Zadatak 441 (H2O, gimnazija)

Koliko je $5\cdot 2^{2010}-3\cdot 2^{2011}+14\cdot 2^{2009}$?

Rješenje 441

Ponovimo!

$$a^n\cdot a^m=a^{n+m}\quad,\quad a^1=a\quad,\quad a^{-n}=\frac{1}{a^n}\quad,\quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c}\quad,\quad \frac{n}{1}=n.$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$5\cdot 2^{2010}-3\cdot 2^{2011}+14\cdot 2^{2009}=5\cdot 2^{2009}\cdot 2^1-3\cdot 2^{2009}\cdot 2^2+14\cdot 2^{2009}=$$

$$=2^{2009}\cdot\left(5\cdot 2^1-3\cdot 2^2+14\right)=2^{2009}\cdot(5\cdot 2-3\cdot 4+14)=2^{2009}\cdot(10-12+14)=2^{2009}\cdot 12=$$

$$=2^{2009}\cdot 4\cdot 3=2^{2009}\cdot 2^2\cdot 3=2^{2011}\cdot 3=3\cdot 2^{2011}.$$

2.inačica

$$5\cdot 2^{2010}-3\cdot 2^{2011}+14\cdot 2^{2009}=5\cdot 2^{2010}-3\cdot 2^{2010}\cdot 2^1+14\cdot 2^{2010}\cdot 2^{-1}=$$

$$=2^{2010}\cdot\left(5-3\cdot 2^1+14\cdot 2^{-1}\right)=2^{2010}\cdot\left(5-3\cdot 2+14\cdot\frac{1}{2}\right)=2^{2010}\cdot\left(5-6+\frac{14}{2}\right)=$$

$$=2^{2010}\cdot(5-6+7)=2^{2010}\cdot 6=2^{2010}\cdot 2\cdot 3=2^{2010}\cdot 2^1\cdot 3=2^{2011}\cdot 3=3\cdot 2^{2011}.$$

3.inačica

$$5\cdot 2^{2010}-3\cdot 2^{2011}+14\cdot 2^{2009}=5\cdot 2^{2011}\cdot 2^{-1}-3\cdot 2^{2011}+14\cdot 2^{2011}\cdot 2^{-2}=$$

$$=2^{2011}\cdot\left(5\cdot 2^{-1}-3+14\cdot 2^{-2}\right)=2^{2011}\cdot\left(5\cdot\frac{1}{2}-3+14\cdot\frac{1}{2^2}\right)=2^{2011}\cdot\left(5\cdot\frac{1}{2}-3+14\cdot\frac{1}{4}\right)=$$

$$=2^{2011}\cdot\left(\frac{5}{2}-3+\frac{14}{4}\right)=2^{2011}\cdot\left(\frac{5}{2}-3+\frac{14}{4}\right)=2^{2011}\cdot\left(\frac{5}{2}-3+\frac{7}{2}\right)=2^{2011}\cdot\left(\frac{5}{2}-\frac{3}{1}+\frac{7}{2}\right)=$$

$$=2^{2011}\cdot\frac{5-6+7}{2}=2^{2011}\cdot\frac{6}{2}=2^{2011}\cdot 3=3\cdot 2^{2011}.$$

Vježba 441

Koliko je $10\cdot 2^{2009}-3\cdot 2^{2011}+7\cdot 2^{2010}$?

Rezultat: $3\cdot 2^{2011}.$

Zadatak 442 (Marija, srednja škola)

Izraz $\left(\frac{1}{a}-\frac{1}{b}\right)\cdot\frac{a^2+a\cdot b}{a^2-b^2}$ jednak je:

$A.\ \frac{a}{b}$ $\quad B.\ \frac{1}{a\cdot b}$ $\quad C.\ -\frac{1}{a\cdot b}$ $\quad D.\ -\frac{1}{b}$

Rješenje 442

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad (a-b)\cdot(a+b)=a^2-b^2\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(\frac{1}{a}-\frac{1}{b}\right)\cdot\frac{a^2+a\cdot b}{a^2-b^2}=\frac{b-a}{a\cdot b}\cdot\frac{a^2+a\cdot b}{a^2-b^2}=\frac{-(a-b)}{a\cdot b}\cdot\frac{a^2+a\cdot b}{a^2-b^2}=\frac{-(a-b)}{a\cdot b}\cdot\frac{a\cdot(a+b)}{(a-b)\cdot(a+b)}=$$

$$=\frac{-(a-b)}{a\cdot b}\cdot\frac{a\cdot(a+b)}{(a-b)\cdot(a+b)}=\frac{-1}{b}\cdot\frac{1}{1}=-\frac{1}{b}.$$

Odgovor je pod D.

Vježba 442

Izraz $\left(\frac{1}{a}-\frac{1}{b}\right):\frac{a^2-b^2}{a^2+a\cdot b}$ jednak je:

$A.\ \frac{a}{b}$ $\quad B.\ \frac{1}{a\cdot b}$ $\quad C.\ -\frac{1}{a\cdot b}$ $\quad D.\ -\frac{1}{b}$

Rezultat: D.

Zadatak 443 (Mira, srednja škola)

Ako je $A=(1-x)\cdot\left(1+x^2\right)$, $B=(1+x)\cdot\left(1+x^4\right)$, onda je vrijednost polinoma $A\cdot B-1$ za $x=\sqrt[4]{8}$ jednaka:

$A.\ -64$ $\quad B.\ 7$ $\quad C.\ 32$ $\quad D.\ -11$

Rješenje 443

Ponovimo!

$$a\cdot b=b\cdot a\quad,\quad (a-b)\cdot(a+b)=a^2-b^2\quad,\quad \left(a^n\right)^m=a^{n\cdot m}\quad,\quad \left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}.$$

$$\sqrt[n\cdot x]{a^{m\cdot x}}=\sqrt[n]{a^m}.$$

$$A\cdot B-1=(1-x)\cdot\left(1+x^2\right)\cdot(1+x)\cdot\left(1+x^4\right)-1=(1-x)\cdot(1+x)\cdot\left(1+x^2\right)\cdot\left(1+x^4\right)-1=$$

$$=\left(1-x^2\right)\cdot\left(1+x^2\right)\cdot\left(1+x^4\right)-1=\left(1-\left(x^2\right)^2\right)\cdot\left(1+x^4\right)-1=\left(1-x^4\right)\cdot\left(1+x^4\right)-1=1-\left(x^4\right)^2-1=$$

$$=1-x^8-1=1-x^8-1=-x^8=-\left(\sqrt[4]{8}\right)^8=-\sqrt[4]{8^8}=-8^2=-64.$$

Odgovor je pod A.

