \left(\sqrt{a}\right)^2=a.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \ , \ n\neq 0 \ , \ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=a \Rightarrow \frac{\cos^2 x+\sin^2 x}{\cos^2 x\cdot\sin^2 x}=a \Rightarrow \frac{1}{\cos^2 x\cdot\sin^2 x}=a \Rightarrow \left[\begin{array}{l}\text{proširimo}\\ \text{razlomak s } 4\end{array}\right] \Rightarrow$$

$$\Rightarrow \frac{4}{4\cdot\cos^2 x\cdot\sin^2 x}=a \Rightarrow \frac{4}{(2\cdot\cos x\cdot\sin x)^2}=a \Rightarrow \frac{4}{(\sin 2x)^2}=a \Rightarrow \frac{4}{\left(\sin 2\cdot\frac{\pi}{8}\right)^2}=a \Rightarrow$$

$$\Rightarrow \frac{4}{\left(\sin 2\cdot\frac{\pi}{8}\right)^2}=a \Rightarrow \frac{4}{\left(\sin\frac{\pi}{4}\right)^2}=a \Rightarrow \frac{4}{\left(\frac{\sqrt{2}}{2}\right)^2}=a \Rightarrow \frac{4}{\frac{\left(\sqrt{2}\right)^2}{2^2}}=a \Rightarrow \frac{4}{\frac{2}{4}}=a \Rightarrow$$

$$\Rightarrow \frac{\frac{4}{1}}{\frac{2}{4}}=a \Rightarrow \frac{16}{2}=a \Rightarrow a=8.$$

Odgovor je pod A.

Vježba 273

Broj $\frac{\pi}{4}$ rješenje je jednadžbe $\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=a$ ako je:

$A.\ a=8 \qquad B.\ a=6 \qquad C.\ a=4 \qquad D.\ a=2$

Rezultat: C.

Zadatak 274 (Matea, gimnazija)

Ako je $\sin x+\cos x=a$, tada je $\sin^4 x+\cos^4 x$ jednako:

$$A.\ \frac{a^4-1}{2} \qquad B.\ \frac{a^4+1}{2} \qquad C.\ \frac{1-2\cdot a^2-a^4}{2} \qquad D.\ \frac{1+2\cdot a^2-a^4}{2}$$

Rješenje 274

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \cos^2\alpha+\sin^2\alpha=1 \quad , \quad (a\cdot b)^n=a^n\cdot b^n.$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad n=\frac{n}{1} \quad , \quad \left(a^n\right)^m=a^{n\cdot m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Najprije preoblikujemo zadanu jednakost.

$$\sin x+\cos x=a \Rightarrow \sin x+\cos x=a \ /^2 \Rightarrow (\sin x+\cos x)^2=a^2 \Rightarrow$$

$$\Rightarrow \sin^2 x+2\cdot\sin x\cdot\cos x+\cos^2 x=a^2 \Rightarrow 2\cdot\sin x\cdot\cos x=a^2-1 \Rightarrow$$

$$\Rightarrow 2\cdot\sin x\cdot\cos x=a^2-1 \ /\cdot\frac{1}{2} \Rightarrow \sin x\cdot\cos x=\frac{a^2-1}{2}.$$

Sada računamo:

$$\sin^4 x+\cos^4 x=\sin^4 x+2\cdot\sin^2 x\cdot\cos^2 x+\cos^4 x-2\cdot\sin^2 x\cdot\cos^2 x=$$

$$=\left(\sin^4 x+2\cdot\sin^2 x\cdot\cos^2 x+\cos^4 x\right)-2\cdot\sin^2 x\cdot\cos^2 x=$$

$$=\left(\left(\sin^2 x\right)^2+2\cdot\sin^2 x\cdot\cos^2 x+\left(\cos^2 x\right)^2\right)-2\cdot\sin^2 x\cdot\cos^2 x=$$

$$=\left(\sin^2 x+\cos^2 x\right)^2-2\cdot\sin^2 x\cdot\cos^2 x=1^2-2\cdot\sin^2 x\cdot\cos^2 x=1-2\cdot\sin^2 x\cdot\cos^2 x=$$

$$=1-2\cdot(\sin x\cdot\cos x)^2=\left[\sin x\cdot\cos x=\frac{a^2-1}{2}\right]=1-2\cdot\left(\frac{a^2-1}{2}\right)^2=1-2\cdot\frac{\left(a^2-1\right)^2}{2^2}=$$

$$=1-2\cdot\frac{a^4-2\cdot a^2+1}{4}=1-2\cdot\frac{a^4-2\cdot a^2+1}{4}=1-\frac{a^4-2\cdot a^2+1}{2}=\frac{1}{1}-\frac{a^4-2\cdot a^2+1}{2}=$$

$$=\frac{2-\left(a^4-2\cdot a^2+1\right)}{2}=\frac{2-a^4+2\cdot a^2-1}{2}=\frac{1+2\cdot a^2-a^4}{2}.$$

Odgovor je pod D.

