

**Zadatak 161 (Ana, gimnazija)**

Dokažite da za sve realne brojeve x, y i z vrijedi nejednakost:

$$x^2 + y^2 + 3 \cdot z^2 - 2 \cdot x \cdot z - 2 \cdot y \cdot z - 2 \cdot z + 1 \geq 0.$$

**Rješenje 161**

Ponovimo!

$$a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2, \quad a^2 \geq 0, \quad a \in \mathbb{R}.$$

$$\begin{aligned} x^2 + y^2 + 3 \cdot z^2 - 2 \cdot x \cdot z - 2 \cdot y \cdot z - 2 \cdot z + 1 &= x^2 - 2 \cdot x \cdot z + z^2 + y^2 - 2 \cdot y \cdot z + z^2 + z^2 - 2 \cdot z + 1 = \\ &= (x^2 - 2 \cdot x \cdot z + z^2) + (y^2 - 2 \cdot y \cdot z + z^2) + (z^2 - 2 \cdot z + 1) = (x-z)^2 + (y-z)^2 + (z-1)^2 \geq 0. \end{aligned}$$

Dobivena nejednakost istinita je za sve realne brojeve x, y i z pa je i polazna nejednakost istinita za sve x, y i z. Jednakost vrijedi ako i samo ako je  $x = y = z = 1$ .**Vježba 161**Dokažite da za sve realne brojeve x, y i z vrijedi nejednakost:  $x^2 + y^2 + 2 \cdot z^2 \geq 2 \cdot z \cdot (x + y)$ .

**Rezultat:**  $(x-z)^2 + (y-z)^2 \geq 0 \Rightarrow x^2 + y^2 + 2 \cdot z^2 \geq 2 \cdot z \cdot (x + y).$

**Zadatak 162 (Miro, maturant gimnazija)**Izračunajte vrijednost izraza  $x^{100} + 100 \cdot y$  ako je  $x^2 + 4 \cdot y^2 + 2 \cdot x - 8 \cdot y + 5 = 0$ .**Rješenje 162**

Ponovimo!

$$a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2, \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2, \quad a^2 = 0 \Leftrightarrow a = 0$$

$$a^2 + b^2 = 0 \Leftrightarrow a = b = 0, \quad (-a)^{2 \cdot n} = a^{2 \cdot n}.$$

Zadani izraz

$$x^2 + 4 \cdot y^2 + 2 \cdot x - 8 \cdot y + 5 = 0$$

transformiramo pa dobijemo vrijednosti za x i y:

$$\begin{aligned} x^2 + 4 \cdot y^2 + 2 \cdot x - 8 \cdot y + 5 = 0 &\Rightarrow x^2 + 2 \cdot x + 1 + 4 \cdot y^2 - 8 \cdot y + 4 = 0 \Rightarrow \\ \Rightarrow (x^2 + 2 \cdot x + 1) + (4 \cdot y^2 - 8 \cdot y + 4) = 0 &\Rightarrow (x+1)^2 + (2 \cdot y - 2)^2 = 0 \Rightarrow \left. \begin{array}{l} x+1=0 \\ 2 \cdot y - 2 = 0 \end{array} \right\} \Rightarrow \\ &\Rightarrow \left. \begin{array}{l} x = -1 \\ 2 \cdot y = 2 \quad /:2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = -1 \\ y = 1 \end{array} \right\}. \end{aligned}$$

Na kraju dobivamo da je

$$x^{100} + 100 \cdot y = (-1)^{100} + 100 \cdot 1 = 1 + 100 = 101.$$

**Vježba 162**Izračunajte vrijednost izraza  $x^{2008} + 2008 \cdot y$  ako je  $x^2 + 4 \cdot y^2 + 2 \cdot x - 8 \cdot y + 5 = 0$ .

**Rezultat:** 2009.

**Zadatak 163 (Karolina, gimnazija)**Izračunajte vrijednost izraza  $x \cdot (x + 2) + y \cdot (y - 2) - 2 \cdot x \cdot y$ , ako je  $x - y = 5$ .**Rješenje 163**

Ponovimo!

$$a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2.$$

Zadani izraz

$$x \cdot (x + 2) + y \cdot (y - 2) - 2 \cdot x \cdot y$$

transformiramo da dobijemo članove  $x - y$ :

$$x \cdot (x + 2) + y \cdot (y - 2) - 2 \cdot x \cdot y = x^2 + 2 \cdot x + y^2 - 2 \cdot y - 2 \cdot x \cdot y = x^2 - 2 \cdot x \cdot y + y^2 + 2 \cdot x - 2 \cdot y =$$

$$= \left( x^2 - 2 \cdot x \cdot y + y^2 \right) + 2 \cdot (x - y) = (x - y)^2 + 2 \cdot (x - y) = 5^2 + 2 \cdot 5 = 25 + 10 = 35.$$

### Vježba 163

Izračunajte vrijednost izraza  $x \cdot (x + 2) + y \cdot (y - 2) - 2 \cdot x \cdot y$ , ako je  $x - y = 1$ .

**Rezultat:** 3.

### Zadatak 164 (Tea, TUPŠ)

Ako je  $x + \frac{1}{x} = 3$ , izračunajte  $x^2 + \frac{1}{x^2}$ .

### Rješenje 164

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

$$\begin{aligned} x + \frac{1}{x} = 3 &\Rightarrow x + \frac{1}{x} = 3 \quad / \cdot 2 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 3^2 \Rightarrow x^2 + 2 \cdot x \cdot \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 9 \Rightarrow x^2 + 2 + \frac{1}{x^2} = 9 \Rightarrow \\ &\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 \Rightarrow x^2 + \frac{1}{x^2} = 7. \end{aligned}$$

### Vježba 164

Ako je  $x + \frac{1}{x} = 2$ , izračunajte  $x^2 + \frac{1}{x^2}$ .

**Rezultat:** 2.

### Zadatak 165 (Tea, TUPŠ)

Ako je  $\frac{a}{b} + \frac{b}{a} = 3$ , izračunajte  $\frac{a^3}{b^3} + \frac{b^3}{a^3}$ .