Vježba 443

Ako je $A=(1-x)\cdot\left(1+x^2\right)$, $B=(1+x)\cdot\left(1+x^4\right)$, onda je vrijednost polinoma $A\cdot B-1$ za $x=\sqrt[8]{8}$ jednaka :

$A.\ -7 \qquad B.\ -8 \qquad C.\ 8 \qquad D.\ 16$

Rezultat: B.

Zadatak 444 (Branka, strukovna škola)

Napiši u što jednostavnijem obliku: $\frac{a^{2\cdot n}-1}{a^{2\cdot n}-b^{2\cdot n}}\cdot\frac{a^n-b^n}{a^n-1}.$

Rješenje 444

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{a^{2\cdot n}-1}{a^{2\cdot n}-b^{2\cdot n}}\cdot\frac{a^n-b^n}{a^n-1}=\frac{\left(a^n\right)^2-1}{\left(a^n\right)^2-\left(b^n\right)^2}\cdot\frac{a^n-b^n}{a^n-1}=\frac{\left(a^n-1\right)\cdot\left(a^n+1\right)}{\left(a^n-b^n\right)\cdot\left(a^n+b^n\right)}\cdot\frac{a^n-b^n}{a^n-1}=$$

$$=\frac{\left(a^n-1\right)\cdot\left(a^n+1\right)}{\left(a^n-b^n\right)\cdot\left(a^n+b^n\right)}\cdot\frac{a^n-b^n}{a^n-1}=\frac{a^n+1}{a^n+b^n}.$$

Vježba 444

Napiši u što jednostavnijem obliku: $\frac{a^{2\cdot n}-b^{2\cdot n}}{a^{2\cdot n}-1}\cdot\frac{a^n-1}{a^n-b^n}.$

Rezultat: $\frac{a^n+b^n}{a^n+1}.$

Zadatak 445 (Tomislav, strukovna škola)

Izračunajte: $\frac{(3\cdot a-6)^2}{a^2-2\cdot a}.$

Rješenje 445

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{(3\cdot a-6)^2}{a^2-2\cdot a}=\frac{(3\cdot a)^2-2\cdot 3\cdot a\cdot 6+6^2}{a\cdot(a-2)}=\frac{9\cdot a^2-36\cdot a+36}{a\cdot(a-2)}=\frac{9\cdot\left(a^2-4\cdot a+4\right)}{a\cdot(a-2)}=$$

$$=\frac{9\cdot(a-2)^2}{a\cdot(a-2)}=\frac{9\cdot(a-2)^2}{a\cdot(a-2)}=\frac{9\cdot(a-2)}{a}.$$

2.inačica

$$\frac{(3\cdot a-6)^2}{a^2-2\cdot a}=\frac{(3\cdot(a-2))^2}{a\cdot(a-2)}=\frac{3^2\cdot(a-2)^2}{a\cdot(a-2)}=\frac{9\cdot(a-2)^2}{a\cdot(a-2)}=\frac{9\cdot(a-2)^2}{a\cdot(a-2)}=\frac{9\cdot(a-2)}{a}.$$

Vježba 445

Izračunajte: $\frac{(2\cdot a-4)^2}{a^2-2\cdot a}$.

Rezultat: $\frac{4\cdot(a-2)}{a}$.

Zadatak 446 (Martina, strukovna škola)

Pojednostavnite izraz: $2\cdot\sqrt{12}+3\cdot\sqrt{3}+6\cdot\sqrt{\frac{1}{3}}$.

$A.\ 6\cdot\sqrt{3}$ $\quad B.\ 7\cdot\sqrt{3}$ $\quad C.\ 8\cdot\sqrt{3}$ $\quad D.\ 9\cdot\sqrt{3}$

Rješenje 446

Ponovimo!

$\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}$, $\sqrt{a^2}=a$, $a\ge 0$, $a\cdot\sqrt{b}=\sqrt{a^2\cdot b}$, $a\cdot\sqrt{n}+b\cdot\sqrt{n}=(a+b)\cdot\sqrt{n}$.

$$a\cdot\frac{b}{c}=\frac{a\cdot b}{c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\ne 0\ ,\ n\ne 1.$$

$$2\cdot\sqrt{12}+3\cdot\sqrt{3}+6\cdot\sqrt{\frac{1}{3}}=2\cdot\sqrt{4\cdot 3}+3\cdot\sqrt{3}+2\cdot 3\cdot\sqrt{\frac{1}{3}}=2\cdot\sqrt{4}\cdot\sqrt{3}+3\cdot\sqrt{3}+2\cdot\sqrt{3^2\cdot\frac{1}{3}}=$$

$$=2\cdot\sqrt{2^2}\cdot\sqrt{3}+3\cdot\sqrt{3}+2\cdot\sqrt{\frac{3^2}{3}}=2\cdot 2\cdot\sqrt{3}+3\cdot\sqrt{3}+2\cdot\sqrt{\frac{3^2}{3}}=$$

$$=4\cdot\sqrt{3}+3\cdot\sqrt{3}+2\cdot\sqrt{3}=4\cdot\sqrt{3}+3\cdot\sqrt{3}+2\cdot\sqrt{3}=(4+3+2)\cdot\sqrt{3}=9\cdot\sqrt{3}.$$

Odgovor je pod D.

Vježba 446

Pojednostavnite izraz: $3\cdot\sqrt{20}+4\cdot\sqrt{5}+10\cdot\sqrt{\frac{1}{5}}$.

$A.\ 12\cdot\sqrt{5}$ $\quad B.\ 11\cdot\sqrt{5}$ $\quad C.\ 10\cdot\sqrt{5}$ $\quad D.\ 9\cdot\sqrt{5}$

Rezultat: A.

Zadatak 447 (Ivana, ekonomska škola)

Ako je $a+2\cdot b=3,\ b+2\cdot c=5,\ c+2\cdot a=7,$ koliko je $a+b+c$?

