Zadatak 121 (Anamarija, gimnazija)

Pojednostavnite: $\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}$.

Rješenje 121

Ponovimo!

Razlika kvadrata:

$$(a-b)\cdot(a+b)=a^2-b^2.$$

Množenje korijena:

$$\sqrt[n]{a}\cdot\sqrt[m]{b}=\sqrt[n\cdot m]{a^m\cdot b^n}.$$

$$\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}\cdot\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt[4]{3}+\sqrt[4]{2}}=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\left(\sqrt[4]{3}\right)^2-\left(\sqrt[4]{2}\right)^2}=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt{3}-\sqrt{2}}=$$

$$=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{3-2}=$$

$$=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{1}=\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)=\sqrt[4]{3}\cdot\sqrt{3}+\sqrt[4]{3}\cdot\sqrt{2}+\sqrt[4]{2}\cdot\sqrt{3}+\sqrt[4]{2}\cdot\sqrt{2}=$$

$$=\sqrt[4]{3\cdot 3^2}+\sqrt[4]{3\cdot 2^2}+\sqrt[4]{2\cdot 3^2}+\sqrt[4]{2\cdot 2^2}=\sqrt[4]{27}+\sqrt[4]{12}+\sqrt[4]{18}+\sqrt[4]{8}.$$

Vježba 121

Pojednostavnite: $\frac{1}{\sqrt[4]{3}+\sqrt[4]{2}}$.

Rezultat: $\sqrt[4]{27}+\sqrt[4]{12}-\sqrt[4]{18}-\sqrt[4]{8}.$

Zadatak 122 (Iva, gimnazija)

Pojednostavnite: $\frac{a^2-4}{2\cdot a-4}$.

Rješenje 122

Ponovimo!

Razlika kvadrata:

$$(a-b)\cdot(a+b)=a^2-b^2.$$

$$\frac{a^2-4}{2\cdot a-4}=\begin{bmatrix}\text{u brojniku je razlika kvadrata,}\\ \text{u nazivniku izlučimo broj 2}\end{bmatrix}=\frac{(a-2)\cdot(a+2)}{2\cdot(a-2)}=\begin{bmatrix}\text{kratimo faktore}\\ a-2\end{bmatrix}=\frac{(a-2)\cdot(a+2)}{2\cdot(a-2)}=\frac{a+2}{2}.$$

Vježba 122

Pojednostavnite: $\frac{a^2-9}{2\cdot a-6}$.

Rezultat: $\frac{a+3}{2}.$

Zadatak 123 (Anamarija, gimnazija)

Izračunajte: $\left(\frac{x-1}{x+1}-1\right):\left(\frac{x+1}{x-1}-1\right)$.

Rješenje 123

U obje zagrade je računska operacija oduzimanje pa zato moramo naći najmanji zajednički nazivnik za svaku zagradu:

$$\left(\frac{x-1}{x+1}-1\right):\left(\frac{x+1}{x-1}-1\right)=\frac{x-1-1\cdot(x+1)}{x+1}:\frac{x+1-1\cdot(x-1)}{x-1}=\frac{x-1-x-1}{x+1}:\frac{x+1-x+1}{x-1}=$$

$$=\frac{x-1-x-1}{x+1}:\frac{x+1-x+1}{x-1}=\frac{-2}{x+1}:\frac{2}{x-1}=\frac{-2}{x+1}\cdot\frac{x-1}{2}=\frac{-2}{x+1}\cdot\frac{x-1}{2}=\frac{-1}{x+1}\cdot\frac{x-1}{1}=\frac{-(x-1)}{x+1}=\frac{-x+1}{x+1}=\frac{1-x}{1+x}.$$

Vježba 123

Izračunajte: $\left(\frac{x-1}{x+1}-1\right):\frac{2}{x+1}.$

Rezultat: $-1.$

Zadatak 124 (Nena, Sarah, hotelijerska škola - THK)

Rastavite na faktore: $a\cdot(a+b)-c\cdot(b+c).$

Rješenje 124

Ponovimo!

Zakon distribucije množenja prema zbrajanju: $a\cdot(b+c)=a\cdot b+a\cdot c$, $a\cdot b+a\cdot c=a\cdot(b+c).$

Razlika kvadrata: $a^2-b^2=(a-b)\cdot(a+b)$, $(a-b)\cdot(a+b)=a^2-b^2.$

$$a\cdot(a+b)-c\cdot(b+c)=a^2+a\cdot b-b\cdot c-c^2=\left[\begin{array}{l}\text{grupiramo prvi i četvrti član, dobijemo razliku kvadrata}\\ \text{grupiramo drugi i treći član, izlučimo b}\end{array}\right]=$$

$$=a^2-c^2+a\cdot b-b\cdot c=(a-c)\cdot(a+c)+b\cdot(a-c)=\left[\text{izlučimo zagradu } (a-c)\right]=(a-c)\cdot(a+c+b).$$

Vježba 124

Rastavite na faktore: $a\cdot(a+b)+c\cdot(b-c).$

Rezultat: $(a+c)\cdot(a+b-c).$

Zadatak 125 (Nena, Sarah, hotelijerska škola - THK)

Rastavite na faktore: $a^2-b^2+2\cdot b\cdot c-c^2.$

Rješenje 125

Ponovimo!

