Zadatak 521 (Mihaela, gimnazija)

Dokaži da je broj $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}$ manji od 3.

Rješenje 521

Ponovimo!

$$\left(\sqrt{a}\right)^2=a \quad , \quad a\ge b \Rightarrow a^2\ge b^2 \;,\; a,b\ge 0.$$

Tvrdnju dokazujemo neizravno (indirektno). Pretpostavit ćemo suprotno, tj. da je zadani broj veći od 3 ili jednak 3. Tada se nizom transformacija dobije:

$$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}\ge 3 \Rightarrow \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}\ge 3 \;/^2 \Rightarrow$$

$$\Rightarrow \left(\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}\right)^2\ge 3^2 \Rightarrow 6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}\ge 9 \Rightarrow$$

$$\Rightarrow \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}\ge 9-6 \Rightarrow \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}\ge 3 \Rightarrow \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}\ge 3 \;/^2 \Rightarrow$$

$$\Rightarrow \left(\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}\right)^2\ge 3^2 \Rightarrow 6+\sqrt{6+\sqrt{6+\sqrt{6}}}\ge 9 \Rightarrow$$

$$\Rightarrow \sqrt{6+\sqrt{6+\sqrt{6}}}\ge 9-6 \Rightarrow \sqrt{6+\sqrt{6+\sqrt{6}}}\ge 3 \Rightarrow \sqrt{6+\sqrt{6+\sqrt{6}}}\ge 3 \;/^2 \Rightarrow$$

$$\Rightarrow \left(\sqrt{6+\sqrt{6+\sqrt{6}}}\right)^2\ge 3^2 \Rightarrow 6+\sqrt{6+\sqrt{6}}\ge 9 \Rightarrow \sqrt{6+\sqrt{6}}\ge 9-6 \Rightarrow \sqrt{6+\sqrt{6}}\ge 3 \Rightarrow$$

$$\Rightarrow \sqrt{6+\sqrt{6}}\ge 3 \;/^2 \Rightarrow \left(\sqrt{6+\sqrt{6}}\right)^2\ge 3^2 \Rightarrow 6+\sqrt{6}\ge 9 \Rightarrow \sqrt{6}\ge 9-6 \Rightarrow \sqrt{6}\ge 3 \Rightarrow$$

$$\Rightarrow \sqrt{6}\ge 3 \;/^2 \Rightarrow \left(\sqrt{6}\right)^2\ge 3^2 \Rightarrow 6\ge 9. \quad \rightarrow\leftarrow$$

Ova tvrdnja nije točna pa je zadana nejednakost istinita.

Vježba 521

Dokaži da je broj $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$ manji od 3.

Rezultat: Dokaz analogan.

Zadatak 522 (Leon, srednja škola)

Izračunaj koliko je $x^{100}+x^{101}+x^{102}+x^{103}+x^{104}$, ako je $x^3=-1$.

Rješenje 522

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$x^{100}+x^{101}+x^{102}+x^{103}+x^{104}=x^{100}+x^{100}\cdot x^1+x^{100}\cdot x^2+x^{100}\cdot x^3+x^{100}\cdot x^4=$$

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$$= x^{100} + x^{100} \cdot x^{1} + x^{100} \cdot x^{2} + x^{100} \cdot x^{3} + x^{100} \cdot x^{4} = x^{100} \cdot \left(1 + x^{1} + x^{2} + x^{3} + x^{4}\right) =$$

$$= x^{100} \cdot \left(1 + x + x^{2} + x^{3} + x^{4}\right) = \left[x^{3} = -1\right] = x^{100} \cdot \left(1 + x + x^{2} - 1 + x^{4}\right) =$$

$$= x^{100} \cdot \left(1 + x + x^{2} - 1 + x^{4}\right) = x^{100} \cdot \left(x + x^{2} + x^{4}\right) = x^{100} \cdot x \cdot \left(1 + x + x^{3}\right) =$$

$$= \left[x^{3} = -1\right] = x^{100} \cdot x \cdot (1 + x - 1) = x^{100} \cdot x \cdot (1 + x - 1) = x^{100} \cdot x \cdot x = x^{100} \cdot x^{1} \cdot x^{1} = x^{102} =$$

$$= \left(x^{3}\right)^{34} = \left[x^{3} = -1\right] = (-1)^{34} = 1.$$

Vježba 522

Izračunaj koliko je $x^{104} + x^{103} + x^{102} + x^{101} + x^{100}$, ako je $x^{3} + 1 = 0$.

Rezultat: 1.

Zadatak 523 (Smješko, gimnazija)

Dokazati: $\left(\dfrac{b^{-1} + a^{-1}}{a \cdot b^{-1} + b \cdot a^{-1}}\right)^{-1} + \left(\dfrac{a^{-1} + b^{-1}}{2}\right)^{-1} - \dfrac{b^{-1} - a^{-1}}{a^{-1} \cdot b^{-1}} = 2 \cdot b, \ (a \neq 0, \ b \neq 0).$

Rješenje 523

Ponovimo!