Rješenje 447

Ponovimo!

$$\left.\begin{matrix} a=b \\ c=d \end{matrix}\right\} \Rightarrow a+c=b+d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left.\begin{matrix} a+2\cdot b=3 \\ b+2\cdot c=5 \\ c+2\cdot a=7 \end{matrix}\right\} \Rightarrow \left[\begin{matrix}\text{zbrojimo} \\ \text{jednakosti}\end{matrix}\right] \Rightarrow a+2\cdot b+b+2\cdot c+c+2\cdot a=3+5+7 \Rightarrow$$

$$\Rightarrow 3\cdot a+3\cdot b+3\cdot c=15 \Rightarrow 3\cdot(a+b+c)=15 \Rightarrow 3\cdot(a+b+c)=15 \ /:3 \Rightarrow a+b+c=5.$$

Vježba 447

Ako je $a+2\cdot b=5,\ b+2\cdot c=8,\ c+2\cdot a=5,$ koliko je $a+b+c$?

Rezultat: 6.

Zadatak 448 (Ivana, ekonomska škola)

Čemu je jednak b ako je $k=\frac{c}{a+b}$?

$A.\ b=\frac{c-a\cdot k}{k}$ $\quad B.\ b=\frac{a\cdot k-c}{k}$ $\quad C.\ b=\frac{k}{c-a\cdot k}$ $\quad D.\ b=\frac{k}{a\cdot k-c}$

Rješenje 448

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c} \quad , \quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c}.$$

1.inačica

$$k=\frac{c}{a+b} \Rightarrow k=\frac{c}{a+b} \ /\cdot(a+b) \Rightarrow k\cdot(a+b)=c \Rightarrow k\cdot a+k\cdot b=c \Rightarrow k\cdot b=c-k\cdot a \Rightarrow$$

$$\Rightarrow k\cdot b=c-k\cdot a \ /:k \Rightarrow b=\frac{c-k\cdot a}{k} \Rightarrow b=\frac{c-a\cdot k}{k}.$$

Odgovor je pod A.

2.inačica

$$k=\frac{c}{a+b} \Rightarrow k=\frac{c}{a+b} \ /\cdot(a+b) \Rightarrow k\cdot(a+b)=c \Rightarrow k\cdot(a+b)=c \ /:k \Rightarrow a+b=\frac{c}{k} \Rightarrow$$

$$\Rightarrow b=\frac{c}{k}-a \Rightarrow b=\frac{c}{k}-\frac{a}{1} \Rightarrow b=\frac{c-a\cdot k}{k}.$$

Odgovor je pod A.

3.inačica

$$k=\frac{c}{a+b} \Rightarrow \frac{k}{1}=\frac{c}{a+b} \Rightarrow \frac{1}{k}=\frac{a+b}{c} \Rightarrow \frac{a+b}{c}=\frac{1}{k} \Rightarrow \frac{a+b}{c}=\frac{1}{k} \;/\cdot c \Rightarrow$$

$$\Rightarrow a+b=\frac{c}{k} \Rightarrow b=\frac{c}{k}-a \Rightarrow b=\frac{c}{k}-\frac{a}{1} \Rightarrow b=\frac{c-a\cdot k}{k}.$$

Odgovor je pod A.

Vježba 448

Čemu je jednak b ako je $k=\frac{c}{a+b}$?

$A.\ a=\frac{c-b\cdot k}{k}$ $\quad B.\ a=\frac{b\cdot k-c}{k}$ $\quad C.\ a=\frac{k}{c-b\cdot k}$ $\quad D.\ a=\frac{k}{b\cdot k-c}$

Rezultat: A.

Zadatak 449 (Mihaela, gimnazija)

Ako je $\frac{3\cdot a-b}{a+3\cdot b}=2,$ koliko je $\frac{a-3\cdot b}{3\cdot a+b}$?

$A.\ \frac{1}{2}$ $\quad B.\ \frac{2}{11}$ $\quad C.\ \frac{3}{4}$ $\quad D.\ \frac{1}{4}$

Rješenje 449

Ponovimo!

$$n=\frac{n}{1}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

Transformiramo zadanu jednakost.

$$\frac{3\cdot a-b}{a+3\cdot b}=2 \Rightarrow \frac{3\cdot a-b}{a+3\cdot b}=\frac{2}{1} \Rightarrow \frac{3\cdot a-b}{a+3\cdot b}=\frac{2}{1} \;/\cdot(a+3\cdot b) \Rightarrow 3\cdot a-b=2\cdot(a+3\cdot b) \Rightarrow$$

$$\Rightarrow 3\cdot a-b=2\cdot a+6\cdot b \Rightarrow 3\cdot a-2\cdot a=6\cdot b+b \Rightarrow a=7\cdot b.$$

Tada je:

$$\frac{a-3\cdot b}{3\cdot a+b}=\left[a=7\cdot b\right]=\frac{7\cdot b-3\cdot b}{3\cdot 7\cdot b+b}=\frac{4\cdot b}{21\cdot b+b}=\frac{4\cdot b}{22\cdot b}=\frac{4\cdot b}{22\cdot b}=\frac{4}{22}=\frac{4}{22}=\frac{2}{11}.$$

Odgovor je pod B.

Vježba 449

Ako je $\frac{a+3\cdot b}{3\cdot a-b}=\frac{1}{2},$ koliko je $\frac{a-3\cdot b}{3\cdot a+b}$?

$A.\ \frac{1}{2}$ $\quad B.\ \frac{2}{11}$ $\quad C.\ \frac{3}{4}$ $\quad D.\ \frac{1}{4}$

Rezultat: B.