Razlika kvadrata: $a^2-b^2=(a-b)\cdot(a+b)$, $(a-b)\cdot(a+b)=a^2-b^2.$

Kvadrat razlike: $(a-b)^2=a^2-2\cdot a\cdot b+b^2$, $a^2-2\cdot a\cdot b+b^2=(a-b)^2.$

$$a^2-b^2+2\cdot b\cdot c-c^2=\left[\begin{array}{l}\text{iz zadnja tri člana izlučimo predznak minus,}\\ \text{dobije se kvadrata razlike}\end{array}\right]=a^2-\left(b^2-2\cdot b\cdot c+c^2\right)=$$

$$=a^2-(b-c)^2=\left[\begin{array}{c}\text{razlika kvadrata}\\ I=a\ ,\ II=b-c\end{array}\right]=(a-(b-c))\cdot(a+(b-c))=(a-b+c)\cdot(a+b-c).$$

Vježba 125

Rastavite na faktore: $b^2-2\cdot b\cdot c+c^2-a^2.$

Rezultat: $(b-c-a)\cdot(b-c+a).$

Zadatak 126 (Nena, Sarah, hotelijerska škola - THK)

Rastavite na faktore: $2\cdot(x+1)^2-18\cdot(2-x)^2.$

Rješenje 126

Ponovimo!

Zakon distribucije množenja prema zbrajanju: $a\cdot(b+c)=a\cdot b+a\cdot c$, $a\cdot b+a\cdot c=a\cdot(b+c).$

Razlika kvadrata: $a^2-b^2=(a-b)\cdot(a+b)$, $(a-b)\cdot(a+b)=a^2-b^2.$

$$2\cdot(x+1)^2-18\cdot(2-x)^2=\left[\text{izlučimo broj 2}\right]=2\cdot\left[(x+1)^2-9\cdot(2-x)^2\right]=$$

$$=\begin{bmatrix} \text{razlika kvadrata} \\ I = x+1 \ , \ II = 3\cdot(2-x) \end{bmatrix} = 2\cdot\left[(x+1)-3\cdot(2-x)\right]\cdot\left[(x+1)+3\cdot(2-x)\right] = 2\cdot[x+1-6+3\cdot x]\cdot[x+1+6-3\cdot x] =$$

$$= 2\cdot[4\cdot x-5]\cdot[7-2\cdot x].$$

Vježba 126

Rastavite na faktore: $2\cdot(x+1)^2 - 2\cdot(2-x)^2$.

Rezultat: $6\cdot(2\cdot x-1)$.

Zadatak 127 (4A, hotelijerska škola)

Rastavite na faktore algebarski izraz: : $6\cdot x^2 - 5\cdot x\cdot y - 4\cdot y^2$.

Rješenje 127

$$6\cdot x^2 - 5\cdot x\cdot y - 4\cdot y^2 = 6\cdot x^2 - 5\cdot x\cdot y - 4\cdot y^2 = 6\cdot x^2 + 3\cdot x\cdot y - 8\cdot x\cdot y - 4\cdot y^2 =$$

$$= 3\cdot x\cdot(2\cdot x+y) - 4\cdot y\cdot(2\cdot x+y) = (2\cdot x+y)(3\cdot x-4\cdot y).$$

Vježba 127

Rastavite na faktore algebarski izraz: : $4\cdot a^2 - 12\cdot a\cdot b + 9\cdot b^2 - 2\cdot a + 3\cdot b$.

Rezultat: $4\cdot a^2 - 12\cdot a\cdot b + 9\cdot b^2 - 2\cdot a + 3\cdot b = 4\cdot a^2 - 6\cdot a\cdot b - 6\cdot a\cdot b + 9\cdot b^2 - 2\cdot a + 3\cdot b =$
$= 2\cdot a\cdot(2\cdot a-3\cdot b) - 3\cdot b\cdot(2\cdot a-3\cdot b) - (2\cdot a-3\cdot b) = (2\cdot a-3\cdot b)\cdot(2\cdot a-3\cdot b-1).$

Zadatak 128 (4A, hotelijerska škola)

Rastavite na faktore algebarski izraz: : $x^3 + 5\cdot x^2 + 3\cdot x - 9$.