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} \quad , \quad a^{1} = a \quad , \quad a^{-1} = \frac{1}{a} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \quad , \quad (a - b) \cdot (a + b) = a^{2} - b^{2}.$$

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Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$\left(\frac{b^{-1} + a^{-1}}{a \cdot b^{-1} + b \cdot a^{-1}}\right)^{-1} + \left(\frac{a^{-1} + b^{-1}}{2}\right)^{-1} - \frac{b^{-1} - a^{-1}}{a^{-1} \cdot b^{-1}} =$$

$$= \frac{a \cdot b^{-1} + b \cdot a^{-1}}{b^{-1} + a^{-1}} + \frac{2}{a^{-1} + b^{-1}} - \frac{b^{-1} - a^{-1}}{a^{-1} \cdot b^{-1}} = \frac{\frac{a}{b} + \frac{b}{a}}{\frac{1}{b} + \frac{1}{a}} + \frac{2}{\frac{1}{a} + \frac{1}{b}} - \frac{\frac{1}{b} - \frac{1}{a}}{\frac{1}{a} \cdot \frac{1}{b}} =$$

$$= \frac{\frac{a^{2} + b^{2}}{a \cdot b}}{\frac{a + b}{a \cdot b}} + \frac{\frac{2}{1}}{\frac{b + a}{a \cdot b}} - \frac{\frac{a - b}{a \cdot b}}{\frac{1}{a \cdot b}} = \frac{\frac{a^{2} + b^{2}}{a \cdot b}}{\frac{a + b}{a \cdot b}} + \frac{\frac{2}{1}}{\frac{b + a}{a \cdot b}} - \frac{\frac{a - b}{a \cdot b}}{\frac{1}{a \cdot b}} = \frac{\frac{a^{2} + b^{2}}{1}}{\frac{a + b}{1}} + \frac{\frac{2}{1}}{\frac{b + a}{a \cdot b}} - \frac{\frac{a - b}{1}}{\frac{1}{1}} =$$

$$=\frac{a^2+b^2}{a+b}+\frac{2\cdot a\cdot b}{a+b}-\frac{a-b}{1}=\frac{a^2+b^2+2\cdot a\cdot b-(a-b)\cdot(a+b)}{a+b}=\frac{a^2+b^2+2\cdot a\cdot b-\left(a^2-b^2\right)}{a+b}=$$

$$=\frac{a^2+b^2+2\cdot a\cdot b-a^2+b^2}{a+b}=\frac{a^2+b^2+2\cdot a\cdot b-a^2+b^2}{a+b}=\frac{b^2+2\cdot a\cdot b+b^2}{a+b}=$$

$$=\frac{2\cdot a\cdot b+2\cdot b^2}{a+b}=\frac{2\cdot b\cdot(a+b)}{a+b}=\frac{2\cdot b\cdot(a+b)}{a+b}=2\cdot b.$$

Vježba 523

Dokazati: $\left(\frac{b^{-1}+a^{-1}}{a\cdot b^{-1}+b\cdot a^{-1}}\right)^{-1}+\left(\frac{a^{-1}+b^{-1}}{2}\right)^{-1}+\frac{a^{-1}-b^{-1}}{a^{-1}\cdot b^{-1}}=2\cdot b,\ (a\neq 0,\ b\neq 0).$

Rezultat: Dokaz analogan.

Zadatak 524 (Lussy, gimnazija)

Izraz $\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}$ za a ≠ b ≠ 0 identičan je razlomku:

$A.\ \frac{b^2}{b-a}$ $\qquad B.\ \frac{a^2}{a-b}$ $\qquad C.\ \frac{1}{a^2-b^2}$ $\qquad D.\ \frac{1}{a\cdot b}$

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Rješenje 524

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad\frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad(a-b)\cdot(a+b)=a^2-b^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}=\left(\frac{b}{\frac{1}{1}-\frac{a}{b}}+\frac{a}{\frac{1}{1}-\frac{b}{a}}\right)\cdot\frac{1}{\frac{1}{1}-\frac{a^2}{b^2}}=\left(\frac{b}{\frac{b-a}{b}}+\frac{a}{\frac{a-b}{a}}\right)\cdot\frac{1}{\frac{b^2-a^2}{b^2}}=$$

$$=\left(\frac{\frac{b}{1}}{\frac{b-a}{b}}-\frac{\frac{a}{1}}{\frac{b-a}{a}}\right)\cdot\frac{\frac{1}{1}}{\frac{b^2-a^2}{b^2}}=\left(\frac{b^2}{b-a}-\frac{a^2}{b-a}\right)\cdot\frac{b^2}{b^2-a^2}=\frac{b^2-a^2}{b-a}\cdot\frac{b^2}{b^2-a^2}=$$

$$=\frac{b^2-a^2}{b-a}\cdot\frac{b^2}{b^2-a^2}=\frac{1}{b-a}\cdot\frac{b^2}{1}=\frac{b^2}{b-a}.$$

Odgovor je pod A.

Vježba 524

Izraz $\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{-1}{\frac{a^2}{b^2}-1}$ za a ≠ b ≠ 0 identičan je razlomku:

$A.\ \frac{b^2}{b-a}$ $\quad B.\ \frac{a^2}{a-b}$ $\quad C.\ \frac{1}{a^2-b^2}$ $\quad D.\ \frac{1}{a\cdot b}$

Rezultat: A.