Zadatak 450 (Mihaela, gimnazija)

Ako je $\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2}$, onda je $\frac{a}{b}-\frac{b}{a}$ jednako:

$A.\ \frac{1}{4}$ $\quad B.\ 2$ $\quad C.\ \frac{1}{2}$ $\quad D.\ 8$

Rješenje 450

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c} \quad , \quad n=\frac{n}{1}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

1.inačica

$$\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2} \Rightarrow \frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2} \ /\cdot 2\cdot(a-b)\cdot(a+b) \Rightarrow$$

$$\Rightarrow 2\cdot(a+b)^2-2\cdot(a-b)^2=(a-b)\cdot(a+b) \Rightarrow$$

$$\Rightarrow 2\cdot\left(a^2+2\cdot a\cdot b+b^2\right)-2\cdot\left(a^2-2\cdot a\cdot b+b^2\right)=a^2-b^2 \Rightarrow$$

$$\Rightarrow 2\cdot a^2+4\cdot a\cdot b+2\cdot b^2-2\cdot a^2+4\cdot a\cdot b-2\cdot b^2=a^2-b^2 \Rightarrow$$

$$\Rightarrow 2\cdot a^2+4\cdot a\cdot b+2\cdot b^2-2\cdot a^2+4\cdot a\cdot b-2\cdot b^2=a^2-b^2 \Rightarrow 4\cdot a\cdot b+4\cdot a\cdot b=a^2-b^2 \Rightarrow$$

$$\Rightarrow 8\cdot a\cdot b=a^2-b^2 \Rightarrow a^2-b^2=8\cdot a\cdot b \Rightarrow a^2-b^2=8\cdot a\cdot b \ /\cdot\frac{1}{a\cdot b} \Rightarrow \frac{a^2-b^2}{a\cdot b}=8.$$

Odgovor je pod D.

2.inačica

$$\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2} \Rightarrow \frac{(a+b)^2-(a-b)^2}{(a-b)\cdot(a+b)}=\frac{1}{2} \Rightarrow \frac{a^2+2\cdot a\cdot b+b^2-\left(a^2-2\cdot a\cdot b+b^2\right)}{a^2-b^2}=\frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}{a^2-b^2}=\frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{2\cdot a\cdot b+2\cdot a\cdot b}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{4\cdot a\cdot b}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{a^2-b^2}{4\cdot a\cdot b}=\frac{2}{1} \Rightarrow \frac{a^2-b^2}{4\cdot a\cdot b}=2 \Rightarrow \frac{a^2-b^2}{4\cdot a\cdot b}=2 \ /\cdot 4 \Rightarrow$$

$$\Rightarrow \frac{a^2-b^2}{4\cdot a\cdot b}=8.$$

Odgovor je pod D.

3.inačica

$$\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2} \Rightarrow \frac{(a+b)^2-(a-b)^2}{(a-b)\cdot(a+b)}=\frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{((a+b)-(a-b))\cdot((a+b)+(a-b))}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{(a+b-a+b)\cdot(a+b+a-b)}{a^2-b^2}=\frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{(a+b-a+b)\cdot(a+b+a-b)}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{2\cdot b\cdot 2\cdot a}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{4\cdot a\cdot b}{a^2-b^2}=\frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{a^2-b^2}{4\cdot a\cdot b}=\frac{2}{1} \Rightarrow \frac{a^2-b^2}{4\cdot a\cdot b}=2 \Rightarrow \frac{a^2-b^2}{4\cdot a\cdot b}=2 \;/\cdot 4 \Rightarrow \frac{a^2-b^2}{4\cdot a\cdot b}=8.$$

Odgovor je pod D.

Vježba 450

Ako je $\frac{a+b}{a-b}+\frac{b-a}{a+b}=\frac{1}{2}$, onda je $\frac{a}{b}-\frac{b}{a}$ jednako:

$$A.\ \frac{1}{4} \qquad B.\ 2 \qquad C.\ \frac{1}{2} \qquad D.\ 8$$

Rezultat: D.

Zadatak 451 (MC, gimnazija)

Ako je $\sqrt{x+y}+\sqrt{x-y}=10$ i $\sqrt{x^2-y^2}=9$, onda je:

$$A.\ x=9 \qquad B.\ x=20 \qquad C.\ x=99 \qquad D.\ x=41$$

Rješenje 451

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\sqrt{x+y}+\sqrt{x-y}=10 \Rightarrow \sqrt{x+y}+\sqrt{x-y}=10 \;/^2 \Rightarrow \left(\sqrt{x+y}+\sqrt{x-y}\right)^2=10^2 \Rightarrow$$

$$\Rightarrow \left(\sqrt{x+y}\right)^2+2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2=100 \Rightarrow$$

$$\Rightarrow x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y=100 \Rightarrow x+y+2\cdot\sqrt{x^2-y^2}+x-y=100 \Rightarrow$$

$$\Rightarrow x+y+2\cdot\sqrt{x^2-y^2}+x-y=100 \Rightarrow x+2\cdot\sqrt{x^2-y^2}+x=100 \Rightarrow$$

$$\Rightarrow 2\cdot x+2\cdot\sqrt{x^2-y^2}=100 \Rightarrow 2\cdot x+2\cdot\sqrt{x^2-y^2}=100 \;/:2 \Rightarrow x+\sqrt{x^2-y^2}=50 \Rightarrow$$

$$\Rightarrow \left[\begin{array}{c}\text{uvjet}\\ \sqrt{x^2-y^2}=9\end{array}\right] \Rightarrow x+9=50 \Rightarrow x=50-9 \Rightarrow x=41.$$

Odgovor je pod D.

Vježba 451

Ako je $\sqrt{x+y}+\sqrt{x-y}=10$ i $\sqrt{x^2-y^2}=10$, onda je:

$$A.\ x=19 \qquad B.\ x=40 \qquad C.\ x=95 \qquad D.\ x=50$$

Rezultat: B.

Zadatak 452 (Marija, gimnazija)

Ako je $\frac{1}{a\cdot(b+1)}+\frac{1}{b\cdot(a+1)}=\frac{1}{(a+1)\cdot(b+1)}$, koliko je $\frac{1}{a}+\frac{1}{b}$?

Rješenje 452

Ponovimo!

$$\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{1}{a\cdot(b+1)}+\frac{1}{b\cdot(a+1)}=\frac{1}{(a+1)\cdot(b+1)}\ \Rightarrow\ \frac{1}{a\cdot(b+1)}+\frac{1}{b\cdot(a+1)}=\frac{1}{(a+1)\cdot(b+1)}\ /\cdot(a+1)\cdot(b+1)\ \Rightarrow$$

$$\Rightarrow\ \frac{a+1}{a}+\frac{b+1}{b}=1\ \Rightarrow\ \frac{a}{a}+\frac{1}{a}+\frac{b}{b}+\frac{1}{b}=1\ \Rightarrow\ \frac{a}{a}+\frac{1}{a}+\frac{b}{b}+\frac{1}{b}=1\ \Rightarrow\ 1+\frac{1}{a}+1+\frac{1}{b}=1\ \Rightarrow$$

$$\Rightarrow\ 1+\frac{1}{a}+1+\frac{1}{b}=1\ \Rightarrow\ \frac{1}{a}+1+\frac{1}{b}=0\ \Rightarrow\ \frac{1}{a}+\frac{1}{b}=-1.$$

Vježba 452

Ako je $\frac{b\cdot(a+1)+a\cdot(b+1)}{a\cdot b\cdot(a+1)\cdot(b+1)}-\frac{1}{(a+1)\cdot(b+1)}=0$, koliko je $\frac{1}{a}+\frac{1}{b}$?