Rješenje 128

Ponovimo!

Razlika kubova: $a^3 - b^3 = (a-b)\cdot\left(a^2 + a\cdot b + b^2\right)$. Razlika kvadrata: $a^2 - b^2 = (a-b)\cdot(a+b)$.

$$x^3 + 5\cdot x^2 + 3\cdot x - 9 = x^3 + 5\cdot x^2 + 3\cdot x - 9 = x^3 + 5\cdot x^2 + 3\cdot x - 1 - 5 - 3 = x^3 - 1 + 5\cdot x^2 - 5 + 3\cdot x - 3 =$$

$$= (x-1)\cdot\left(x^2 + x + 1\right) + 5\cdot\left(x^2 - 1\right) + 3\cdot(x-1) = (x-1)\cdot\left(x^2 + x + 1\right) + 5\cdot(x-1)\cdot(x+1) + 3\cdot(x-1) =$$

$$= (x-1)\cdot\left(x^2 + x + 1 + 5\cdot(x+1) + 3\right) = (x-1)\cdot\left(x^2 + x + 1 + 5\cdot x + 5 + 3\right) = (x-1)\cdot\left(x^2 + 6\cdot x + 9\right) = (x-1)\cdot(x+3)^2.$$

Vježba 128

Rastavite na faktore algebarski izraz: : $x^3 + x^2 + 4$.

Rezultat: $x^3 + x^2 + 4 = x^3 + x^2 + 8 - 4 = x^3 + 8 + x^2 - 4 = (x+2)\cdot\left(x^2 - 2\cdot x + 4\right) + (x-2)\cdot(x+2) =$

$= (x+2)\cdot\left(x^2 - 2\cdot x + 4 + x - 2\right) = (x+2)\cdot\left(x^2 - x + 2\right).$

Zadatak 129 (Anamarija, gimnazija)

Dokažite da vrijednost razlomka ne ovisi o vrijednosti broja n: $\dfrac{16^{n-1}}{8^{n-2}\cdot 2^{n+2}}$.

Rješenje 129

Ponovimo!

$$\left(a^n\right)^m = a^{n\cdot m} \quad , \quad a^n\cdot a^m = a^{n+m} \quad , \quad a^n\cdot b^n = (a\cdot b)^n.$$

1.inačica

$$\frac{16^{n-1}}{8^{n-2}\cdot 2^{n+2}} = \frac{\left(2^4\right)^{n-1}}{\left(2^3\right)^{n-2}\cdot 2^{n+2}} = \frac{2^{4n-4}}{2^{3n-6}\cdot 2^{n+2}} = \frac{2^{4n-4}}{2^{3n-6+n+2}} = \frac{2^{4n-4}}{2^{4n-4}} = 1.$$

2.inačica

$$\frac{16^{n-1}}{8^{n-2}\cdot 2^{n+2}}=\frac{16^{n}\cdot 16^{-1}}{8^{n}\cdot 8^{-2}\cdot 2^{n}\cdot 2^{2}}=\frac{16^{n}\cdot 16^{-1}}{(8\cdot 2)^{n}\cdot 8^{-2}\cdot 2^{2}}=\frac{16^{n}\cdot 16^{-1}}{16^{n}\cdot 8^{-2}\cdot 2^{2}}=\frac{16^{n}\cdot 16^{-1}}{16^{n}\cdot 8^{-2}\cdot 2^{2}}=\frac{16^{-1}}{8^{-2}\cdot 2^{2}}=$$

$$=\frac{8^{2}}{16\cdot 2^{2}}=\frac{64}{16\cdot 4}=\frac{64}{64}=1.$$

Vježba 129

Dokažite da vrijednost razlomka ne ovisi o vrijednosti broja n: $\frac{25^{n+1}\cdot 5^{n+2}}{125^{n-1}}$.

Rezultat: 5^7.

Zadatak 130 (Deny, strojarska škola)

Ako je a > 1, izračunajte $\frac{a-1}{a+1}\cdot\frac{\sqrt{a^{4}-2\cdot a^{2}+1}}{a^{2}-2\cdot a+1}+5.$

Rješenje 130

Ponovimo!