Zadatak 525 (Lussy, gimnazija)

Pojednostavni: $\left(1-\frac{a^2-b^2-c^2}{2\cdot b\cdot c}\right):\left(\frac{1}{a\cdot b}+\frac{1}{b\cdot c}+\frac{1}{a\cdot c}\right).$

$A.\ \frac{c\cdot(a+b-c)}{2}$ $\quad B.\ \frac{b\cdot(a+c-b)}{2}$ $\quad C.\ \frac{a\cdot(b+c-a)}{2}$ $\quad D.\ \frac{1}{a\cdot b\cdot c}$

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Rješenje 525

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad a^2-b^2=(a-b)\cdot(a+b).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Preoblikujemo zadani izraz.

$$\left(1-\frac{a^2-b^2-c^2}{2\cdot b\cdot c}\right):\left(\frac{1}{a\cdot b}+\frac{1}{b\cdot c}+\frac{1}{a\cdot c}\right)=\left(\frac{1}{1}-\frac{a^2-b^2-c^2}{2\cdot b\cdot c}\right):\left(\frac{1}{a\cdot b}+\frac{1}{b\cdot c}+\frac{1}{a\cdot c}\right)=$$

$$=\frac{2\cdot b\cdot c-\left(a^2-b^2-c^2\right)}{2\cdot b\cdot c}:\frac{c+a+b}{a\cdot b\cdot c}=\frac{2\cdot b\cdot c-a^2+b^2+c^2}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=$$

$$=\frac{b^2+2\cdot b\cdot c+c^2-a^2}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=\frac{(b+c)^2-a^2}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=\frac{(b+c-a)\cdot(b+c+a)}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=$$

$$=\frac{(b+c-a)\cdot(a+b+c)}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=\frac{b+c-a}{2}\cdot\frac{a}{1}=\frac{a\cdot(b+c-a)}{2}.$$

Odgovor je pod C.

Vježba 525

Pojednostavni: $\left(1+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}\right):\left(\frac{1}{a\cdot b}+\frac{1}{b\cdot c}+\frac{1}{a\cdot c}\right).$

$A.\ \frac{c\cdot(a+b-c)}{2}$ $\quad B.\ \frac{b\cdot(a+c-b)}{2}$ $\quad C.\ \frac{a\cdot(b+c-a)}{2}$ $\quad D.\ \frac{1}{a\cdot b\cdot c}$

Rezultat: C.

Zadatak 526 (Lussy, gimnazija)

Jednostavniji zapis algebarskog izraza $\frac{x^{-3}+y^{-3}}{x^{-2}-y^{-2}}\cdot\left(\frac{x}{y}+\frac{y}{x}-1\right)^{-1}$ jest:

$A.\ \frac{1}{y-x}$ $\quad B.\ x^{-1}-y^{-1}$ $\quad C.\ \frac{x-y}{x\cdot y}$ $\quad D.\ \frac{1}{x\cdot y}$

Rješenje 526

Ponovimo!

$$a^{-n}=\frac{1}{a^n}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad a^1=a.$$

$$\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^2-b^2=(a-b)\cdot(a+b).$$

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Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Preoblikujemo zadani izraz.

$$\frac{x^{-3}+y^{-3}}{x^{-2}-y^{-2}}\cdot\left(\frac{x}{y}+\frac{y}{x}-1\right)^{-1}=\frac{\frac{1}{x^3}+\frac{1}{y^3}}{\frac{1}{x^2}-\frac{1}{y^2}}\cdot\left(\frac{x^2+y^2-x\cdot y}{x\cdot y}\right)^{-1}=$$

$$=\frac{\frac{y^3+x^3}{x^3\cdot y^3}}{\frac{y^2-x^2}{x^2\cdot y^2}}\cdot\frac{x\cdot y}{x^2+y^2-x\cdot y}=\frac{\frac{x^3+y^3}{x^3\cdot y^3}}{\frac{y^2-x^2}{x^2\cdot y^2}}\cdot\frac{x\cdot y}{x^2-x\cdot y+y^2}=\frac{\frac{x^3+y^3}{x\cdot y}}{\frac{y^2-x^2}{1}}\cdot\frac{x\cdot y}{x^2-x\cdot y+y^2}=$$

$$=\frac{x^3+y^3}{x\cdot y\cdot\left(y^2-x^2\right)}\cdot\frac{x\cdot y}{x^2-x\cdot y+y^2}=\frac{x^3+y^3}{x\cdot y\cdot\left(y^2-x^2\right)}\cdot\frac{x\cdot y}{x^2-x\cdot y+y^2}=$$

$$=\frac{x^3+y^3}{y^2-x^2}\cdot\frac{1}{x^2-x\cdot y+y^2}=$$

$$=\frac{(x+y)\cdot\left(x^2-x\cdot y+y^2\right)}{(y-x)\cdot(y+x)}\cdot\frac{1}{x^2-x\cdot y+y^2}=\frac{(x+y)\cdot\left(x^2-x\cdot y+y^2\right)}{(y-x)\cdot(y+x)}\cdot\frac{1}{x^2-x\cdot y+y^2}=$$

$$=\frac{1}{y-x}\cdot\frac{1}{1}=\frac{1}{y-x}.$$

Odgovor je pod A.

Vježba 526

Jednostavniji zapis algebarskog izraza $\frac{x^{-3}+y^{-3}}{x^{-2}-y^{-2}}:\left(\frac{x}{y}+\frac{y}{x}-1\right)$ jest:

$A.\ \frac{1}{y-x}$ $\quad B.\ x^{-1}-y^{-1}$ $\quad C.\ \frac{x-y}{x\cdot y}$ $\quad D.\ \frac{1}{x\cdot y}$

Rezultat: C.