### Rješenje 165

Ponovimo!

$$(x+y)^3 = x^3 + 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + y^3, \quad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}.$$

$$\begin{aligned} \frac{a}{b} + \frac{b}{a} = 3 &\Rightarrow \frac{a}{b} + \frac{b}{a} = 3 \quad / \cdot 3 \Rightarrow \left(\frac{a}{b} + \frac{b}{a}\right)^3 = 3^3 \Rightarrow \left(\frac{a}{b}\right)^3 + 3 \cdot \left(\frac{a}{b}\right)^2 \cdot \frac{b}{a} + 3 \cdot \frac{a}{b} \cdot \left(\frac{b}{a}\right)^2 + \left(\frac{b}{a}\right)^3 = 27 \Rightarrow \\ \Rightarrow \frac{a^3}{b^3} + 3 \cdot \frac{a^2}{b^2} \cdot \frac{b}{a} + 3 \cdot \frac{a}{b} \cdot \frac{b^2}{a^2} + \frac{b^3}{a^3} = 27 &\Rightarrow \frac{a^3}{b^3} + 3 \cdot \frac{a}{b} + 3 \cdot \frac{b}{a} + \frac{b^3}{a^3} = 27 \Rightarrow \frac{a^3}{b^3} + 3 \cdot \left(\frac{a}{b} + \frac{b}{a}\right) + \frac{b^3}{a^3} = 27 \Rightarrow \\ \Rightarrow \frac{a^3}{b^3} + 3 \cdot 3 + \frac{b^3}{a^3} = 27 &\Rightarrow \frac{a^3}{b^3} + 9 + \frac{b^3}{a^3} = 27 \Rightarrow \frac{a^3}{b^3} + \frac{b^3}{a^3} = 27 - 9 \Rightarrow \frac{a^3}{b^3} + \frac{b^3}{a^3} = 18. \end{aligned}$$

### Vježba 165

Ako je  $\frac{a}{b} + \frac{b}{a} = 2$ , izračunajte  $\frac{a^3}{b^3} + \frac{b^3}{a^3}$ .

**Rezultat:** 2.

### Zadatak 166 (Mario, gimnazija)

Pojednostavnite izraz:  $\sqrt{5 + \sqrt{24}}$ .

### Rješenje 166

Ponovimo!

$$\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A + \sqrt{A^2 - B}}{2}} \pm \sqrt{\frac{A - \sqrt{A^2 - B}}{2}}$$

Sada računamo:

$$\begin{aligned} \sqrt{5 + \sqrt{24}} &= \sqrt{\frac{5 + \sqrt{5^2 - 24}}{2}} + \sqrt{\frac{5 - \sqrt{5^2 - 24}}{2}} = \sqrt{\frac{5 + \sqrt{25 - 24}}{2}} + \sqrt{\frac{5 - \sqrt{25 - 24}}{2}} = \\ &= \sqrt{\frac{5 + \sqrt{1}}{2}} + \sqrt{\frac{5 - \sqrt{1}}{2}} = \sqrt{\frac{5+1}{2}} + \sqrt{\frac{5-1}{2}} = \sqrt{\frac{6}{2}} + \sqrt{\frac{4}{2}} = \sqrt{3} + \sqrt{2} = \sqrt{2} + \sqrt{3}. \end{aligned}$$

### Vježba 166

Pojednostavnite izraz:  $\sqrt{6 + \sqrt{20}}$ .

**Rezultat:**  $1 + \sqrt{5}$ .

### Zadatak 167 (Mira, srednja škola)

Ako je  $\frac{1}{a \cdot (b+1)} + \frac{1}{b \cdot (a+1)} = \frac{1}{(a+1) \cdot (b+1)}$ , koliko je  $\frac{1}{a} + \frac{1}{b}$ ?

### Rješenje 167

Ponovimo!

$$\frac{x+y}{n} = \frac{x}{n} + \frac{y}{n}$$

Sada računamo:

$$\begin{aligned} \frac{1}{a \cdot (b+1)} + \frac{1}{b \cdot (a+1)} &= \frac{1}{(a+1) \cdot (b+1)} \Rightarrow \left[ \begin{array}{l} \text{množimo jednakost} \\ \text{sa } (a+1) \cdot (b+1) \end{array} \right] \Rightarrow \\ \Rightarrow \frac{1}{a \cdot (b+1)} + \frac{1}{b \cdot (a+1)} &= \frac{1}{(a+1) \cdot (b+1)} \cdot (a+1) \cdot (b+1) \Rightarrow \frac{a+1}{a} + \frac{b+1}{b} = 1 \Rightarrow \frac{a}{a} + \frac{1}{a} + \frac{b}{b} + \frac{1}{b} = 1 \Rightarrow \\ \Rightarrow 1 + \frac{1}{a} + 1 + \frac{1}{b} &= 1 \Rightarrow 1 + \frac{1}{a} + 1 + \frac{1}{b} = 1 \Rightarrow 1 + \frac{1}{a} + \frac{1}{b} = 0 \Rightarrow \frac{1}{a} + \frac{1}{b} = -1. \end{aligned}$$

### Vježba 167

Ako je  $\frac{1}{a \cdot (b+1)} + \frac{1}{b \cdot (a+1)} = \frac{2}{(a+1) \cdot (b+1)}$ , koliko je  $\frac{1}{a} + \frac{1}{b}$ ?