$$a^{2}-2\cdot a\cdot b+b^{2}=(a-b)^{2}\quad,\quad \sqrt{a^{2}}=|a|\quad,\quad |a|=a \text{ za } a\ge 0\quad,\quad a^{2}-b^{2}=(a-b)\cdot(a+b).$$

$$\frac{a-1}{a+1}\cdot\frac{\sqrt{a^{4}-2\cdot a^{2}+1}}{a^{2}-2\cdot a+1}+5=\frac{a-1}{a+1}\cdot\frac{\sqrt{\left(a^{2}-1\right)^{2}}}{(a-1)^{2}}+5=\frac{a-1}{a+1}\cdot\frac{\left|a^{2}-1\right|}{(a-1)^{2}}+5=\left[\begin{array}{l} a>1\Rightarrow a^{2}-1>0\\ \left|a^{2}-1\right|=a^{2}-1\end{array}\right]=$$

$$=\frac{a-1}{a+1}\cdot\frac{a^{2}-1}{(a-1)^{2}}+5=\frac{a-1}{a+1}\cdot\frac{(a-1)\cdot(a+1)}{(a-1)\cdot(a-1)}+5=1+5=6.$$

Vježba 130

Ako je a > 1, izračunajte $(a+1)\cdot\frac{\sqrt{a^{2}-2\cdot a+1}}{a^{2}-1}+5.$

Rezultat: 6.

Zadatak 131 (Jasenka, srednja škola)

Izračunajte $\sqrt{\left(1-\sqrt{2}\right)^{2}}+\sqrt{\left(\sqrt{2}-\sqrt{3}\right)^{2}}+\sqrt{\left(\sqrt{3}-2\right)^{2}}.$

Rješenje 131

Ponovimo!

$$\sqrt{a^{2}}=|a|\quad,\quad |a|=a \text{ za } a\ge 0\quad,\quad |a|=-a \text{ za } a<0.$$

$$\sqrt{\left(1-\sqrt{2}\right)^{2}}+\sqrt{\left(\sqrt{2}-\sqrt{3}\right)^{2}}+\sqrt{\left(\sqrt{3}-2\right)^{2}}=\left|1-\sqrt{2}\right|+\left|\sqrt{2}-\sqrt{3}\right|+\left|\sqrt{3}-2\right|.$$

Pogledajmo kakvi su po predznacima (pozitivni ili negativni) izrazi pod znakom apsolutne vrijednosti:

$$\left.\begin{array}{l} 1-\sqrt{2}\approx 1-1.41=-0.41<0 \text{ negativno}\\ \sqrt{2}-\sqrt{3}\approx 141-1.73=-0.32<0 \text{ negativno}\\ \sqrt{3}-2\approx 1.73-2=-0.27<0 \text{ negativno}\end{array}\right\}.$$

Po definiciji apsolutne vrijednosti je:

$$|a|=-a \text{ za } a<0.$$

Zato je:

$$\sqrt{\left(1-\sqrt{2}\right)^2}+\sqrt{\left(\sqrt{2}-\sqrt{3}\right)^2}+\sqrt{\left(\sqrt{3}-2\right)^2}=\left|1-\sqrt{2}\right|+\left|\sqrt{2}-\sqrt{3}\right|+\left|\sqrt{3}-2\right|=$$

$$=-\left(1-\sqrt{2}\right)-\left(\sqrt{2}-\sqrt{3}\right)-\left(\sqrt{3}-2\right)=-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+2=-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+2=-1+2=1.$$

Vježba 131

Izračunajte $\sqrt{\left(1-\sqrt{2}\right)^2}$

Rezultat: $\sqrt{2}-1.$

Zadatak 132 (Jasenka, srednja škola)

Izračunajte $\dfrac{\left(\sqrt{8}-3\right)+\left(1-\sqrt{2}\right)}{\left(\sqrt{2}-2\right)+\sqrt{18}}.$

Rješenje 132

Ponovimo!

$$\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad(a-b)\cdot(a+b)=a^2-b^2.$$

$$\frac{\left(\sqrt{8}-3\right)+\left(1-\sqrt{2}\right)}{\left(\sqrt{2}-2\right)+\sqrt{18}}=\left[\begin{array}{l}\text{riješimo}\\\text{se zagrada}\end{array}\right]=\frac{\sqrt{8}-3+1-\sqrt{2}}{\sqrt{2}-2+\sqrt{18}}=\left[\begin{array}{l}\text{djelomično}\\\text{korjenujemo}\end{array}\right]=\frac{\sqrt{4\cdot2}-2-\sqrt{2}}{\sqrt{2}-2+\sqrt{9\cdot2}}=\frac{\sqrt{4}\cdot\sqrt{2}-2-\sqrt{2}}{\sqrt{2}-2+\sqrt{9}\cdot\sqrt{2}}=$$