Rezultat: – 1.

Zadatak 453 (4A, TUPŠ)

Čemu je jednako z iz formule $s=\frac{h}{m}\cdot(t-z)$?

$A.\ z=h\cdot t-m\cdot s$ $\qquad B.\ z=h\cdot t+m\cdot s$ $\qquad C.\ z=\frac{h\cdot t-m\cdot s}{h}$ $\qquad D.\ z=\frac{h\cdot t+m\cdot s}{h}$

Rješenje 453

Ponovimo!

$$a\cdot\frac{b}{c}=\frac{a\cdot b}{c}\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$s=\frac{h}{m}\cdot(t-z)\ \Rightarrow\ s=\frac{h}{m}\cdot(t-z)\ /\cdot m\ \Rightarrow\ m\cdot s=h\cdot(t-z)\ \Rightarrow\ m\cdot s=h\cdot t-h\cdot z\ \Rightarrow$$

$$\Rightarrow\ h\cdot z=h\cdot t-m\cdot s\ \Rightarrow\ h\cdot z=h\cdot t-m\cdot s\ /\cdot\frac{1}{h}\ \Rightarrow\ z=\frac{h\cdot t-m\cdot s}{h}.$$

Odgovor je pod C.

2.inačica

$$s=\frac{h}{m}\cdot(t-z)\ \Rightarrow\ s=\frac{h}{m}\cdot(t-z)\ /\cdot\frac{m}{h}\ \Rightarrow\ \frac{m\cdot s}{h}=t-z\ \Rightarrow\ z=t-\frac{m\cdot s}{h}\ \Rightarrow$$

$$\Rightarrow\ z=\frac{t}{1}-\frac{m\cdot s}{h}\ \Rightarrow\ z=\frac{h\cdot t-m\cdot s}{h}.$$

Odgovor je pod C.

Vježba 453

Čemu je jednako z iz formule $s=\frac{1}{m}\cdot(z-h\cdot t)$?

$A.\ z=h\cdot t-m\cdot s$ $\quad B.\ z=h\cdot t+m\cdot s$ $\quad C.\ z=\frac{h\cdot t-m\cdot s}{h}$ $\quad D.\ z=\frac{h\cdot t+m\cdot s}{h}$

Rezultat: B.

Zadatak 454 (4A, TUPŠ)

Skratite razlomak: $\frac{2\cdot a^2-a\cdot b+2\cdot a-b}{4\cdot a^2-b^2}$.

Rješenje 454

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad(a\cdot b)^n=a^n\cdot b^n.$$

$$a^1=a\quad,\quad a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{2\cdot a^2-a\cdot b+2\cdot a-b}{4\cdot a^2-b^2}=\left[\begin{array}{l}\text{u brojniku koristimo metodu grupiranja}\\ \text{u nazivniku je razlika kvadrata}\end{array}\right]=\frac{\left(2\cdot a^2-a\cdot b\right)+(2\cdot a-b)}{(2\cdot a)^2-b^2}=$$

$$=\frac{a\cdot(2\cdot a-b)+(2\cdot a-b)}{(2\cdot a-b)\cdot(2\cdot a+b)}=\frac{a\cdot(2\cdot a-b)+(2\cdot a-b)}{(2\cdot a-b)\cdot(2\cdot a+b)}=\frac{(2\cdot a-b)\cdot(a+1)}{(2\cdot a-b)\cdot(2\cdot a+b)}=\frac{(2\cdot a-b)\cdot(a+1)}{(2\cdot a-b)\cdot(2\cdot a+b)}=$$

$$=\frac{(2\cdot a-b)\cdot(a+1)}{(2\cdot a-b)\cdot(2\cdot a+b)}=\frac{a+1}{2\cdot a+b}.$$

2.inačica

$$\frac{2\cdot a^2-a\cdot b+2\cdot a-b}{4\cdot a^2-b^2}=\left[\begin{array}{l}\text{u brojniku koristimo metodu grupiranja}\\ \text{u nazivniku je razlika kvadrata}\end{array}\right]=\frac{\left(2\cdot a^2+2\cdot a\right)+(-a\cdot b-b)}{(2\cdot a)^2-b^2}=$$

$$=\frac{2\cdot a\cdot(a+1)-b\cdot(a+1)}{(2\cdot a-b)\cdot(2\cdot a+b)}=\frac{2\cdot a\cdot(a+1)-b\cdot(a+1)}{(2\cdot a-b)\cdot(2\cdot a+b)}=\frac{(a+1)\cdot(2\cdot a-b)}{(2\cdot a-b)\cdot(2\cdot a+b)}=\frac{(a+1)\cdot(2\cdot a-b)}{(2\cdot a-b)\cdot(2\cdot a+b)}=$$

$$=\frac{(a+1)\cdot(2\cdot a-b)}{(2\cdot a-b)\cdot(2\cdot a+b)}=\frac{a+1}{2\cdot a+b}.$$

Vježba 454

Skratite razlomak: $\frac{2\cdot a^2+a\cdot b+2\cdot a+b}{4\cdot a^2-b^2}.$

Rezultat: $\frac{a+1}{2\cdot a-b}.$

Zadatak 455 (4A, TUPŠ)

Operacija $\otimes$ s realnim brojevima definirana je pravilom $a\otimes b=a-2\cdot b+2.$ Izračunajte koliko je $2\otimes 5.$

Rješenje 455

$$2\otimes 5=\begin{bmatrix} a\otimes b=a-2\cdot b+2 \\ a=2\ \ ,\ \ b=5 \end{bmatrix}=2-2\cdot 5+2=2-10+2=-6.$$

Vježba 455

Operacija $\otimes$ s realnim brojevima definirana je pravilom $a\otimes b=a-2\cdot b+2.$ Izračunajte koliko je $5\otimes 2.$

Rezultat: 3.