**Rezultat:** 0.

### Zadatak 168 (Deny, gimnazija)

Pojednostavnite izraz:

$$\frac{a-b}{a+b+\sqrt{(a+b)^2-(a-b)^2}} + \frac{2 \cdot (a \cdot \sqrt{a} - b \cdot \sqrt{b})}{(\sqrt{a} + \sqrt{b}) \cdot [a+b-\sqrt{(a+b)^2-(a-b)^2}]}$$

### Rješenje 168

Ponovimo!

$$(x+y)^2 = x^2 + 2 \cdot x \cdot y + y^2, \quad (x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2, \quad x^3 - y^3 = (x-y) \cdot (x^2 + x \cdot y + y^2)$$

$$x^2 - y^2 = (x-y) \cdot (x+y), \quad x-y = (\sqrt{x} - \sqrt{y}) \cdot (\sqrt{x} + \sqrt{y}), \quad \sqrt{a^2} = |a|, \quad |a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

$$\sqrt{x} \cdot \sqrt{y} = \sqrt{x \cdot y}$$

$$\frac{a-b}{a+b+\sqrt{(a+b)^2-(a-b)^2}} + \frac{2 \cdot (a \cdot \sqrt{a} - b \cdot \sqrt{b})}{(\sqrt{a} + \sqrt{b}) \cdot [a+b-\sqrt{(a+b)^2-(a-b)^2}]} =$$

$$\begin{aligned}
&= \frac{a-b}{a+b+\sqrt{a^2+2\cdot a\cdot b+b^2-(a^2-2\cdot a\cdot b+b^2)}} + \\
&\quad \frac{2\cdot(\sqrt{a^2\cdot a}-\sqrt{b^2\cdot b})}{(\sqrt{a}+\sqrt{b})\cdot\left[a+b-\sqrt{a^2+2\cdot a\cdot b+b^2-(a^2-2\cdot a\cdot b+b^2)}\right]} = \\
&= \frac{a-b}{a+b+\sqrt{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}} + \\
&\quad \frac{2\cdot(\sqrt{a^3}-\sqrt{b^3})}{(\sqrt{a}+\sqrt{b})\cdot\left[a+b-\sqrt{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}\right]} = \\
&= \frac{a-b}{a+b+\sqrt{4\cdot a\cdot b}} + \frac{2\cdot\left((\sqrt{a})^3-(\sqrt{b})^3\right)}{(\sqrt{a}+\sqrt{b})\cdot\left[a+b-\sqrt{4\cdot a\cdot b}\right]} = \frac{a-b}{a+b+2\cdot\sqrt{a\cdot b}} + \frac{2\cdot\left((\sqrt{a})^3-(\sqrt{b})^3\right)}{(\sqrt{a}+\sqrt{b})\cdot\left[a+b-2\cdot\sqrt{a\cdot b}\right]} = \\
&= \frac{a-b}{a+2\cdot\sqrt{a\cdot b}+b} + \frac{2\cdot(\sqrt{a}-\sqrt{b})\cdot\left((\sqrt{a})^2+\sqrt{a}\cdot\sqrt{b}+(\sqrt{b})^2\right)}{(\sqrt{a}+\sqrt{b})\cdot\left[a-2\cdot\sqrt{a\cdot b}+b\right]} = \\
&= \frac{a-b}{(\sqrt{a}+\sqrt{b})^2} + \frac{2\cdot(\sqrt{a}-\sqrt{b})\cdot(a+\sqrt{a\cdot b}+b)}{(\sqrt{a}+\sqrt{b})\cdot(\sqrt{a}-\sqrt{b})^2} = \frac{(\sqrt{a}-\sqrt{b})\cdot(\sqrt{a}+\sqrt{b})}{(\sqrt{a}+\sqrt{b})^2} + \frac{2\cdot(\sqrt{a}-\sqrt{b})\cdot(a+\sqrt{a\cdot b}+b)}{(\sqrt{a}+\sqrt{b})\cdot(\sqrt{a}-\sqrt{b})^2} = \\
&= \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} + \frac{2\cdot(a+\sqrt{a\cdot b}+b)}{(\sqrt{a}+\sqrt{b})\cdot(\sqrt{a}-\sqrt{b})} = \left[ \begin{array}{l} \text{zbrajamo razlomke svodeći} \\ \text{ih na zajednički nazivnik} \end{array} \right] = \frac{(\sqrt{a}-\sqrt{b})^2+2\cdot(a+\sqrt{a\cdot b}+b)}{(\sqrt{a}+\sqrt{b})\cdot(\sqrt{a}-\sqrt{b})} = \\
&= \frac{(\sqrt{a})^2-2\cdot\sqrt{a}\cdot\sqrt{b}+(\sqrt{b})^2+2\cdot a+2\cdot\sqrt{a\cdot b}+2\cdot b}{(\sqrt{a})^2-(\sqrt{b})^2} = \frac{a-2\cdot\sqrt{a\cdot b}+b+2\cdot a+2\cdot\sqrt{a\cdot b}+2\cdot b}{a-b} = \\
&= \frac{3\cdot a+3\cdot b}{a-b} = \frac{3\cdot(a+b)}{a-b}.
\end{aligned}$$

### Vježba 168

Pojednostavnite izraz:

$$\frac{a-b}{a+b+\sqrt{(a+b)^2-(a-b)^2}}.$$

**Rezultat:**  $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}.$

### Zadatak 169 (Nena, gimnazija)

Zapiši u obliku potencije s bazom 6:  $3^{2\cdot n-1} \cdot 4^{n+1} + 9^{n+1} \cdot 2^{2\cdot n-1} + 6^{2\cdot n-1}.$

### Rješenje 169

Ponovimo!

$$(a^n)^m = a^{n\cdot m}, \quad a^n \cdot a^m = a^{n+m}, \quad a^n \cdot b^n = (a\cdot b)^n, \quad a^{-n} = \frac{1}{a^n}.$$

1. inačica

$$\begin{aligned}
 & 3^{2 \cdot n - 1} \cdot 4^{n+1} + 9^{n+1} \cdot 2^{2 \cdot n - 1} + 6^{2 \cdot n - 1} = 3^{2 \cdot n - 1} \cdot (2^2)^{n+1} + (3^2)^{n+1} \cdot 2^{2 \cdot n - 1} + 6^{2 \cdot n - 1} = \\
 & = 3^{2 \cdot n - 1} \cdot 2^{2 \cdot n + 2} + 3^{2 \cdot n + 2} \cdot 2^{2 \cdot n - 1} + 6^{2 \cdot n - 1} = 3^{2 \cdot n - 1} \cdot 2^{2 \cdot n - 1} \cdot 2^3 + 3^{2 \cdot n - 1} \cdot 3^3 \cdot 2^{2 \cdot n - 1} + 6^{2 \cdot n - 1} = \\
 & = (3 \cdot 2)^{2 \cdot n - 1} \cdot 2^3 + (3 \cdot 2)^{2 \cdot n - 1} \cdot 3^3 + 6^{2 \cdot n - 1} = 6^{2 \cdot n - 1} \cdot 2^3 + 6^{2 \cdot n - 1} \cdot 3^3 + 6^{2 \cdot n - 1} = \\
 & = 6^{2 \cdot n - 1} \cdot 8 + 6^{2 \cdot n - 1} \cdot 27 + 6^{2 \cdot n - 1} = 6^{2 \cdot n - 1} \cdot (8 + 27 + 1) = 6^{2 \cdot n - 1} \cdot 36 = 6^{2 \cdot n - 1} \cdot 6^2 = 6^{2 \cdot n + 1}.
 \end{aligned}$$