$$=\frac{2\cdot\sqrt{2}-2-\sqrt{2}}{\sqrt{2}-2+3\cdot\sqrt{2}}=\frac{\sqrt{2}-2}{4\cdot\sqrt{2}-2}=\left[\begin{array}{l}\text{racionalizacija}\\\text{nazivnika}\end{array}\right]=\frac{\sqrt{2}-2}{4\cdot\sqrt{2}-2}\cdot\frac{4\cdot\sqrt{2}+2}{4\cdot\sqrt{2}+2}=\frac{4\cdot\left(\sqrt{2}\right)^2+2\cdot\sqrt{2}-8\cdot\sqrt{2}-4}{\left(4\cdot\sqrt{2}\right)^2-2^2}=$$

$$=\frac{4\cdot2+2\cdot\sqrt{2}-8\cdot\sqrt{2}-4}{16\cdot2-4}=\frac{8-6\cdot\sqrt{2}-4}{32-4}=\frac{4-6\cdot\sqrt{2}}{28}=\frac{2\cdot\left(2-3\cdot\sqrt{2}\right)}{28}=\frac{2-3\cdot\sqrt{2}}{14}.$$

Vježba 132

Izračunajte $\dfrac{1}{\sqrt{2}-1}.$

Rezultat: $\sqrt{2}+1.$

Zadatak 133 (Jasenka, srednja škola)

Izračunajte $\left(\sqrt{3}-1\right)^2\cdot\left(4+2\cdot\sqrt{3}\right).$

Rješenje 133

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad(a-b)\cdot(a+b)=a^2-b^2.$$

$$\left(\sqrt{3}-1\right)^2\cdot\left(4+2\cdot\sqrt{3}\right)=\left(\left(\sqrt{3}\right)^2-2\cdot\sqrt{3}\cdot1+1^2\right)\cdot\left(4+2\cdot\sqrt{3}\right)=\left(3-2\cdot\sqrt{3}+1\right)\cdot\left(4+2\cdot\sqrt{3}\right)=$$

$$=\left(4-2\cdot\sqrt{3}\right)\cdot\left(4+2\cdot\sqrt{3}\right)=\left[\begin{array}{l}\text{razlika}\\\text{kvadrata}\end{array}\right]=4^2-\left(2\cdot\sqrt{3}\right)^2=16-4\cdot3=16-12=4.$$

Vježba 133

Izračunajte $\left(\sqrt{2}-1\right)^2\cdot\left(3+2\cdot\sqrt{2}\right).$

Rezultat: 1.

Zadatak 134 (Jasenka, srednja škola)

Izračunajte $\frac{\sqrt{3}}{2\cdot\sqrt{3}-3}$.

Rješenje 134

Ponovimo!

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left(\sqrt{a}\right)^2=a.$$

1.inačica

$$\frac{\sqrt{3}}{2\cdot\sqrt{3}-3}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{\sqrt{3}}{2\cdot\sqrt{3}-3}\cdot\frac{2\cdot\sqrt{3}+3}{2\cdot\sqrt{3}+3}=\frac{\sqrt{3}\cdot\left(2\cdot\sqrt{3}+3\right)}{\left(2\cdot\sqrt{3}\right)^2-3^2}=\frac{2\cdot\left(\sqrt{3}\right)^2+3\cdot\sqrt{3}}{4\cdot 3-9}=\frac{2\cdot 3+3\cdot\sqrt{3}}{12-9}=$$

$$=\frac{6+3\cdot\sqrt{3}}{3}=\frac{3\cdot\left(2+\sqrt{3}\right)}{3}=2+\sqrt{3}.$$

2.inačica

$$\frac{\sqrt{3}}{2\cdot\sqrt{3}-3}=\frac{\sqrt{3}}{2\cdot\sqrt{3}-\left(\sqrt{3}\right)^2}=\frac{\sqrt{3}}{\sqrt{3}\cdot\left(2-\sqrt{3}\right)}=\frac{\sqrt{3}}{\sqrt{3}\cdot\left(2-\sqrt{3}\right)}=\frac{1}{2-\sqrt{3}}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=$$

$$=\frac{1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}=\frac{2+\sqrt{3}}{4-3}=\frac{2+\sqrt{3}}{1}=2+\sqrt{3}.$$

Vježba 134

Izračunajte $\frac{\sqrt{2}}{\sqrt{2}-2}$.

Rezultat: $-\left(1+\sqrt{2}\right)$.

Zadatak 135 (Jasenka, srednja škola)

Izračunajte $\left(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}\right)^2$.