Zadatak 527 (Tibor, srednja škola)

Ako je $x^2-x+2=0$, onda je $x^4-2\cdot x^3+x^2+7$ jednako:

$A.\ 11$ $\quad B.\ 13$ $\quad C.\ 15$ $\quad D.\ 17$

Rješenje 527

Ponovimo!

$$a^1=a\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

Svojstvo udruživanja ili asocijativnosti

$$(a+b)+c=a+(b+c).$$

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Najprije preoblikujemo zadanu jednakost.

$$x^2-x+2=0\ \Rightarrow\ x^2-x=-2\ \Rightarrow\ x^2-x=-2\ /^2\ \Rightarrow\ \left(x^2-x\right)^2=(-2)^2\ \Rightarrow$$

$$\Rightarrow\ x^4-2\cdot x^3+x^2=4.$$

Sada je:

$$x^4-2\cdot x^3+x^2+7=\left(x^4-2\cdot x^3+x^2\right)+7=4+7=11.$$

Odgovor je pod A.

Vježba 527

Ako je $x^2-x+2=0$, onda je $x^4-2\cdot x^3+x^2+11$ jednako:

$A.\ 11$ $\quad B.\ 13$ $\quad C.\ 15$ $\quad D.\ 17$

Rezultat: C.

Zadatak 528 (Domy, gimnazija)

Dokazati nejednakost: $a\cdot b+a\cdot c+b\cdot c\le a^2+b^2+c^2.$

Rješenje 528

Ponovimo!

$$a^2 \geq 0\ ,\ a \in R \quad , \quad (x-y)^2 = x^2 - 2\cdot x\cdot y + y^2 \quad , \quad \left.\begin{matrix} x \geq 0 \\ y \geq 0 \end{matrix}\right\} \Rightarrow x+y \geq 0.$$

Polazimo od očiglednih nejednakosti.

$$\left.\begin{matrix} (a-b)^2 \geq 0 \\ (a-c)^2 \geq 0 \\ (b-c)^2 \geq 0 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{zbrojimo} \\ \text{nejednakosti} \end{bmatrix} \Rightarrow (a-b)^2 + (a-c)^2 + (b-c)^2 \geq 0 \Rightarrow$$

$$\Rightarrow a^2 - 2\cdot a\cdot b + b^2 + a^2 - 2\cdot a\cdot c + c^2 + b^2 - 2\cdot b\cdot c + c^2 \geq 0 \Rightarrow$$

$$\Rightarrow 2\cdot a^2 + 2\cdot b^2 + 2\cdot c^2 - 2\cdot a\cdot b - 2\cdot a\cdot c - 2\cdot b\cdot c \geq 0 \Rightarrow$$

$$\Rightarrow 2\cdot a^2 + 2\cdot b^2 + 2\cdot c^2 - 2\cdot a\cdot b - 2\cdot a\cdot c - 2\cdot b\cdot c \geq 0\ /:2 \Rightarrow$$

$$\Rightarrow a^2 + b^2 + c^2 - a\cdot b - a\cdot c - b\cdot c \geq 0 \Rightarrow$$

$$\Rightarrow a^2 + b^2 + c^2 \geq a\cdot b + a\cdot c + b\cdot c \Rightarrow a\cdot b + a\cdot c + b\cdot c \leq a^2 + b^2 + c^2.$$

Vježba 528

Dokazati nejednakost: $a\cdot(a-b) + b\cdot(b-c) + c\cdot(c-a) \geq 0.$

Rezultat: Dokaz analogan.

Zadatak 529 (Domy, gimnazija)

Izračunaj: $\frac{1}{1-x} + \frac{1}{1+x} + \frac{2}{1+x^2} + \frac{4}{1+x^4}.$

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Rješenje 529

Ponovimo!

Svojstvo udruživanja ili asocijativnosti

$$(a+b)+c = a+(b+c).$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a\cdot d + b\cdot c}{b\cdot d} \quad , \quad (a-b)\cdot(a+b) = a^2 - b^2 \quad , \quad \left(a^n\right)^m = a^{n\cdot m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

$$\frac{1}{1-x} + \frac{1}{1+x} + \frac{2}{1+x^2} + \frac{4}{1+x^4} = \left(\frac{1}{1-x} + \frac{1}{1+x}\right) + \frac{2}{1+x^2} + \frac{4}{1+x^4} = \frac{1+x+1-x}{(1-x)\cdot(1+x)} + \frac{2}{1+x^2} + \frac{4}{1+x^4} =$$

$$= \frac{1+x+1-x}{(1-x)\cdot(1+x)} + \frac{2}{1+x^2} + \frac{4}{1+x^4} = \frac{2}{1-x^2} + \frac{2}{1+x^2} + \frac{4}{1+x^4} = \left(\frac{2}{1-x^2} + \frac{2}{1+x^2}\right) + \frac{4}{1+x^4} =$$

$$= \frac{2\cdot\left(1+x^2\right) + 2\cdot\left(1-x^2\right)}{\left(1-x^2\right)\cdot\left(1+x^2\right)} + \frac{4}{1+x^4} = \frac{2+2\cdot x^2 + 2 - 2\cdot x^2}{1-\left(x^2\right)^2} + \frac{4}{1+x^4} =$$

$$= \frac{2+2\cdot x^2 + 2 - 2\cdot x^2}{1-x^4} + \frac{4}{1+x^4} = \frac{4}{1-x^4} + \frac{4}{1+x^4} =$$

$$=\frac{4\cdot\left(1+x^4\right)+4\cdot\left(1-x^4\right)}{\left(1-x^4\right)\cdot\left(1+x^4\right)}=\frac{4+4\cdot x^4+4-4\cdot x^4}{1-\left(x^4\right)^2}=\frac{4+4\cdot x^4+4-4\cdot x^4}{1-\left(x^4\right)^2}=\frac{8}{1-x^8}.$$