2. inačica

$$\begin{aligned}
 & 3^{2 \cdot n - 1} \cdot 4^{n+1} + 9^{n+1} \cdot 2^{2 \cdot n - 1} + 6^{2 \cdot n - 1} = 3^{2 \cdot n} \cdot 3^{-1} \cdot 4^n \cdot 4 + 9^n \cdot 9 \cdot 2^{2 \cdot n} \cdot 2^{-1} + 6^{2 \cdot n} \cdot 6^{-1} = \\
 & = (3^2)^n \cdot \frac{1}{3} \cdot 4^n \cdot 4 + 9^n \cdot 9 \cdot (2^2)^n \cdot \frac{1}{2} + (6^2)^n \cdot \frac{1}{6} = 9^n \cdot 4^n \cdot \frac{4}{3} + 9^n \cdot 4^n \cdot \frac{9}{2} + 36^n \cdot \frac{1}{6} = \\
 & = (9 \cdot 4)^n \cdot \frac{4}{3} + (9 \cdot 4)^n \cdot \frac{9}{2} + 36^n \cdot \frac{1}{6} = 36^n \cdot \frac{4}{3} + 36^n \cdot \frac{9}{2} + 36^n \cdot \frac{1}{6} = 36^n \cdot \left( \frac{4}{3} + \frac{9}{2} + \frac{1}{6} \right) = \\
 & = 36^n \cdot \frac{8 + 27 + 1}{6} = 36^n \cdot \frac{36}{6} = 36^n \cdot 6 = (6^2)^n \cdot 6 = 6^{2 \cdot n} \cdot 6 = 6^{2 \cdot n + 1}.
 \end{aligned}$$

### Vježba 169

Prikaži u obliku potencije s bazom 10 sljedeći brojevni izraz:  $2^6 \cdot 5^4 + 6 \cdot 10^4$ .

**Rezultat:**  $10^5$ .

### Zadatak 170 (Ines, gimnazija)

Skratite: 
$$\frac{a^{n+2} - 2 \cdot a^n + a^{n-2}}{a^{n+2} - a^{n+1} + a^{n-1} - a^{n-2}}.$$

### Rješenje 170

Ponovimo!

$$\begin{aligned}
 x^n \cdot x^m &= x^{n+m}, & (x-y)^2 &= x^2 - 2 \cdot x \cdot y + y^2, & x^2 - y^2 &= (x-y) \cdot (x+y). \\
 x^3 + y^3 &= (x+y) \cdot (x^2 - x \cdot y + y^2), & x^{-n} &= \frac{1}{x^n}.
 \end{aligned}$$

1. inačica

$$\begin{aligned}
 & \frac{a^{n+2} - 2 \cdot a^n + a^{n-2}}{a^{n+2} - a^{n+1} + a^{n-1} - a^{n-2}} = \left[ \begin{array}{l} \text{u brojniku i nazivniku} \\ \text{izlučimo } a^{n-2} \end{array} \right] = \frac{a^{n-2} \cdot (a^4 - 2 \cdot a^2 + 1)}{a^{n-2} \cdot (a^4 - a^3 + a - 1)} = \\
 & = \frac{a^{n-2} \cdot (a^4 - 2 \cdot a^2 + 1)}{a^{n-2} \cdot (a^4 - a^3 + a - 1)} = \frac{a^4 - 2 \cdot a^2 + 1}{a^4 - a^3 + a - 1} = \frac{(a^2 - 1)^2}{a^3 \cdot (a-1) + (a-1)} = \frac{(a^2 - 1)^2}{(a-1) \cdot (a^3 + 1)} = \\
 & = \frac{(a^2 - 1) \cdot (a^2 - 1)}{(a-1) \cdot (a+1) \cdot (a^2 - a + 1)} = \frac{(a^2 - 1) \cdot (a^2 - 1)}{(a^2 - 1) \cdot (a^2 - a + 1)} = \frac{(a^2 - 1) \cdot (a^2 - 1)}{(a^2 - 1) \cdot (a^2 - a + 1)} = \frac{a^2 - 1}{a^2 - a + 1}.
 \end{aligned}$$

2. inačica

$$\frac{a^{n+2} - 2 \cdot a^n + a^{n-2}}{a^{n+2} - a^{n+1} + a^{n-1} - a^{n-2}} = \frac{a^n \cdot a^2 - 2 \cdot a^n + a^n \cdot a^{-2}}{a^n \cdot a^2 - a^n \cdot a^1 + a^n \cdot a^{-1} - a^n \cdot a^{-2}} = \left[ \begin{array}{l} \text{u brojnici i nazivniku} \\ \text{izlučimo } a^n \end{array} \right] =$$