Rješenje 135

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2.$$

$$\left(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}\right)^2=\left(\sqrt{2+\sqrt{3}}\right)^2+2\cdot\sqrt{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{3}}+\left(\sqrt{2-\sqrt{3}}\right)^2=$$

$$=2+\sqrt{3}+2\cdot\sqrt{\left(2+\sqrt{3}\right)\cdot\left(2-\sqrt{3}\right)}+2-\sqrt{3}=2+\sqrt{3}+2\cdot\sqrt{\left(2+\sqrt{3}\right)\cdot\left(2-\sqrt{3}\right)}+2-\sqrt{3}=$$

$$=4+2\cdot\sqrt{\left(2+\sqrt{3}\right)\cdot\left(2-\sqrt{3}\right)}=4+2\cdot\sqrt{2^2-\left(\sqrt{3}\right)^2}=4+2\cdot\sqrt{4-3}=4+2\cdot\sqrt{1}=4+2\cdot 1=6.$$

Vježba 135

Izračunajte $\left(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right)^2$.

Rezultat: 2.

Zadatak 136 (Jasenka, srednja škola)

Izračunajte $\frac{1}{2-\sqrt{2}+\sqrt{3}-\sqrt{6}}$.

Rješenje 136

Ponovimo!

$$\left(\sqrt{a}\right)^2 = a \quad , \quad (a-b)\cdot(a+b) = a^2 - b^2.$$

$$\frac{1}{2-\sqrt{2}+\sqrt{3}-\sqrt{6}} = \frac{1}{\left(\sqrt{2}\right)^2-\sqrt{2}+\sqrt{3}-\sqrt{6}} = \frac{1}{\left(\sqrt{2}\right)^2-\sqrt{2}+\sqrt{3}-\sqrt{3\cdot 2}} = \frac{1}{\sqrt{2}\cdot\left(\sqrt{2}-1\right)-\sqrt{3}\cdot\left(\sqrt{2}-1\right)} =$$

$$= \frac{1}{\left(\sqrt{2}-1\right)\cdot\left(\sqrt{2}-\sqrt{3}\right)} = \begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix} = \frac{1}{\left(\sqrt{2}-1\right)\cdot\left(\sqrt{2}-\sqrt{3}\right)}\cdot\frac{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}+\sqrt{3}\right)}{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}+\sqrt{3}\right)} =$$

$$= \frac{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}+\sqrt{3}\right)}{\left(\left(\sqrt{2}\right)^2-1\right)\cdot\left(\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2\right)} = \frac{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}+\sqrt{3}\right)}{(2-1)\cdot(2-3)} = \frac{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}+\sqrt{3}\right)}{1\cdot(-1)} = -\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}+\sqrt{3}\right).$$

Vježba 136

Izračunajte $\frac{1}{\sqrt{6}-\sqrt{3}}$.

Rezultat: $\frac{\sqrt{6}+\sqrt{3}}{3}$.

Zadatak 137 (Jasenka, srednja škola)

Izračunajte $\frac{\sqrt{x}+1}{1+\sqrt{x}+x} : \frac{1}{x^2-\sqrt{x}}$.

Rješenje 137

Ponovimo!

$$a^3-b^3 = (a-b)\cdot\left(a^2+a\cdot b+b^2\right) \quad , \quad a^2 = \left(\sqrt{a}\right)^4 \quad , \quad (a-b)\cdot(a+b) = a^2-b^2.$$

$$\frac{\sqrt{x}+1}{1+\sqrt{x}+x} : \frac{1}{x^2-\sqrt{x}} = \frac{\sqrt{x}+1}{1+\sqrt{x}+x}\cdot\frac{x^2-\sqrt{x}}{1} = \frac{\sqrt{x}+1}{1+\sqrt{x}+x}\cdot\frac{\left(\sqrt{x}\right)^4-\sqrt{x}}{1} = \frac{\sqrt{x}+1}{1+\sqrt{x}+x}\cdot\frac{\sqrt{x}\cdot\left(\left(\sqrt{x}\right)^3-1\right)}{1} =$$

$$= \frac{\sqrt{x}+1}{1+\sqrt{x}+x}\cdot\frac{\sqrt{x}\cdot\left(\sqrt{x}-1\right)\left(\left(\sqrt{x}\right)^2+\sqrt{x}\cdot 1+1^2\right)}{1} = \frac{\sqrt{x}+1}{1+\sqrt{x}+x}\cdot\frac{\sqrt{x}\cdot\left(\sqrt{x}-1\right)\left(x+\sqrt{x}+1\right)}{1} =$$

$$= \frac{\sqrt{x}+1}{1+\sqrt{x}+x}\cdot\frac{\sqrt{x}\cdot\left(\sqrt{x}-1\right)\left(x+\sqrt{x}+1\right)}{1} = \sqrt{x}\cdot\left(\sqrt{x}+1\right)\cdot\left(\sqrt{x}-1\right) = \sqrt{x}\cdot\left(\left(\sqrt{x}\right)^2-1\right) = \sqrt{x}\cdot(x-1).$$

Vježba 137

Izračunajte $\left(x^2-\sqrt{x}\right) : \frac{1+\sqrt{x}+x}{\sqrt{x}+1}$.