Vježba 529

Izračunaj: $\frac{1}{1-x}+\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\frac{8}{1+x^8}.$

Rezultat: $\frac{16}{1+x^{16}}.$

Zadatak 530 (Domy, gimnazija)

Pojednostavni: $\dfrac{x+x^2+x^3+...+x^n}{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+...+\frac{1}{x^n}}.$

Rješenje 530

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{1}{a}+\frac{1}{a^2}+\frac{1}{a^n}=\frac{a^{n-1}+a^{n-2}+1}{a^n} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$a^1=a \quad , \quad n=\frac{n}{1} \quad , \quad a^n:a^m=a^{n-m} \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{x+x^2+x^3+...+x^n}{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+...+\frac{1}{x^n}}=\frac{x+x^2+x^3+...+x^{n-2}+x^{n-1}+x^n}{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+...+\frac{1}{x^{n-2}}+\frac{1}{x^{n-1}}+\frac{1}{x^n}}=$$

$$=\frac{x\cdot\left(1+x+x^2+...+x^{n-3}+x^{n-2}+x^{n-1}\right)}{\frac{x^{n-1}+x^{n-2}+x^{n-3}+...+x^2+x+1}{x^n}}=\frac{\frac{x\cdot\left(1+x+x^2+...+x^{n-3}+x^{n-2}+x^{n-1}\right)}{1}}{\frac{x^{n-1}+x^{n-2}+x^{n-3}+...+x^2+x+1}{x^n}}=$$

$$=\frac{\frac{x\cdot\left(1+x+x^2+...+x^{n-3}+x^{n-2}+x^{n-1}\right)}{1}}{\frac{x^{n-1}+x^{n-2}+x^{n-3}+...+x^2+x+1}{x^n}}=\frac{\frac{x}{1}}{\frac{1}{x^n}}=\frac{x^n\cdot x}{1}=x^n\cdot x=x^n\cdot x^1=x^{n+1}.$$

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Vježba 530

Pojednostavni: $\dfrac{x+x^2+x^3+...+x^{10}}{\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+...+\frac{1}{x^{10}}}$.

Rezultat: x^{11}.

Zadatak 531 (Zvone, gimnazija)

Pojednostavni dvojni razlomak: $\dfrac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}-2}$.

Rješenje 531

Ponovimo!

$$n=\frac{n}{1}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad (a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad a^1=a.$$

$$a^n\cdot a^m=a^{n+m}\quad,\quad \frac{a}{b}\cdot b\cdot a=\frac{a}{b}\cdot\frac{b\cdot a}{1}=a^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}-2}=\frac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}-\frac{2}{1}}=\frac{\frac{x^2-y^2}{x\cdot y}}{\frac{x^2+y^2-2\cdot x\cdot y}{x\cdot y}}=\frac{\frac{x^2-y^2}{x\cdot y}}{\frac{x^2+y^2-2\cdot x\cdot y}{x\cdot y}}=\frac{\frac{x^2-y^2}{1}}{\frac{x^2+y^2-2\cdot x\cdot y}{1}}=$$

$$=\frac{x^2-y^2}{x^2+y^2-2\cdot x\cdot y}=\frac{x^2-y^2}{x^2-2\cdot x\cdot y+y^2}=\frac{(x-y)\cdot(x+y)}{(x-y)^2}=\frac{(x-y)\cdot(x+y)}{(x-y)^2}=\frac{x+y}{x-y}.$$

2.inačica

$$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}-2}=\left[\begin{array}{l}\text{razlomak}\\ \text{proširimo s } x\cdot y\end{array}\right]=\frac{\left(\frac{x}{y}-\frac{y}{x}\right)\cdot x\cdot y}{\left(\frac{x}{y}+\frac{y}{x}-2\right)\cdot x\cdot y}=$$

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$$=\frac{x^2-y^2}{x^2+y^2-2\cdot x\cdot y}=\frac{x^2-y^2}{x^2-2\cdot x\cdot y+y^2}=\frac{(x-y)\cdot(x+y)}{(x-y)^2}=\frac{(x-y)\cdot(x+y)}{(x-y)^2}=\frac{x+y}{x-y}.$$

Vježba 531

Pojednostavni dvojni razlomak: $\dfrac{\frac{x}{y}+\frac{y}{x}-2}{\frac{x}{y}-\frac{y}{x}}$.

Rezultat: $\dfrac{x-y}{x+y}$.

Zadatak 532 (Zvone, gimnazija)

Pojednostavni dvojni razlomak: $\dfrac{1-\frac{a^3}{b^3}}{1-\frac{a^2}{b^2}}$.

Rješenje 532

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad\frac{a}{b}\cdot b=a.$$

$$x^3-y^3=(x-y)\cdot\left(x^2+x\cdot y+y^2\right)\quad,\quad x^2-y^2=(x-y)\cdot(x+y).$$