$$\begin{aligned}
&= \frac{a^n \cdot (a^2 - 2 + a^{-2})}{a^n \cdot (a^2 - a + a^{-1} - a^{-2})} = \frac{a^n \cdot (a^2 - 2 + a^{-2})}{a^n \cdot (a^2 - a + a^{-1} - a^{-2})} = \frac{a^2 - 2 + a^{-2}}{a^2 - a + a^{-1} - a^{-2}} = \frac{a^2 - 2 + \frac{1}{a^2}}{a^2 - a + \frac{1}{a} - \frac{1}{a^2}} = \\
&= \frac{\frac{a^4 - 2 \cdot a^2 + 1}{a^2}}{\frac{a^4 - a^3 + a - 1}{a^2}} = \frac{a^4 - 2 \cdot a^2 + 1}{a^4 - a^3 + a - 1} = \frac{a^4 - 2 \cdot a^2 + 1}{a^3 \cdot (a - 1) + (a - 1)} = \frac{(a^2 - 1)^2}{(a - 1) \cdot (a^3 + 1)} = \\
&= \frac{(a^2 - 1) \cdot (a^2 - 1)}{(a - 1) \cdot (a + 1) \cdot (a^2 - a + 1)} = \frac{(a^2 - 1) \cdot (a^2 - 1)}{(a^2 - 1) \cdot (a^2 - a + 1)} = \frac{(a^2 - 1) \cdot (a^2 - 1)}{(a^2 - 1) \cdot (a^2 - a + 1)} = \frac{a^2 - 1}{a^2 - a + 1}.
\end{aligned}$$

### Vježba 170

Skratite:  $\frac{a^{n+2} - a^{n+1} + a^{n-1} - a^{n-2}}{a^{n+2} - 2 \cdot a^n + a^{n-2}}$ .

**Rezultat:**  $\frac{a^2 - a + 1}{a^2 - 1}$ .

### Zadatak 171 (Goran, gimnazija)

Pojednostavnite:  $\frac{\sqrt{5-2 \cdot \sqrt{6}}}{(4\sqrt{3} + 4\sqrt{2}) \cdot (4\sqrt{3} - 4\sqrt{2})}$ .

### Rješenje 171

Ponovimo!

$$(a+b) \cdot (a-b) = a^2 - b^2, \quad (\sqrt{a})^2 = a, \quad a \geq 0, \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}, \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

$$a \cdot \sqrt{b} = \sqrt{a^2 \cdot b}, \quad \frac{n}{1} = n, \quad \sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A + \sqrt{A^2 - B}}{2}} \pm \sqrt{\frac{A - \sqrt{A^2 - B}}{2}}.$$

1. inačica

$$\begin{aligned}
&\frac{\sqrt{5-2 \cdot \sqrt{6}}}{(4\sqrt{3} + 4\sqrt{2}) \cdot (4\sqrt{3} - 4\sqrt{2})} = \frac{\sqrt{5-2 \cdot \sqrt{6}}}{(4\sqrt{3})^2 - (4\sqrt{2})^2} = \frac{\sqrt{5-2 \cdot \sqrt{6}}}{\sqrt{3} - \sqrt{2}} = \left[ \begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \frac{\sqrt{5-2 \cdot \sqrt{6}}}{\sqrt{3} - \sqrt{2}} \cdot \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \\
&= \frac{\sqrt{5-2 \cdot \sqrt{6}} \cdot (\sqrt{3} + \sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{\sqrt{(5-2 \cdot \sqrt{6}) \cdot (\sqrt{3} + \sqrt{2})^2}}{3-2} = \frac{\sqrt{(5-2 \cdot \sqrt{6}) \cdot ((\sqrt{3})^2 + 2 \cdot \sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^2)}}{1} = \\
&= \sqrt{(5-2 \cdot \sqrt{6}) \cdot (3+2 \cdot \sqrt{6}+2)} = \sqrt{(5-2 \cdot \sqrt{6}) \cdot (5+2 \cdot \sqrt{6})} = \sqrt{5^2 - (2 \cdot \sqrt{6})^2} = \sqrt{25-24} = \sqrt{1} = 1.
\end{aligned}$$

2. inačica

$$\frac{\sqrt{5-2 \cdot \sqrt{6}}}{(4\sqrt{3} + 4\sqrt{2}) \cdot (4\sqrt{3} - 4\sqrt{2})} = \frac{\sqrt{5-\sqrt{2^2 \cdot 6}}}{(4\sqrt{3})^2 - (4\sqrt{2})^2} = \frac{\sqrt{5-\sqrt{24}}}{\sqrt{3} - \sqrt{2}} =$$

$$\begin{aligned}
&= \frac{\sqrt{\frac{5+\sqrt{5^2-24}}{2}} - \sqrt{\frac{5-\sqrt{5^2-24}}{2}}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{\frac{5+\sqrt{25-24}}{2}} - \sqrt{\frac{5-\sqrt{25-24}}{2}}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{\frac{5+\sqrt{1}}{2}} - \sqrt{\frac{5-\sqrt{1}}{2}}}{\sqrt{3}-\sqrt{2}} = \\
&= \frac{\sqrt{\frac{5+1}{2}} - \sqrt{\frac{5-1}{2}}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{\frac{6}{2}} - \sqrt{\frac{4}{2}}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 1.
\end{aligned}$$

### Vježba 171

Pojednostavnite: 
$$\frac{(4\sqrt{3}+4\sqrt{2}) \cdot (4\sqrt{3}-4\sqrt{2})}{\sqrt{5-2} \cdot \sqrt{6}}.$$

**Rezultat:** 1.

### Zadatak 172 (Ema, gimnazija)

Skrati razlomak: 
$$\frac{a^3 \cdot c - 2 \cdot a^2 \cdot c^2 + a \cdot c^3 - a \cdot b^2 \cdot c}{(a^2 + c^2 - b^2)^2 - 4 \cdot a^2 \cdot c^2}.$$