Rezultat: $\sqrt{x}\cdot(x-1)$.

Zadatak 138 (Jasenka, srednja škola)

Izračunajte $\frac{36}{\left(3\cdot\sqrt{2}-2\cdot\sqrt{3}\right)^2}$.

Rješenje 138

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2.$$

$$\frac{36}{\left(3\cdot\sqrt{2}-2\cdot\sqrt{3}\right)^2}=\frac{36}{\left(3\cdot\sqrt{2}\right)^2-2\cdot3\cdot\sqrt{2}\cdot2\cdot\sqrt{3}+\left(2\cdot\sqrt{3}\right)^2}=\frac{36}{9\cdot2-12\cdot\sqrt{6}+4\cdot3}=\frac{36}{18-12\cdot\sqrt{6}+12}=$$

$$=\frac{36}{30-12\cdot\sqrt{6}}=\frac{36}{6\cdot\left(5-2\cdot\sqrt{6}\right)}=\frac{36}{6\cdot\left(5-2\cdot\sqrt{6}\right)}=\frac{6}{5-2\cdot\sqrt{6}}=\left[\begin{matrix}\text{racionalizacija}\\ \text{nazivnika}\end{matrix}\right]=\frac{6}{5-2\cdot\sqrt{6}}\cdot\frac{5+2\cdot\sqrt{6}}{5+2\cdot\sqrt{6}}=$$

$$=\frac{6\cdot\left(5+2\cdot\sqrt{6}\right)}{5^2-\left(2\cdot\sqrt{6}\right)^2}=\frac{6\cdot\left(5+2\cdot\sqrt{6}\right)}{25-4\cdot6}=\frac{6\cdot\left(5+2\cdot\sqrt{6}\right)}{25-24}=\frac{6\cdot\left(5+2\cdot\sqrt{6}\right)}{1}=6\cdot\left(5+2\cdot\sqrt{6}\right).$$

Vježba 138

Izračunajte $\frac{6}{\left(3\cdot\sqrt{2}-2\cdot\sqrt{3}\right)^2}$.

Rezultat: $5+2\cdot\sqrt{6}.$

Zadatak 139 (Jasenka, srednja škola)

Koliko je $x^2-x\cdot y+y^2$ za $x=\frac{1}{\sqrt{3}-\sqrt{2}}$, $y=\frac{1}{\sqrt{3}+\sqrt{2}}$.

Rješenje 139

Ponovimo!

$$(a\pm b)^2=a^2\pm2\cdot a\cdot b+b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2.$$

1.inačica

Posebno izračunamo x i y, a zatim njihove vrijednosti uvrstimo u zadani izraz:

$$\left.\begin{matrix}x=\frac{1}{\sqrt{3}-\sqrt{2}}\\ y=\frac{1}{\sqrt{3}+\sqrt{2}}\end{matrix}\right\}\Rightarrow\left[\begin{matrix}\text{racionalizacija}\\ \text{nazivnika}\end{matrix}\right]\Rightarrow\left.\begin{matrix}x=\frac{1}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\\ y=\frac{1}{\sqrt{3}+\sqrt{2}}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\end{matrix}\right\}\Rightarrow\left.\begin{matrix}x=\frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}\\ y=\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{matrix}x=\frac{\sqrt{3}+\sqrt{2}}{3-2}\\ y=\frac{\sqrt{3}-\sqrt{2}}{3-2}\end{matrix}\right\}\Rightarrow\left.\begin{matrix}x=\frac{\sqrt{3}+\sqrt{2}}{1}\\ y=\frac{\sqrt{3}-\sqrt{2}}{1}\end{matrix}\right\}\Rightarrow\left.\begin{matrix}x=\sqrt{3}+\sqrt{2}\\ y=\sqrt{3}-\sqrt{2}\end{matrix}\right\}.$$

Vrijednost izraza iznosi:

$$x^2-x\cdot y+y^2=\left(\sqrt{3}+\sqrt{2}\right)^2-\left(\sqrt{3}+\sqrt{2}\right)\cdot\left(\sqrt{3}-\sqrt{2}\right)+\left(\sqrt{3}-\sqrt{2}\right)^2=$$

$$=\left(\sqrt{3}\right)^2+2\cdot\sqrt{3}\cdot\sqrt{2}+\left(\sqrt{2}\right)^2-\left(\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2\right)+\left(\sqrt{3}\right)^2-2\cdot\sqrt{3}\cdot\sqrt{2}+\left(\sqrt{2}\right)^2=$$

$$=3+2\cdot\sqrt{6}+2-(3-2)+3-2\cdot\sqrt{6}+2=3+2\cdot\sqrt{6}+2-(3-2)+3-2\cdot\sqrt{6}+2=3+2-1+3+2=9.$$