Zadatak 456 (Marija, srednja škola)

Vrijednost brojevnog izraza $\left(x^2-1\right)^3\cdot\left(x^2+1\right)^3$ za $x=\sqrt[4]{3}$ jednaka je:

$A.\ 8 \qquad B.\ 3\cdot\sqrt{3} \qquad C.\ 4 \qquad D.\ 2\cdot\sqrt{2}$

Rješenje 456

Ponovimo!

$$a^n\cdot b^n=(a\cdot b)^n \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad \left(\sqrt[n]{a}\right)^n=a.$$

$$\left(x^2-1\right)^3\cdot\left(x^2+1\right)^3=\left(\left(x^2-1\right)\cdot\left(x^2+1\right)\right)^3=\left(\left(x^2\right)^2-1^2\right)^3=\left(x^4-1\right)^3=$$

$$=\left(\left(\sqrt[4]{3}\right)^4-1\right)^3=(3-1)^3=2^3=8.$$

Vježba 456

Vrijednost brojevnog izraza $\left(x^2-2\right)^3\cdot\left(x^2+2\right)^3$ za $x=\sqrt[4]{3}$ jednaka je:

$A.\ -8 \qquad B.\ -3\cdot\sqrt{3} \qquad C.\ -1 \qquad D.\ \sqrt{2}$

Rezultat: C.

Zadatak 457 (Liljana, srednja škola)

Skrati razlomak: $\frac{(2\cdot a+3\cdot b)^2-(3\cdot a+2\cdot b)^2}{a^4-b^4}.$

Rješenje 457

Ponovimo!

$$(x+y)^2 = x^2 + 2\cdot x\cdot y + y^2 \quad , \quad (x\cdot y)^n = x^n\cdot y^n \quad , \quad \left(x^n\right)^m = x^{n\cdot m}.$$

$$x^2 - y^2 = (x-y)\cdot(x+y).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

1.inačica

$$\frac{(2\cdot a+3\cdot b)^2-(3\cdot a+2\cdot b)^2}{a^4-b^4} = \frac{\left(4\cdot a^2+12\cdot a\cdot b+9\cdot b^2\right)-\left(9\cdot a^2+12\cdot a\cdot b+4\cdot b^2\right)}{\left(a^2\right)^2-\left(b^2\right)^2} =$$

$$= \frac{4\cdot a^2+12\cdot a\cdot b+9\cdot b^2-9\cdot a^2-12\cdot a\cdot b-4\cdot b^2}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)} = \frac{4\cdot a^2+12\cdot a\cdot b+9\cdot b^2-9\cdot a^2-12\cdot a\cdot b-4\cdot b^2}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)} =$$

$$= \frac{4\cdot a^2+9\cdot b^2-9\cdot a^2-4\cdot b^2}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)} = \frac{-5\cdot a^2+5\cdot b^2}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)} = \frac{-5\cdot\left(a^2-b^2\right)}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)} =$$

$$= \frac{-5\cdot\left(a^2-b^2\right)}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)} = \frac{-5}{a^2+b^2}.$$

2.inačica

$$\frac{(2\cdot a+3\cdot b)^2-(3\cdot a+2\cdot b)^2}{a^4-b^4} = \frac{\left((2\cdot a+3\cdot b)-(3\cdot a+2\cdot b)\right)\cdot\left((2\cdot a+3\cdot b)+(3\cdot a+2\cdot b)\right)}{\left(a^2\right)^2-\left(b^2\right)^2} =$$

$$= \frac{(2\cdot a+3\cdot b-3\cdot a-2\cdot b)\cdot(2\cdot a+3\cdot b+3\cdot a+2\cdot b)}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)} = \frac{(-a+b)\cdot(5\cdot a+5\cdot b)}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)} =$$

$$= \frac{-(a-b)\cdot 5\cdot(a+b)}{(a-b)\cdot(a+b)\cdot\left(a^2+b^2\right)} = \frac{-5\cdot(a-b)\cdot(a+b)}{(a-b)\cdot(a+b)\cdot\left(a^2+b^2\right)} =$$

$$= \frac{-5\cdot(a-b)\cdot(a+b)}{(a-b)\cdot(a+b)\cdot\left(a^2+b^2\right)} = \frac{-5}{a^2+b^2}.$$

Vježba 457

Skrati razlomak: $\dfrac{a^4-b^4}{(2\cdot a+3\cdot b)^2-(3\cdot a+2\cdot b)^2}.$

Rezultat: $-\dfrac{a^2+b^2}{5}.$

Zadatak 458 (Alen, srednja škola)

Dokaži da je: $\frac{2 \cdot \left(\sqrt{2}+\sqrt{6}\right)}{3 \cdot \sqrt{2+\sqrt{3}}}=\frac{4}{3}.$

Rješenje 458

Ponovimo!

$$\sqrt{a \cdot b}=\sqrt{a} \cdot \sqrt{b} \quad , \quad a^2+2 \cdot a \cdot b+b^2=(a+b)^2 \quad , \quad \sqrt{a^2}=a \ , \ a \geq 0 \quad , \quad \sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}.$$

$$n=\frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a \cdot d}{b \cdot c} \quad , \quad \left(\sqrt{a}\right)^2=a.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c)=a \cdot b+a \cdot c \quad , \quad a \cdot b+a \cdot c=a \cdot (b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a \cdot n}{b \cdot n} \ , \ n \neq 0 \ , \ n \neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n}=\frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

$$\frac{2 \cdot \left(\sqrt{2}+\sqrt{6}\right)}{3 \cdot \sqrt{2+\sqrt{3}}}=\frac{2 \cdot \left(\sqrt{2}+\sqrt{2 \cdot 3}\right)}{3 \cdot \sqrt{\frac{2+\sqrt{3}}{1}}}=\left[\begin{array}{l}\text{razlomak pod korijenom} \\ \text{proširimo s 2}\end{array}\right]=\frac{2 \cdot \left(\sqrt{2}+\sqrt{2} \cdot \sqrt{3}\right)}{3 \cdot \sqrt{\frac{2 \cdot \left(2+\sqrt{3}\right)}{2 \cdot 1}}}=$$