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

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Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\ne 0\ ,\ n\ne 1.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\ne 0\ ,\ n\ne 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Svojstvo zamjene ili komutativnosti za zbrajanje i množenje realnih brojeva

$$a\cdot b=b+a\quad,\quad a\cdot b=b\cdot a.$$

1.inačica

$$\frac{1-\frac{a^3}{b^3}}{1-\frac{a^2}{b^2}}=\frac{\frac{1}{1}-\frac{a^3}{b^3}}{\frac{1}{1}-\frac{a^2}{b^2}}=\frac{\frac{b^3-a^3}{b^3}}{\frac{b^2-a^2}{b^2}}=\frac{\frac{b^3-a^3}{b^3}}{\frac{b^2-a^2}{b^2}}=\frac{\frac{b^3-a^3}{b}}{\frac{b^2-a^2}{1}}=\frac{b^3-a^3}{b\cdot\left(b^2-a^2\right)}=$$

$$=\frac{(b-a)\cdot\left(b^2+b\cdot a+a^2\right)}{b\cdot(b-a)\cdot(b+a)}=\frac{(b-a)\cdot\left(b^2+b\cdot a+a^2\right)}{b\cdot(b-a)\cdot(b+a)}=\frac{b^2+b\cdot a+a^2}{b\cdot(b+a)}=\frac{a^2+a\cdot b+b^2}{b\cdot(a+b)}.$$

2.inačica

$$\frac{1-\frac{a^3}{b^3}}{1-\frac{a^2}{b^2}}=\frac{1-\left(\frac{a}{b}\right)^3}{1-\left(\frac{a}{b}\right)^2}=\frac{\left(1-\frac{a}{b}\right)\cdot\left(1+\frac{a}{b}+\left(\frac{a}{b}\right)^2\right)}{\left(1-\frac{a}{b}\right)\cdot\left(1+\frac{a}{b}\right)}=\frac{\left(1-\frac{a}{b}\right)\cdot\left(1+\frac{a}{b}+\left(\frac{a}{b}\right)^2\right)}{\left(1-\frac{a}{b}\right)\cdot\left(1+\frac{a}{b}\right)}=\frac{1+\frac{a}{b}+\left(\frac{a}{b}\right)^2}{1+\frac{a}{b}}=$$

$$=\frac{1+\frac{a}{b}+\frac{a^2}{b^2}}{1+\frac{a}{b}}=\frac{\frac{1}{1}+\frac{a}{b}+\frac{a^2}{b^2}}{\frac{1}{1}+\frac{a}{b}}=\frac{\frac{b^2+b\cdot a+a^2}{b^2}}{\frac{b+a}{b}}=\frac{\frac{b^2+b\cdot a+a^2}{b^2}}{\frac{b+a}{b}}=\frac{\frac{b^2+b\cdot a+a^2}{b}}{\frac{b+a}{1}}=$$

$$=\frac{b^2+b\cdot a+a^2}{b\cdot(b+a)}=\frac{a^2+a\cdot b+b^2}{b\cdot(a+b)}.$$

3.inačica

$$\frac{1-\frac{a^3}{b^3}}{1-\frac{a^2}{b^2}}=\left[\begin{array}{l}\text{razlomak}\\ \text{proširimo s } b^3\end{array}\right]=\frac{\left(1-\frac{a^3}{b^3}\right)\cdot b^3}{\left(1-\frac{a^2}{b^2}\right)\cdot b^2}=\frac{b^3-a^3}{b^3-b\cdot a^2}=\frac{b^3-a^3}{b\cdot\left(b^2-a^2\right)}=$$

$$=\frac{(b-a)\cdot\left(b^2+b\cdot a+a^2\right)}{b\cdot(b-a)\cdot(b+a)}=\frac{(b-a)\cdot\left(b^2+b\cdot a+a^2\right)}{b\cdot(b-a)\cdot(b+a)}=\frac{b^2+b\cdot a+a^2}{b\cdot(b+a)}=\frac{a^2+a\cdot b+b^2}{b\cdot(a+b)}.$$

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Vježba 532

Pojednostavni dvojni razlomak: $\dfrac{1-\frac{a^2}{b^2}}{1-\frac{a^3}{b^3}}$.

Rezultat: $\dfrac{b\cdot(a+b)}{a^2+a\cdot b+b^2}$.

Zadatak 533 (Ana, ekonomska škola)

Skrati razlomak: $\dfrac{3\cdot a+a\cdot b+b+3}{4\cdot a+a\cdot b+b+4}$.

Rješenje 533

Ponovimo!

Svojstvo udruživanja ili asocijativnosti

Za svaka tri realna broja vrijedi

$$(a+b)+c=a+(b+c)\quad,\quad(a\cdot b)\cdot c=a\cdot(b\cdot c).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

1.inačica

$$\frac{3\cdot a+a\cdot b+b+3}{4\cdot a+a\cdot b+b+4}=\left[\begin{array}{c}\text{metoda grupiranja}\\ \\ \text{Kako grupirati?}\end{array}\right]\left[\begin{array}{l}\text{u brojniku grupiramo prvi i drugi te treći i četvrti član}\\ \text{u nazivniku grupiramo prvi i drugi te treći i četvrti član}\end{array}\right]=$$

$$=\frac{(3\cdot a+a\cdot b)+(b+3)}{(4\cdot a+a\cdot b)+(b+4)}=\frac{a\cdot(3+b)+(b+3)}{a\cdot(4+b)+(b+4)}=\frac{a\cdot(b+3)+(b+3)}{a\cdot(b+4)+(b+4)}=\frac{a\cdot(b+3)+(b+3)}{a\cdot(b+4)+(b+4)}=$$

$$=\frac{(b+3)\cdot(a+1)}{(b+4)\cdot(a+1)}=\frac{(b+3)\cdot(a+1)}{(b+4)\cdot(a+1)}=\frac{b+3}{b+4}.$$