2.inačica

$$\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=2\cdot\cos\frac{\left(x+\frac{\pi}{6}\right)+\left(x-\frac{\pi}{6}\right)}{2}\cdot\sin\frac{\left(x+\frac{\pi}{6}\right)-\left(x-\frac{\pi}{6}\right)}{2}=$$

$$=2\cdot\cos\frac{x+\frac{\pi}{6}+x-\frac{\pi}{6}}{2}\cdot\sin\frac{x+\frac{\pi}{6}-x+\frac{\pi}{6}}{2}=2\cdot\cos\frac{x+\frac{\pi}{6}+x-\frac{\pi}{6}}{2}\cdot\sin\frac{x+\frac{\pi}{6}-x+\frac{\pi}{6}}{2}=$$

$$=2\cdot\cos\frac{x+x}{2}\cdot\sin\frac{\frac{\pi}{6}+\frac{\pi}{6}}{2}=2\cdot\cos\frac{2\cdot x}{2}\cdot\sin\frac{2\cdot\frac{\pi}{6}}{2}=2\cdot\cos\frac{2\cdot x}{2}\cdot\sin\frac{2\cdot\frac{\pi}{6}}{2}=$$

$$=2\cdot\cos x\cdot\sin\frac{\pi}{6}=2\cdot\cos x\cdot\frac{1}{2}=2\cdot\cos x\cdot\frac{1}{2}=\cos x.$$

Vježba 274

Ako je $\sin x+\cos x=1,$ tada je $\sin^4 x+\cos^4 x$ jednako:

$A.\ 1 \qquad B.\ 2 \qquad C.\ 3 \qquad D.\ 4$

Rezultat: A.

Zadatak 275 (Ivan, tehnička škola)

Ako je $\frac{5\cdot\sin\alpha-4\cdot\cos\alpha}{7\cdot\sin\alpha-5\cdot\cos\alpha}=1,$ tada je tg α jednako:

$A.\ \frac{1}{2} \qquad B.\ -\frac{1}{2} \qquad C.\ \frac{1}{3} \qquad D.\ -\frac{1}{3}$

Rješenje 275

Ponovimo!

$$\frac{a}{b}=\frac{\frac{a}{c}}{\frac{b}{c}}\quad,\quad\frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}\quad,\quad tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}.$$

$$\frac{5\cdot\sin\alpha-4\cdot\cos\alpha}{7\cdot\sin\alpha-5\cdot\cos\alpha}=1\ \Rightarrow\ \frac{\frac{5\cdot\sin\alpha-4\cdot\cos\alpha}{\cos\alpha}}{\frac{7\cdot\sin\alpha-5\cdot\cos\alpha}{\cos\alpha}}=1\ \Rightarrow\ \frac{\frac{5\cdot\sin\alpha-4\cdot\cos\alpha}{\cos\alpha}}{\frac{7\cdot\sin\alpha-5\cdot\cos\alpha}{\cos\alpha}}=1\ \Rightarrow$$

$$\Rightarrow\ \frac{\frac{5\cdot\sin\alpha}{\cos\alpha}-\frac{4\cdot\cos\alpha}{\cos\alpha}}{\frac{7\cdot\sin\alpha}{\cos\alpha}-\frac{5\cdot\cos\alpha}{\cos\alpha}}=1\ \Rightarrow\ \frac{\frac{5\cdot\sin\alpha}{\cos\alpha}-\frac{4\cdot\cos\alpha}{\cos\alpha}}{\frac{7\cdot\sin\alpha}{\cos\alpha}-\frac{5\cdot\cos\alpha}{\cos\alpha}}=1\ \Rightarrow\ \frac{5\cdot tg\,\alpha-4}{7\cdot tg\,\alpha-5}=1\ \Rightarrow$$