2.inačica

Odmah uvrstimo vrijednosti za x i y u zadani izraz:

$$\left.\begin{matrix}x=\frac{1}{\sqrt{3}-\sqrt{2}}\ ,\ y=\frac{1}{\sqrt{3}+\sqrt{2}}\\ x^2-x\cdot y+y^2=?\end{matrix}\right\}\Rightarrow\left(\frac{1}{\sqrt{3}-\sqrt{2}}\right)^2-\frac{1}{\sqrt{3}-\sqrt{2}}\cdot\frac{1}{\sqrt{3}+\sqrt{2}}+\left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)^2=$$

$$=\frac{1}{\left(\sqrt{3}-\sqrt{2}\right)^2}-\frac{1}{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}+\frac{1}{\left(\sqrt{3}+\sqrt{2}\right)^2}=$$

$$=\frac{1}{\left(\sqrt{3}\right)^2-2\cdot\sqrt{3}\cdot\sqrt{2}+\left(\sqrt{2}\right)^2}-\frac{1}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}+\frac{1}{\left(\sqrt{3}\right)^2+2\cdot\sqrt{3}\cdot\sqrt{2}+\left(\sqrt{2}\right)^2}=$$

$$=\frac{1}{3-2\cdot\sqrt{6}+2}-\frac{1}{3-2}+\frac{1}{3+2\cdot\sqrt{6}+2}=\frac{1}{5-2\cdot\sqrt{6}}-\frac{1}{1}+\frac{1}{5+2\cdot\sqrt{6}}=\left[\begin{matrix}\text{zbrajanje}\\ \text{razlomaka}\end{matrix}\right]=$$

$$=\frac{5+2\cdot\sqrt{6}-\left(5-2\cdot\sqrt{6}\right)\cdot\left(5+2\cdot\sqrt{6}\right)+5-2\cdot\sqrt{6}}{\left(5-2\cdot\sqrt{6}\right)\cdot\left(5+2\cdot\sqrt{6}\right)}=\frac{5+2\cdot\sqrt{6}-\left(5^2-\left(2\cdot\sqrt{6}\right)^2\right)+5-2\cdot\sqrt{6}}{5^2-\left(2\cdot\sqrt{6}\right)^2}=$$

$$=\frac{5-(25-4\cdot6)+5}{25-4\cdot6}=\frac{5-(25-24)+5}{25-24}=\frac{5-1+5}{1}=9.$$

Vježba 139

Koliko je x^2+y^2 za $x=\frac{1}{\sqrt{2}-1}$, $y=\frac{1}{\sqrt{2}+1}$.

Rezultat: 6.

Zadatak 140 (Maturant, gimnazija)

Nađite vrijednost algebarskog izraza $\left(x+\frac{x\cdot y+y^2}{x+y}\right)\cdot\left(\frac{a^3+8}{x^2-y^2}:\frac{a^2-2\cdot a+4}{x-y}\right)$ za $|x|\neq|y|$.

Rješenje 140

Ponovimo!

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right)\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

$$\left(x+\frac{x\cdot y+y^2}{x+y}\right)\cdot\left(\frac{a^3+8}{x^2-y^2}:\frac{a^2-2\cdot a+4}{x-y}\right)=\frac{x\cdot(x+y)+x\cdot y+y^2}{x+y}\cdot\left(\frac{(a+2)\cdot\left(x^2-2\cdot a+4\right)}{(x-y)\cdot(x+y)}:\frac{a^2-2\cdot a+4}{x-y}\right)=$$

$$=\frac{x\cdot(x+y)+x\cdot y+y^2}{x+y}\cdot\frac{(a+2)\cdot\left(x^2-2\cdot a+4\right)}{(x-y)\cdot(x+y)}\cdot\frac{x-y}{a^2-2\cdot a+4}=\frac{x^2+x\cdot y+x\cdot y+y^2}{x+y}\cdot\frac{a+2}{x+y}\cdot1=$$

$$=\frac{x^2+2\cdot x\cdot y+y^2}{x+y}\cdot\frac{a+2}{x+y}=\frac{(x+y)^2}{(x+y)^2}\cdot(a+2)=a+2.$$

Vježba 140

Nađite vrijednost algebarskog izraza $\frac{a^3+8}{x^2-y^2}:\frac{a^2-2\cdot a+4}{x-y}$ za $|x|\neq|y|$.

Rezultat: $\frac{a+2}{x+y}$.