2.inačica

$$\frac{3\cdot a+a\cdot b+b+3}{4\cdot a+a\cdot b+b+4}=\left[\begin{array}{c}\text{metoda grupiranja}\\ \\ \text{Kako grupirati?}\end{array}\right]\left[\begin{array}{l}\text{u brojniku grupiramo prvi i četvrti te drugi i treći član}\\ \text{u nazivniku grupiramo prvi i četvrti te drugi i treći član}\end{array}\right]=$$

$$=\frac{(3\cdot a+3)+(a\cdot b+b)}{(4\cdot a+4)+(a\cdot b+b)}=\frac{3\cdot(a+1)+b\cdot(a+1)}{4\cdot(a+1)+b\cdot(a+1)}=\frac{3\cdot(a+1)+b\cdot(a+1)}{4\cdot(a+1)+b\cdot(a+1)}=\frac{(a+1)\cdot(3+b)}{(a+1)\cdot(4+b)}=$$

$$=\frac{(a+1)\cdot(3+b)}{(a+1)\cdot(4+b)}=\frac{3+b}{4+b}=\frac{b+3}{b+4}.$$

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Vježba 533

Skrati razlomak: $\frac{4\cdot a+a\cdot b+b+4}{3\cdot a+a\cdot b+b+3}.$

Rezultat: $\frac{b+4}{b+3}.$

Zadatak 534 (Melita, srednja škola)

Pojednostavnite:

$$1)\ \frac{a}{x}:\frac{b}{y}:\frac{x}{a}:\frac{y}{b} \qquad 2)\ \frac{a}{x}:\left(\frac{b}{y}:\frac{x}{a}:\frac{y}{b}\right) \qquad 3)\ \frac{a}{x}:\left[\frac{b}{y}:\left(\frac{x}{a}:\frac{y}{b}\right)\right] \qquad 4)\ \left(\frac{a}{x}:\frac{b}{y}\right):\left(\frac{x}{a}:\frac{y}{b}\right).$$

Rješenje 534

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$1)\ \frac{a}{x}:\frac{b}{y}:\frac{x}{a}:\frac{y}{b}=\frac{a}{x}\cdot\frac{y}{b}\cdot\frac{a}{x}\cdot\frac{b}{y}=\frac{a}{x}\cdot\frac{y}{b}\cdot\frac{a}{x}\cdot\frac{b}{y}=\frac{a}{x}\cdot\frac{1}{1}\cdot\frac{a}{x}\cdot\frac{1}{1}=\frac{a^2}{x^2}.$$

$$2)\ \frac{a}{x}:\left(\frac{b}{y}:\frac{x}{a}:\frac{y}{b}\right)=\frac{a}{x}:\left(\frac{b}{y}\cdot\frac{a}{x}\cdot\frac{b}{y}\right)=\frac{a}{x}:\left(\frac{b}{y}\cdot\frac{a}{x}\cdot\frac{b}{y}\right)=\frac{a}{x}:\frac{a\cdot b^2}{x\cdot y^2}=\frac{a}{x}\cdot\frac{x\cdot y^2}{a\cdot b^2}=$$

$$=\frac{a}{x}\cdot\frac{x\cdot y^2}{a\cdot b^2}=\frac{1}{1}\cdot\frac{y^2}{b^2}=\frac{y^2}{b^2}.$$

3) $$\frac{a}{x}:\left[\frac{b}{y}:\left(\frac{x}{a}:\frac{y}{b}\right)\right]=\frac{a}{x}:\left[\frac{b}{y}:\left(\frac{x}{a}\cdot\frac{b}{y}\right)\right]=\frac{a}{x}:\left[\frac{b}{y}:\frac{b\cdot x}{a\cdot y}\right]=\frac{a}{x}:\left[\frac{b}{y}\cdot\frac{a\cdot y}{b\cdot x}\right]=\frac{a}{x}:\left[\frac{b}{y}\cdot\frac{a\cdot y}{b\cdot x}\right]=$$

$$=\frac{a}{x}:\left[\frac{1}{1}\cdot\frac{a}{x}\right]=\frac{a}{x}:\frac{a}{x}=1.$$

4) $$\left(\frac{a}{x}:\frac{b}{y}\right):\left(\frac{x}{a}:\frac{y}{b}\right)=\left(\frac{a}{x}\cdot\frac{y}{b}\right):\left(\frac{x}{a}\cdot\frac{b}{y}\right)=\frac{a\cdot y}{b\cdot x}:\frac{b\cdot x}{a\cdot y}=\frac{a\cdot y}{b\cdot x}\cdot\frac{a\cdot y}{b\cdot x}=\frac{a^2\cdot y^2}{b^2\cdot x^2}.$$

Vježba 534

Pojednostavnite:

1) $\frac{2}{5}:\frac{3}{7}:\frac{5}{2}:\frac{7}{3}$ 2) $\frac{2}{5}:\left(\frac{3}{7}:\frac{5}{2}:\frac{7}{3}\right)$ 3) $\frac{2}{5}:\left[\frac{3}{7}:\left(\frac{5}{2}:\frac{7}{3}\right)\right]$ 4) $\left(\frac{2}{5}:\frac{3}{7}\right):\left(\frac{5}{2}:\frac{7}{3}\right).$

Rezultat: 1) $\frac{4}{25}$ 2) $\frac{49}{9}$ 3) 1 4) $\frac{196}{225}.$

Zadatak 535 (Ajax, gimnazija)

Pojednostavnite: $$\frac{\left(a\cdot b^{-3}-a^{-3}\cdot b\right)^{-1}\cdot\left(b^{-2}-a^{-2}\right)^{-1}}{\left(a^{-2}+b^{-2}\right)^{-1}}.$$

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Rješenje 535

Ponovimo!