### Rješenje 172

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2, \quad (x+y)^2 = x^2 + 2 \cdot x \cdot y + y^2, \quad x^2 - y^2 = (x-y) \cdot (x+y).$$

$$\begin{aligned}
&\frac{a^3 \cdot c - 2 \cdot a^2 \cdot c^2 + a \cdot c^3 - a \cdot b^2 \cdot c}{(a^2 + c^2 - b^2)^2 - 4 \cdot a^2 \cdot c^2} = \left[ \begin{array}{l} \text{u brojniku izlučimo } a \cdot c, \\ \text{u nazivniku je razlika kvadrata} \end{array} \right] = \\
&= \frac{a \cdot c \cdot (a^2 - 2 \cdot a \cdot c + c^2 - b^2)}{(a^2 + c^2 - b^2 - 2 \cdot a \cdot c) \cdot (a^2 + c^2 - b^2 + 2 \cdot a \cdot c)} = \frac{a \cdot c \cdot (a^2 - 2 \cdot a \cdot c + c^2 - b^2)}{(a^2 - 2 \cdot a \cdot c + c^2 - b^2) \cdot (a^2 + 2 \cdot a \cdot c + c^2 - b^2)} = \\
&= \frac{a \cdot c \cdot ((a^2 - 2 \cdot a \cdot c + c^2) - b^2)}{((a^2 - 2 \cdot a \cdot c + c^2) - b^2) \cdot ((a^2 + 2 \cdot a \cdot c + c^2) - b^2)} = \frac{a \cdot c \cdot ((a-c)^2 - b^2)}{((a-c)^2 - b^2) \cdot ((a+c)^2 - b^2)} = \\
&= \frac{a \cdot c \cdot (a-c-b) \cdot (a-c+b)}{(a-c-b) \cdot (a-c+b) \cdot (a+c-b) \cdot (a+c+b)} = \frac{a \cdot c \cdot (a-c-b) \cdot (a-c+b)}{(a-c-b) \cdot (a-c+b) \cdot (a+c-b) \cdot (a+c+b)} = \\
&= \frac{a \cdot c}{(a+c-b) \cdot (a+c+b)} = \frac{a \cdot c}{(a+c)^2 - b^2}.
\end{aligned}$$

### Vježba 172

Skrati razlomak: 
$$\frac{4 \cdot a^2 \cdot c^2 - (a^2 + c^2 - b^2)^2}{a \cdot b^2 \cdot c - a \cdot c^3 + 2 \cdot a^2 \cdot c^2 - a^3 \cdot c}.$$

**Rezultat:** 
$$\frac{(a+c)^2 - b^2}{a \cdot c}.$$

**Zadatak 173 (Miki, gimnazija)**Racionaliziraj nazivnik:  $\frac{1}{\sqrt{10} + \sqrt{15} + \sqrt{14} + \sqrt{21}}$ .**Rješenje 173**

Ponovimo!

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \quad , \quad (a+b) \cdot (a-b) = a^2 - b^2 \quad , \quad (\sqrt{a})^2 = a \quad , \quad a \geq 0.$$

$$\begin{aligned} \frac{1}{\sqrt{10} + \sqrt{15} + \sqrt{14} + \sqrt{21}} &= \frac{1}{\sqrt{2 \cdot 5} + \sqrt{3 \cdot 5} + \sqrt{2 \cdot 7} + \sqrt{3 \cdot 7}} = \frac{1}{\sqrt{2} \cdot \sqrt{5} + \sqrt{3} \cdot \sqrt{5} + \sqrt{2} \cdot \sqrt{7} + \sqrt{3} \cdot \sqrt{7}} = \\ &= \frac{1}{\sqrt{5} \cdot (\sqrt{2} + \sqrt{3}) + \sqrt{7} \cdot (\sqrt{2} + \sqrt{3})} = \frac{1}{(\sqrt{2} + \sqrt{3}) \cdot (\sqrt{5} + \sqrt{7})} = \frac{1}{(\sqrt{3} + \sqrt{2}) \cdot (\sqrt{7} + \sqrt{5})} = \\ &= \left[ \begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \frac{1}{(\sqrt{3} + \sqrt{2}) \cdot (\sqrt{7} + \sqrt{5})} \cdot \frac{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{7} - \sqrt{5})}{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{7} - \sqrt{5})} = \\ &= \frac{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{7} - \sqrt{5})}{(\sqrt{3} + \sqrt{2}) \cdot (\sqrt{3} - \sqrt{2}) \cdot (\sqrt{7} + \sqrt{5}) \cdot (\sqrt{7} - \sqrt{5})} = \frac{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{7} - \sqrt{5})}{\left( (\sqrt{3})^2 - (\sqrt{2})^2 \right) \cdot \left( (\sqrt{7})^2 - (\sqrt{5})^2 \right)} = \\ &= \frac{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{7} - \sqrt{5})}{(3-2) \cdot (7-5)} = \frac{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{7} - \sqrt{5})}{1 \cdot 2} = \frac{(\sqrt{3} - \sqrt{2}) \cdot (\sqrt{7} - \sqrt{5})}{2}. \end{aligned}$$

**Vježba 173**Racionaliziraj nazivnik:  $\frac{1}{\sqrt{7} - \sqrt{6}}$ .**Rezultat:**  $\sqrt{7} + \sqrt{6}$ .**Zadatak 174 (Sučo, elektrostrojarska škola)**Izračunaj:  $3 \cdot (2 \cdot a - 3 \cdot b)^2 - 5 \cdot (a - 3 \cdot b) \cdot (a + 3 \cdot b)$ .**Rješenje 174**

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad (x-y) \cdot (x+y) = x^2 - y^2.$$

$$\begin{aligned} 3 \cdot (2 \cdot a - 3 \cdot b)^2 - 5 \cdot (a - 3 \cdot b) \cdot (a + 3 \cdot b) &= \left[ \begin{array}{l} \text{prvu zagradu kvadriramo,} \\ \text{druge dvije zagrade čine razliku kvadrata} \end{array} \right] = \\ &= 3 \cdot \left( (2 \cdot a)^2 - 2 \cdot 2 \cdot a \cdot 3 \cdot b + (3 \cdot b)^2 \right) - 5 \cdot \left( a^2 - (3 \cdot b)^2 \right) = 3 \cdot \left( 4 \cdot a^2 - 12 \cdot a \cdot b + 9 \cdot b^2 \right) - 5 \cdot \left( a^2 - 9 \cdot b^2 \right) = \\ &= 12 \cdot a^2 - 36 \cdot a \cdot b + 27 \cdot b^2 - 5 \cdot a^2 + 45 \cdot b^2 = 7 \cdot a^2 - 36 \cdot a \cdot b + 72 \cdot b^2. \end{aligned}$$