$$\Rightarrow\ \frac{5\cdot tg\,\alpha-4}{7\cdot tg\,\alpha-5}=1\ /\cdot\left(7\cdot tg\,\alpha-5\right)\ \Rightarrow\ 5\cdot tg\,\alpha-4=7\cdot tg\,\alpha-5\ \Rightarrow\ 5\cdot tg\,\alpha-7\cdot tg\,\alpha=-5+4\ \Rightarrow$$

$$\Rightarrow\ -2\cdot tg\,\alpha=-1\ \Rightarrow\ \Rightarrow\ -2\cdot tg\,\alpha=-1\ /:\left(-2\right)\ \Rightarrow\ tg\,\alpha=\frac{1}{2}.$$

Odgovor je pod A.

Vježba 275

Ako je $\dfrac{4\cdot\cos\alpha-5\cdot\sin\alpha}{5\cdot\cos\alpha-7\cdot\sin\alpha}=1$, tada je tg α jednako:

$$A.\ \frac{1}{2}\qquad B.\ -\frac{1}{2}\qquad C.\ \frac{1}{3}\qquad D.\ -\frac{1}{3}$$

Rezultat: A.

Zadatak 276 (Ivan, tehnička škola)

Za dopustive vrijednosti od x vrijednost izraza

$$\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}\ \text{je jednaka:}$$

$$A.\ 1\qquad B.\ 2\qquad C.\ 3\qquad D.\ 4$$

Rješenje 276

Ponovimo!

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right)\quad,\quad a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right).$$

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad \left(a^n\right)^m=a^{n\cdot m}\quad,\quad \cos^2 x+\sin^2 x=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=\frac{(\sin x+\cos x)\cdot\left(\sin^2 x-\sin x\cdot\cos x+\cos^2 x\right)}{\sin x+\cos x}+\frac{(\sin x-\cos x)\cdot\left(\sin^2 x+\sin x\cdot\cos x+\cos^2 x\right)}{\sin x-\cos x}+$$

$$+\frac{\left(\sin^2 x-\cos^2 x\right)\cdot\left(\sin^2 x+\cos^2 x\right)}{\sin^2 x-\cos^2 x}=$$

$$=\frac{(\sin x+\cos x)\cdot\left(\sin^2 x-\sin x\cdot\cos x+\cos^2 x\right)}{\sin x+\cos x}+\frac{(\sin x-\cos x)\cdot\left(\sin^2 x+\sin x\cdot\cos x+\cos^2 x\right)}{\sin x-\cos x}+$$

$$+\frac{\left(\sin^2 x-\cos^2 x\right)\cdot\left(\sin^2 x+\cos^2 x\right)}{\sin^2 x-\cos^2 x}=$$

$$=\sin^2 x-\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\cos^2 x=$$

$$=\sin^2 x-\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\cos^2 x=$$

$$=\sin^2 x+\cos^2 x+\sin^2 x+\cos^2 x+\sin^2 x+\cos^2 x=3\cdot\sin^2 x+3\cdot\cos^2 x=$$

$$= 3 \cdot \left(\sin^2 x + \cos^2 x\right) = 3 \cdot 1 = 3.$$

Odgovor je pod C.

Vježba 276

Za dopustive vrijednosti od x vrijednost izraza

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} - \frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} + \frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} \text{ je jednaka:}$$

$$A.\ 1 \qquad B.\ 2 \qquad C.\ 3 \qquad D.\ 4$$

Rezultat: C.

Zadatak 277 (Božidar, srednja škola)

$$\text{Ako je } x \in \left\langle 0, 90^0 \right\rangle \text{ i } \sin 2x = \frac{3}{4}, \text{ koliko je } \left|\cos x - \sin x\right|?$$

$$A.\ 0 \qquad B.\ \frac{1}{2} \qquad C.\ \frac{1}{3} \qquad D.\ \frac{1}{4}$$

Rješenje 277

Ponovimo!

$$a = b\ ,\ c \in R \ \Rightarrow\ a + c = b + c \quad , \quad \cos^2 \alpha + \sin^2 \alpha = 1 \quad , \quad \sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha.$$

$$(a - b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \quad , \quad \sqrt{a^2} = |a| \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x \ge 0$, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj $-x$ koji je pozitivan. Za svaki x, $x < 0$, je $|x| = -x$.