$$=\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right)}{3 \cdot \sqrt{\frac{4+2 \cdot \sqrt{3}}{2}}}=\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right)}{3 \cdot \sqrt{\frac{1+2 \cdot \sqrt{3}+3}{2}}}=\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right)}{3 \cdot \sqrt{\frac{1^2+2 \cdot \sqrt{3}+\left(\sqrt{3}\right)^2}{2}}}=\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right)}{3 \cdot \sqrt{\frac{\left(1+\sqrt{3}\right)^2}{2}}}=$$

$$=\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right)}{3 \cdot \frac{\sqrt{\left(1+\sqrt{3}\right)^2}}{\sqrt{2}}}=\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right)}{3 \cdot \frac{1+\sqrt{3}}{\sqrt{2}}}=\frac{\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right)}{1}}{3 \cdot \frac{1+\sqrt{3}}{\sqrt{2}}}=\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right) \cdot \sqrt{2}}{3 \cdot \left(1+\sqrt{3}\right)}=$$

$$=\frac{2 \cdot \sqrt{2} \cdot \left(1+\sqrt{3}\right) \cdot \sqrt{2}}{3 \cdot \left(1+\sqrt{3}\right)}=\frac{2 \cdot \sqrt{2} \cdot \sqrt{2}}{3}=\frac{2 \cdot \left(\sqrt{2}\right)^2}{3}=\frac{2 \cdot 2}{3}=\frac{4}{3}.$$ Dokaz gotov.

2.inačica

Pomoću dopuštenih transformacija provjerimo istinitost zadane jednakosti.

$$\frac{2\cdot\left(\sqrt{2}+\sqrt{6}\right)}{3\cdot\sqrt{2+\sqrt{3}}}=\frac{4}{3}\Rightarrow\frac{2\cdot\left(\sqrt{2}+\sqrt{6}\right)}{3\cdot\sqrt{2+\sqrt{3}}}=\frac{4}{3}\ /\cdot\frac{3}{2}\Rightarrow\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}=\frac{2}{1}\Rightarrow$$

$$\Rightarrow\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2+\sqrt{3}}}=\frac{2}{1}\ /\cdot\sqrt{2+\sqrt{3}}\Rightarrow\sqrt{2}+\sqrt{6}=2\cdot\sqrt{2+\sqrt{3}}\Rightarrow\sqrt{2}+\sqrt{6}=2\cdot\sqrt{2+\sqrt{3}}\ /^{2}\Rightarrow$$

$$\Rightarrow\left(\sqrt{2}+\sqrt{6}\right)^{2}=\left(2\cdot\sqrt{2+\sqrt{3}}\right)^{2}\Rightarrow\left(\sqrt{2}\right)^{2}+2\cdot\sqrt{2}\cdot\sqrt{6}+\left(\sqrt{6}\right)^{2}=2^{2}\cdot\left(\sqrt{2+\sqrt{3}}\right)^{2}\Rightarrow$$

$$\Rightarrow 2+2\cdot\sqrt{12}+6=4\cdot\left(2+\sqrt{3}\right)\Rightarrow 8+2\cdot\sqrt{12}=8+4\cdot\sqrt{3}\Rightarrow 8+2\cdot\sqrt{12}=8+4\cdot\sqrt{3}\Rightarrow$$

$$\Rightarrow 2\cdot\sqrt{12}=4\cdot\sqrt{3}\Rightarrow 2\cdot\sqrt{12}=4\cdot\sqrt{3}\ /:2\Rightarrow\sqrt{12}=2\cdot\sqrt{3}\Rightarrow\sqrt{4\cdot 3}=2\cdot\sqrt{3}\Rightarrow$$

$$\Rightarrow\sqrt{4}\cdot\sqrt{3}=2\cdot\sqrt{3}\Rightarrow 2\cdot\sqrt{3}=2\cdot\sqrt{3}.$$

Vježba 458

Dokaži da je: $\frac{3\cdot\sqrt{2+\sqrt{3}}}{2\cdot\left(\sqrt{2}+\sqrt{6}\right)}=\frac{3}{4}.$

Rezultat: Dokaz analogan.

Zadatak 459 (Alen, srednja škola)

Koliko je $\frac{\left(27^{n-1}-7\cdot 27^{n-2}\right)^{2}}{\left(9^{n-1}+9^{n-2}\right)^{3}}$?

Rješenje 459

Ponovimo!

$$a^{n}\cdot a^{m}=a^{n+m}\quad,\quad\left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad a^{-n}=\frac{1}{a^{n}}\quad,\quad a^{1}=a\quad,\quad(a\cdot b)^{n}=a^{n}\cdot b^{n}.$$

$$a^{n}:a^{m}=a^{n-m}\quad,\quad\left(\frac{a}{b}\right)^{n}=\frac{a^{n}}{b^{n}}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{\left(27^{n-1}-7\cdot 27^{n-2}\right)^{2}}{\left(9^{n-1}+9^{n-2}\right)^{3}}=\frac{\left(27^{n-1}-7\cdot 27^{n-1}\cdot 27^{-1}\right)^{2}}{\left(9^{n-1}+9^{n-1}\cdot 9^{-1}\right)^{3}}=\frac{\left(27^{n-1}\cdot\left(1-7\cdot 27^{-1}\right)\right)^{2}}{\left(9^{n-1}\cdot\left(1+9^{-1}\right)\right)^{3}}=$$

$$=\frac{\left(27^{n-1}\right)^2\cdot\left(1-7\cdot 27^{-1}\right)^2}{\left(9^{n-1}\right)^3\cdot\left(1+9^{-1}\right)^3}=\frac{27^{2\cdot n-2}\cdot\left(1-\frac{7}{27}\right)^2}{9^{3\cdot n-3}\cdot\left(1+\frac{1}{9}\right)^3}=\frac{\left(3^3\right)^{2\cdot n-2}\cdot\left(\frac{27-7}{27}\right)^2}{\left(3^2\right)^{3\cdot n-3}\cdot\left(\frac{9+1}{9}\right)^3}=$$

$$=\frac{3^{6\cdot n-6}\cdot\left(\frac{20}{27}\right)^2}{3^{6\cdot n-6}\cdot\left(\frac{10}{9}\right)^3}=\frac{3^{6\cdot n-6}\cdot\left(\frac{20}{27}\right)^2}{3^{6\cdot n-6}\cdot\left(\frac{10}{9}\right)^3}=\frac{\left(\frac{20}{27}\right)^2}{\left(\frac{10}{9}\right)^3}=\frac{\frac{20^2}{27^2}}{\frac{10^3}{9^3}}=\frac{20^2\cdot 9^3}{27^2\cdot 10^3}=\frac{20^2\cdot\left(3^2\right)^3}{\left(3^3\right)^2\cdot 10^3}=\frac{20^2\cdot 3^6}{3^6\cdot 10^3}=$$

$$=\frac{20^2\cdot 3^6}{3^6\cdot 10^3}=\frac{20^2}{10^3}=\frac{400}{1000}=\left[\begin{array}{l}\text{razlomak}\\ \text{kratimo s }200\end{array}\right]=\frac{2}{5}.$$