**Vježba 174**Izračunaj:  $3 \cdot (2 \cdot a - 3 \cdot b)^2 + 5 \cdot (a - 3 \cdot b) \cdot (a + 3 \cdot b)$ .**Rezultat:**  $17 \cdot a^2 - 36 \cdot a \cdot b - 18 \cdot b^2$ .**Zadatak 175 (Sučo, elektrostrojarska škola)**Izračunaj:  $2 \cdot (a - 3 \cdot b)^3 - (2 \cdot a + 3 \cdot b) \cdot (4 \cdot a^2 - 6 \cdot a \cdot b + 9 \cdot b^2)$ .**Rješenje 175**

Ponovimo!



$$(x-y)^3 = x^3 - 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 - y^3, \quad (x+y) \cdot (x^2 - x \cdot y + y^2) = x^3 - y^3.$$

$$\begin{aligned} 2 \cdot (a-3 \cdot b)^3 - (2 \cdot a + 3 \cdot b) \cdot (4 \cdot a^2 - 6 \cdot a \cdot b + 9 \cdot b^2) &= \left[ \begin{array}{l} \text{prvu zagradu kubiramo,} \\ \text{druge dvije zagrade čine zbroj kubova} \end{array} \right] = \\ &= 2 \cdot (a^3 - 3 \cdot a^2 \cdot 3 \cdot b + 3 \cdot a \cdot (3 \cdot b)^2 - (3 \cdot b)^3) - ((2 \cdot a)^3 + (3 \cdot b)^3) = \\ &= 2 \cdot (a^3 - 9 \cdot a^2 \cdot b + 3 \cdot a \cdot 9 \cdot b^2 - 27 \cdot b^3) - (8 \cdot a^3 + 27 \cdot b^3) = \\ &= 2 \cdot (a^3 - 9 \cdot a^2 \cdot b + 27 \cdot a \cdot b^2 - 27 \cdot b^3) - (8 \cdot a^3 + 27 \cdot b^3) = \\ &= 2 \cdot a^3 - 18 \cdot a^2 \cdot b + 54 \cdot a \cdot b^2 - 54 \cdot b^3 - 8 \cdot a^3 - 27 \cdot b^3 = -6 \cdot a^3 - 18 \cdot a^2 \cdot b + 54 \cdot a \cdot b^2 - 81 \cdot b^3 = \\ &= -3 \cdot (2 \cdot a^3 + 6 \cdot a^2 \cdot b - 18 \cdot a \cdot b^2 + 27 \cdot b^3). \end{aligned}$$

### Vježba 175

Izračunaj:  $2 \cdot (a-3 \cdot b)^3 + (2 \cdot a + 3 \cdot b) \cdot (4 \cdot a^2 - 6 \cdot a \cdot b + 9 \cdot b^2)$ .

**Rezultat:**  $10 \cdot a^3 - 18 \cdot a^2 \cdot b + 54 \cdot a \cdot b^2 - 27 \cdot b^3$ .

### Zadatak 176 (Sućo, elektrostrojarska škola)

Rastavi na faktore:  $8 \cdot a^2 - 50 \cdot b^2$ .

### Rješenje 176

Ponovimo!

$$(x+y) \cdot (x-y) = x^2 - y^2.$$

$$8 \cdot a^2 - 50 \cdot b^2 = 2 \cdot (4 \cdot a^2 - 25 \cdot b^2) = 2 \cdot (2 \cdot a - 5 \cdot b) \cdot (2 \cdot a + 5 \cdot b).$$

### Vježba 176

Rastavi na faktore:  $12 \cdot a^2 - 75 \cdot b^2$ .

**Rezultat:**  $3 \cdot (2 \cdot a - 5 \cdot b) \cdot (2 \cdot a + 5 \cdot b)$ .

### Zadatak 177 (Sućo, elektrostrojarska škola)

Skrati razlomak:  $\frac{x \cdot y - y - x + 1}{-1 + 2 \cdot x - x^2}$ .

### Rješenje 177

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

$$\begin{aligned} \frac{x \cdot y - y - x + 1}{-1 + 2 \cdot x - x^2} &= \frac{x \cdot y - y - x + 1}{-x^2 + 2 \cdot x - 1} = \frac{x \cdot y - y - x + 1}{-(x^2 - 2 \cdot x + 1)} = \frac{x \cdot y - y - x + 1}{-(x-1)^2} = \frac{y \cdot (x-1) - (x-1)}{-(x-1)^2} = \\ &= \frac{(x-1) \cdot (y-1)}{-(x-1)^2} = \frac{(x-1) \cdot (y-1)}{-(x-1)^2} = \frac{y-1}{-(x-1)} = \frac{y-1}{-x+1} = \frac{y-1}{1-x}. \end{aligned}$$

### Vježba 177

Skrati razlomak:  $\frac{-1 + 2 \cdot x - x^2}{x \cdot y - y - x + 1}$ .

**Rezultat:**  $\frac{1-x}{y-1}$ .

**Zadatak 178 (Sučo, elektrostrojarska škola)**

$$\text{Izračunaj: } \frac{\frac{a-b}{a+b} + \frac{a+b}{a-b}}{\frac{a+b}{a-b} - \frac{a-b}{a+b}}$$

**Rješenje 178**

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2, \quad (x+y)^2 = x^2 + 2 \cdot x \cdot y + y^2.$$