$$\sin 2x = \frac{3}{4} \ \Rightarrow\ \sin 2x = \frac{3}{4} \ / \cdot (-1) \ \Rightarrow\ -\sin 2x = -\frac{3}{4} \ \Rightarrow\ -\sin 2x = -\frac{3}{4} \ / + 1 \ \Rightarrow$$

$$\Rightarrow\ 1 - \sin 2x = 1 - \frac{3}{4} \ \Rightarrow\ \cos^2 x + \sin^2 x - \sin 2x = \frac{1}{1} - \frac{3}{4} \ \Rightarrow\ \cos^2 x - \sin 2x + \sin^2 x = \frac{4 - 3}{4} \ \Rightarrow$$

$$\Rightarrow\ \cos^2 x - 2 \cdot \sin x \cdot \cos x + \sin^2 x = \frac{1}{4} \ \Rightarrow\ (\cos x - \sin x)^2 = \frac{1}{4} \ \Rightarrow\ (\cos x - \sin x)^2 = \frac{1}{4} \ / \sqrt{\ } \ \Rightarrow$$

$$\Rightarrow\ \sqrt{(\cos x - \sin x)^2} = \sqrt{\frac{1}{4}} \ \Rightarrow\ \left|\cos x - \sin x\right| = \frac{1}{2}.$$

Odgovor je pod C.

Vježba 277

$$\text{Ako je } x \in \left\langle 0, 90^0 \right\rangle \text{ i } \sin 2x = \frac{8}{9}, \text{ koliko je } \left|\cos x - \sin x\right|?$$

$$A.\ 0 \qquad B.\ \frac{1}{2} \qquad C.\ \frac{1}{3} \qquad D.\ \frac{1}{4}$$

Rezultat: C.

Zadatak 278 (Gaby, gimnazija)

Riješi jednadžbu $\sin x \cdot \cos x + tg\ x \cdot ctg\ x = 1.$

Rješenje 278

Ponovimo!

$$tg\ \alpha \cdot ctg\ \alpha = 1 \quad , \quad \sin x = 0 \Rightarrow x = k \cdot \pi \ , \ k \in Z \quad , \quad \cos x = 0 \Rightarrow x = \frac{\pi}{2} + k \cdot \pi \ , \ k \in Z.$$

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a \cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Treba znati:

- Funkcija sinus definirana je za sve realne brojeve (kažemo da je definirana na skupu R).
- Funkcija kosinus definirana je za sve realne brojeve (kažemo da je definirana na skupu R).
- Funkcija tangens definirana je za sve realne brojeve koji su različiti od $\frac{\pi}{2} + k \cdot \pi \ , \ k \in Z.$
- Funkcija kotangens definirana je za sve realne brojeve koji su različiti od $k \cdot \pi \ , \ k \in Z.$

Sada računamo:

$$\sin x \cdot \cos x + tg\ x \cdot ctg\ x = 1 \Rightarrow \left[tg\ x \cdot ctg\ x = 1 \right] \Rightarrow \sin x \cdot \cos x + 1 = 1 \Rightarrow \sin x \cdot \cos x + 1 = 1 \Rightarrow$$

$$\Rightarrow \sin x \cdot \cos x = 0 \Rightarrow \left.\begin{matrix} \sin x = 0 \\ \cos x = 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = \sin^{-1} 0 \\ x = \cos^{-1} 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = k \cdot \pi \ , \ k \in Z \\ x = \frac{\pi}{2} + k \cdot \pi \ , \ k \in Z \end{matrix}\right\}.$$

Budući da za te realne brojeve funkcije tangens i kotangens nisu definirane, jednadžba nema rješenja.

Vježba 278

Riješi jednadžbu $\sin x \cdot \cos x + 1 = tg\ x \cdot ctg\ x.$

Rezultat: Nema rješenja.