2.inačica

$$\frac{\left(27^{n-1}-7\cdot 27^{n-2}\right)^2}{\left(9^{n-1}+9^{n-2}\right)^3}=\frac{\left(27^n\cdot 27^{-1}-7\cdot 27^n\cdot 27^{-2}\right)^2}{\left(9^n\cdot 9^{-1}+9^n\cdot 9^{-2}\right)^3}=\frac{\left(27^n\cdot\left(27^{-1}-7\cdot 27^{-2}\right)\right)^2}{\left(9^n\cdot\left(9^{-1}+9^{-2}\right)\right)^3}=$$

$$=\frac{\left(27^n\right)^2\cdot\left(27^{-1}-7\cdot 27^{-2}\right)^2}{\left(9^n\right)^3\cdot\left(9^{-1}+9^{-2}\right)^3}=\frac{27^{2\cdot n}\cdot\left(\frac{1}{27}-\frac{7}{27^2}\right)^2}{9^{3\cdot n}\cdot\left(\frac{1}{9}+\frac{1}{9^2}\right)^3}=\frac{\left(3^3\right)^{2\cdot n}\cdot\left(\frac{27-7}{27^2}\right)^2}{\left(3^2\right)^{3\cdot n}\cdot\left(\frac{9+1}{9^2}\right)^3}=$$

$$=\frac{3^{6\cdot n}\cdot\left(\frac{20}{27^2}\right)^2}{3^{6\cdot n}\cdot\left(\frac{10}{9^2}\right)^3}=\frac{3^{6\cdot n}\cdot\left(\frac{20}{27^2}\right)^2}{3^{6\cdot n}\cdot\left(\frac{10}{9^2}\right)^3}=\frac{\left(\frac{20}{27^2}\right)^2}{\left(\frac{10}{9^2}\right)^3}=\frac{\frac{20^2}{\left(27^2\right)^2}}{\frac{10^3}{\left(9^2\right)^3}}=\frac{\frac{20^2}{27^4}}{\frac{10^3}{9^6}}=\frac{20^2\cdot 9^6}{27^4\cdot 10^3}=\frac{20^2\cdot\left(3^2\right)^6}{\left(3^3\right)^4\cdot 10^3}=$$

$$=\frac{20^2\cdot 3^{12}}{3^{12}\cdot 10^3}=\frac{20^2\cdot 3^{12}}{3^{12}\cdot 10^3}=\frac{20^2}{10^3}=\frac{400}{1000}=\left[\begin{array}{l}\text{razlomak}\\ \text{kratimo s }200\end{array}\right]=\frac{2}{5}.$$

3.inačica

$$\frac{\left(27^{n-1}-7\cdot 27^{n-2}\right)^2}{\left(9^{n-1}+9^{n-2}\right)^3}=\frac{\left(27^{n-2}\cdot 27^1-7\cdot 27^{n-2}\right)^2}{\left(9^{n-2}\cdot 9^1+9^{n-2}\right)^3}=\frac{\left(27^{n-2}\cdot\left(27^1-7\right)\right)^2}{\left(9^{n-2}\cdot\left(9^1+1\right)\right)^3}=$$

$$=\frac{\left(27^{n-2}\cdot(27-7)\right)^2}{\left(9^{n-2}\cdot(9+1)\right)^3}=\frac{\left(27^{n-2}\cdot 20\right)^2}{\left(9^{n-2}\cdot 10\right)^3}=\frac{\left(27^{n-2}\right)^2\cdot 20^2}{\left(9^{n-2}\right)^3\cdot 10^3}=\frac{27^{2\cdot n-4}\cdot 20^2}{9^{3\cdot n-6}\cdot 10^3}=\frac{\left(3^3\right)^{2\cdot n-4}\cdot 20^2}{\left(3^2\right)^{3\cdot n-6}\cdot 10^3}=$$

$$=\frac{3^{6\cdot n-12}\cdot 20^2}{3^{6\cdot n-12}\cdot 10^3}=\frac{3^{6\cdot n-12}\cdot 20^2}{3^{6\cdot n-12}\cdot 10^3}=\frac{20^2}{10^3}=\frac{400}{1000}=\left[\begin{matrix}\text{razlomak}\\ \text{kratimo s } 200\end{matrix}\right]=\frac{2}{5}.$$

Vježba 459

Koliko je $\dfrac{\left(9^{n-1}+9^{n-2}\right)^3}{\left(27^{n-1}-7\cdot 27^{n-2}\right)^2}$?

Rezultat: $\dfrac{5}{2}.$

Zadatak 460 (Doris, srednja škola)

Napiši u obliku umnoška algebarski izraz $x^2-4\cdot x\cdot y+4\cdot y^2-4\cdot x^2\cdot y^2.$

Rješenje 460

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$x^2-4\cdot x\cdot y+4\cdot y^2-4\cdot x^2\cdot y^2=\left[\begin{matrix}\text{grupiramo prva}\\ \text{tri člana}\end{matrix}\right]=\left(x^2-4\cdot x\cdot y+4\cdot y^2\right)-4\cdot x^2\cdot y^2=$$

$$=(x-2\cdot y)^2-(2\cdot x\cdot y)^2=\left[\begin{matrix}\text{razlika}\\ \text{kvadrata}\end{matrix}\right]=(x-2\cdot y-2\cdot x\cdot y)\cdot(x-2\cdot y+2\cdot x\cdot y).$$

Vježba 460

Napiši u obliku umnoška algebarski izraz $x^2-4\cdot x\cdot y+4\cdot y^2-9\cdot x^2\cdot y^2.$

Rezultat: $(x-2\cdot y-3\cdot x\cdot y)\cdot(x-2\cdot y+3\cdot x\cdot y).$