$$\begin{aligned} \frac{\frac{a-b}{a+b} + \frac{a+b}{a-b}}{\frac{a+b}{a-b} - \frac{a-b}{a+b}} &= \frac{\frac{(a-b) \cdot (a-b) + (a+b) \cdot (a+b)}{(a+b) \cdot (a-b)}}{\frac{(a+b) \cdot (a+b) - (a-b) \cdot (a-b)}{(a-b) \cdot (a+b)}} = \frac{\frac{(a-b)^2 + (a+b)^2}{(a+b) \cdot (a-b)}}{\frac{(a+b)^2 - (a-b)^2}{(a-b) \cdot (a+b)}} = \\ &= \frac{(a-b)^2 + (a+b)^2}{(a+b)^2 - (a-b)^2} = \frac{a^2 - 2 \cdot a \cdot b + b^2 + a^2 + 2 \cdot a \cdot b + b^2}{a^2 + 2 \cdot a \cdot b + b^2 - (a^2 - 2 \cdot a \cdot b + b^2)} = \frac{a^2 - 2 \cdot a \cdot b + b^2 + a^2 + 2 \cdot a \cdot b + b^2}{a^2 + 2 \cdot a \cdot b + b^2 - a^2 + 2 \cdot a \cdot b - b^2} = \\ &= \frac{a^2 - 2 \cdot a \cdot b + b^2 + a^2 + 2 \cdot a \cdot b + b^2}{a^2 + 2 \cdot a \cdot b + b^2 - a^2 + 2 \cdot a \cdot b - b^2} = \frac{2 \cdot a^2 + 2 \cdot b^2}{4 \cdot a \cdot b} = \frac{2 \cdot (a^2 + b^2)}{4 \cdot a \cdot b} = \frac{2 \cdot (a^2 + b^2)}{4 \cdot a \cdot b} = \frac{a^2 + b^2}{2 \cdot a \cdot b}. \end{aligned}$$

**Vježba 178**

$$\text{Izračunaj: } \frac{\frac{a+b}{a-b} - \frac{a-b}{a+b}}{\frac{a-b}{a+b} + \frac{a+b}{a-b}}$$

$$\text{Rezultat: } \frac{2 \cdot a \cdot b}{a^2 + b^2}.$$

**Zadatak 179 (Robertina kumica ☺, ekonomska škola)**

$$\text{Izračunaj: } \sqrt[3]{2 \cdot \sqrt{2}}.$$

**Rješenje 179**

Ponovimo!

$$n\sqrt[a^m]{a} = a^{\frac{m}{n}}, \quad a^n \cdot a^m = a^{n+m}, \quad (a^n)^m = a^{n \cdot m}, \quad a^1 = a, \quad a \cdot \sqrt{b} = \sqrt{a^2 \cdot b}.$$

$$n\sqrt[m]{\sqrt[a]{a}} = n \cdot m \sqrt[a]{a}, \quad n\sqrt[a^m]{a} = n \cdot p \sqrt[a^{m \cdot p}]{a}.$$

1. inačica

Redom pretvaramo korijene u racionalni eksponent, te primjenom svojstava potencije pojednostavimo rezultat:

$$\sqrt[3]{2 \cdot \sqrt{2}} = (2 \cdot \sqrt{2})^{\frac{1}{3}} = \left(2 \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(2^1 \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{3}} = \left(2^{1+\frac{1}{2}}\right)^{\frac{1}{3}} = \left(2^{\frac{3}{2}}\right)^{\frac{1}{3}} = \left(2^{\frac{3}{2}}\right)^{\frac{1}{3}} = 2^{\frac{1}{2}} = \sqrt{2}.$$

2. inačica

Korijene prvo svedemo na jedan korijen:

$$\sqrt[3]{2 \cdot \sqrt{2}} = \sqrt[3]{\sqrt{2 \cdot 2} \cdot 2} = \sqrt[3]{\sqrt{2 \cdot 2} \cdot 2^1} = \sqrt[3]{\sqrt{2^3}} = \sqrt[3]{2^3} = \sqrt[3]{2^3} = \sqrt{2}.$$

### Vježba 179

Izračunaj:  $\sqrt[4]{7 \cdot \sqrt{7}}$ .

**Rezultat:**  $8\sqrt[3]{7}$ .

### Zadatak 180 (Robertina kumica ☺, ekonomska škola)

Izračunaj:  $\sqrt[3]{\sqrt[3]{a^{-4}} \cdot \sqrt{a}}$ .

#### Rješenje 180

Ponovimo!

$$n\sqrt[a]{m} = a^{\frac{m}{n}}, \quad a^n \cdot a^m = a^{n+m}, \quad (a^n)^m = a^{n \cdot m}, \quad a^1 = a, \quad a \cdot \sqrt{b} = \sqrt{a^2 \cdot b}.$$

$$n\sqrt[m]{\sqrt{a}} = n \cdot m\sqrt{a}, \quad n\sqrt[a]{m} = n \cdot p\sqrt[a^{m \cdot p}]{}.$$

1. inačica

Redom pretvaramo korijene u racionalni eksponent, te primjenom svojstava potencije pojednostavimo rezultat:

$$\begin{aligned} \sqrt[3]{\sqrt[3]{a^{-4}} \cdot \sqrt{a}} &= \left( \sqrt[3]{a^{-4}} \cdot \sqrt{a} \right)^{\frac{1}{3}} = \left( a^{-4} \cdot \sqrt{a} \right)^{\frac{1}{3}} = \left( a^{-4} \cdot a^{\frac{1}{2}} \right)^{\frac{1}{3}} = \left( a^{-4 + \frac{1}{2}} \right)^{\frac{1}{3}} = \\ &= \left( a^{-\frac{7}{2}} \right)^{\frac{1}{3}} = \left( a^{-\frac{7}{6}} \right)^{\frac{1}{3}} = a^{-\frac{7}{18}} = 18\sqrt{a^{-7}}. \end{aligned}$$

2. inačica

Korijene prvo svedemo na jedan korijen:

$$\begin{aligned} \sqrt[3]{\sqrt[3]{a^{-4}} \cdot \sqrt{a}} &= \sqrt[3]{\sqrt[3]{\sqrt{(a^{-4})^2} \cdot a}} = \sqrt[3]{\sqrt[3]{\sqrt{a^{-8}} \cdot a}} = \sqrt[3]{\sqrt[3]{\sqrt{a^{-8}} \cdot a^1}} = \sqrt[3]{\sqrt[3]{\sqrt{a^{-7}}}} = \\ &= \sqrt[9]{\sqrt{a^{-7}}} = 18\sqrt{a^{-7}}. \end{aligned}$$

### Vježba 180

Izračunaj:  $\sqrt[5]{\sqrt[3]{a^{-4}} \cdot \sqrt{a}}$ .

**Rezultat:**  $30\sqrt{a^{-7}}$ .