Zadatak 279 (MT, gimnazija)

Riješi jednadžbu $\sin(\cos x) = 0.$

Rješenje 279

Ponovimo!

$$\sin x = 0 \Rightarrow x = k \cdot \pi \ , \ k \in Z \quad , \quad \cos x = 0 \Rightarrow x = \frac{\pi}{2} + k \cdot \pi \ , \ k \in Z.$$

$$\cos : R \to [-1, 1] \quad , \quad |\cos x| \le 1.$$

$$\sin(\cos x) = 0.$$

Uvodimo zamjenu (supstituciju).

$$t = \cos x.$$

$$\sin(\cos x) = 0 \Rightarrow \begin{bmatrix} \text{zamjena} \\ t = \cos x \end{bmatrix} \Rightarrow \sin t = 0 \Rightarrow t = \sin^{-1} 0 \Rightarrow t = k \cdot \pi \ , \ k \in Z.$$

Vraćamo se na zamjenu.

$$\left.\begin{matrix} t = \cos x \\ t = k \cdot \pi \end{matrix}\right\} \Rightarrow \cos x = k \cdot \pi.$$

Budući da je kodomena funkcije kosinus segment $[-1, 1]$, tj. vrijedi

$$|\cos x| \le 1,$$

slijedi da cijeli broj k može biti samo 0, k = 0.

Tada je:

$$\left.\begin{array}{l}\cos x = k\cdot\pi \\ k = 0\end{array}\right\} \Rightarrow \cos x = 0\cdot\pi \Rightarrow \cos x = 0 \Rightarrow x = \cos^{-1} 0 \Rightarrow x = \frac{\pi}{2} + k\cdot\pi \ , \ k\in Z.$$

Vježba 279

$$2\cdot\cos(\sin x) = 0.$$

Rezultat: $x = \frac{\pi}{2} + k\cdot\pi \ , \ k\in Z.$

Zadatak 280 (Sany, gimnazija)

Ako je $\sin x + \sin y = a$ i $\cos x + \cos y = b$, tada je $\cos(x-y)$ jednako:

$$A.\ \frac{a^2+b^2}{a+b} \qquad B.\ \frac{a^2-b^2}{a^2+b^2} \qquad C.\ \frac{a^2+b^2}{2} \qquad D.\ \frac{a^2+b^2-2}{2}$$

Rješenje 280

Ponovimo!

$$(a+b)^2 = a^2 + 2\cdot a\cdot b + b^2 \quad , \quad \sin^2\alpha + \cos^2\alpha = 1 \quad , \quad \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta.$$

$$\left.\begin{array}{l}a = b \\ c = d\end{array}\right\} \Rightarrow a + c = b + d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

$$\left.\begin{array}{l}\sin x + \sin y = a \\ \cos x + \cos y = b\end{array}\right\} \Rightarrow \left.\begin{array}{l}\sin x + \sin y = a \ /^2 \\ \cos x + \cos y = b \ /^2\end{array}\right\} \Rightarrow \left.\begin{array}{l}(\sin x + \sin y)^2 = a^2 \\ (\cos x + \cos y)^2 = b^2\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\sin^2 x + 2\cdot\sin x\cdot\sin y + \sin^2 y = a^2 \\ \cos^2 x + 2\cdot\cos x\cdot\cos y + \cos^2 y = b^2\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{zbrojimo} \\ \text{jednakosti}\end{array}\right] \Rightarrow$$

$$\Rightarrow \sin^2 x + 2\cdot\sin x\cdot\sin y + \sin^2 y + \cos^2 x + 2\cdot\cos x\cdot\cos y + \cos^2 y = a^2 + b^2 \Rightarrow$$

$$\Rightarrow \left(\sin^2 x + \cos^2 x\right) + (2\cdot\cos x\cdot\cos y + 2\cdot\sin x\cdot\sin y) + \left(\sin^2 y + \cos^2 y\right) = a^2 + b^2 \Rightarrow$$

$$\Rightarrow 1 + 2\cdot(\cos x\cdot\cos y + \sin x\cdot\sin y) + 1 = a^2 + b^2 \Rightarrow 1 + 2\cdot\cos(x-y) + 1 = a^2 + b^2 \Rightarrow$$

$$\Rightarrow 2\cdot\cos(x-y) = a^2 + b^2 - 1 - 1 \Rightarrow 2\cdot\cos(x-y) = a^2 + b^2 - 2 \Rightarrow$$

$$\Rightarrow 2\cdot\cos(x-y) = a^2 + b^2 - 2 \ /:2 \Rightarrow \cos(x-y) = \frac{a^2+b^2-2}{2}.$$

Odgovor je pod D.

Vježba 280

Ako je $\sin x + \sin y = 1$ i $\cos x + \cos y = 1$, tada je $\cos(x-y)$ jednako:

$$A.\ 1 \qquad B.\ 0 \qquad C.\ 2 \qquad D.\ \frac{1}{2}$$

Rezultat: B.