$$a^{-n}=\frac{1}{a^n}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad a^1=a.$$

$$a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{\left(a\cdot b^{-3}-a^{-3}\cdot b\right)^{-1}\cdot\left(b^{-2}-a^{-2}\right)^{-1}}{\left(a^{-2}+b^{-2}\right)^{-1}}=\frac{a^{-2}+b^{-2}}{\left(a\cdot b^{-3}-a^{-3}\cdot b\right)\cdot\left(b^{-2}-a^{-2}\right)}=$$

$$=\frac{\frac{1}{a^2}+\frac{1}{b^2}}{\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\cdot\left(\frac{1}{b^2}-\frac{1}{a^2}\right)}=\frac{\frac{b^2+a^2}{a^2\cdot b^2}}{\frac{a^4-b^4}{a^3\cdot b^3}\cdot\frac{a^2-b^2}{a^2\cdot b^2}}=\frac{\frac{a^2+b^2}{a^2\cdot b^2}}{\frac{a^4-b^4}{a^3\cdot b^3}\cdot\frac{a^2-b^2}{a^2\cdot b^2}}=$$

$$=\frac{\frac{a^2+b^2}{1}}{\frac{a^4-b^4}{a^3\cdot b^3}\cdot\frac{a^2-b^2}{1}}=\frac{\frac{a^2+b^2}{1}}{\frac{\left(a^4-b^4\right)\cdot\left(a^2-b^2\right)}{a^3\cdot b^3}}=\frac{a^3\cdot b^3\cdot\left(a^2+b^2\right)}{\left(a^4-b^4\right)\cdot\left(a^2-b^2\right)}=$$

$$=\frac{a^3\cdot b^3\cdot\left(a^2+b^2\right)}{\left(\left(a^2\right)^2-\left(b^2\right)^2\right)\cdot\left(a^2-b^2\right)}=\frac{a^3\cdot b^3\cdot\left(a^2+b^2\right)}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)\cdot\left(a^2-b^2\right)}=$$

$$=\frac{a^3\cdot b^3\cdot\left(a^2+b^2\right)}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)\cdot\left(a^2-b^2\right)}=\frac{a^3\cdot b^3}{\left(a^2-b^2\right)^2}.$$

2.inačica

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$$\frac{\left(a\cdot b^{-3}-a^{-3}\cdot b\right)^{-1}\cdot\left(b^{-2}-a^{-2}\right)^{-1}}{\left(a^{-2}+b^{-2}\right)^{-1}}=\frac{\left(\frac{a}{b^3}-\frac{b}{a^3}\right)^{-1}\cdot\left(\frac{1}{b^2}-\frac{1}{a^2}\right)^{-1}}{\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^{-1}}=$$

$$=\frac{\left(\frac{a^4-b^4}{a^3\cdot b^3}\right)^{-1}\cdot\left(\frac{a^2-b^2}{a^2\cdot b^2}\right)^{-1}}{\left(\frac{b^2+a^2}{a^2\cdot b^2}\right)^{-1}}=\frac{\frac{a^3\cdot b^3}{a^4-b^4}\cdot\frac{a^2\cdot b^2}{a^2-b^2}}{\frac{a^2\cdot b^2}{b^2+a^2}}=\frac{\frac{a^3\cdot b^3}{a^4-b^4}\cdot\frac{a^2\cdot b^2}{a^2-b^2}}{\frac{a^2\cdot b^2}{a^2+b^2}}=$$

$$=\frac{\frac{a^3\cdot b^3}{a^4-b^4}\cdot\frac{1}{a^2-b^2}}{\frac{1}{a^2+b^2}}=\frac{\frac{a^3\cdot b^3}{\left(a^4-b^4\right)\cdot\left(a^2-b^2\right)}}{\frac{1}{a^2+b^2}}=\frac{a^3\cdot b^3\cdot\left(a^2+b^2\right)}{\left(a^4-b^4\right)\cdot\left(a^2-b^2\right)}=$$

$$=\frac{a^3\cdot b^3\cdot\left(a^2+b^2\right)}{\left(\left(a^2\right)^2-\left(b^2\right)^2\right)\cdot\left(a^2-b^2\right)}=\frac{a^3\cdot b^3\cdot\left(a^2+b^2\right)}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)\cdot\left(a^2-b^2\right)}=$$

$$=\frac{a^3\cdot b^3\cdot\left(a^2+b^2\right)}{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)\cdot\left(a^2-b^2\right)}=\frac{a^3\cdot b^3}{\left(a^2-b^2\right)^2}.$$

Vježba 535

Pojednostavnite: $\dfrac{\left(a^{-2}+b^{-2}\right)^{-1}}{\left(a\cdot b^{-3}-a^{-3}\cdot b\right)^{-1}\cdot\left(b^{-2}-a^{-2}\right)^{-1}}$.

Rezultat: $\dfrac{\left(a^{2}-b^{2}\right)^{2}}{a^{3}\cdot b^{3}}$.

Zadatak 536 (Ajax, gimnazija)

Pojednostavnite: $\dfrac{a^{n}\cdot b^{m}+a^{2\cdot n}}{4\cdot a^{2\cdot n}}\cdot\dfrac{2\cdot b^{m}-2\cdot a^{n}}{a^{2\cdot n}-b^{2\cdot m}}$.

Rješenje 536

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad a^{2}-b^{2}=(a-b)\cdot(a+b)\quad,\quad\frac{a^{n}}{a^{m}}=a^{n-m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

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$$\frac{a^{n}\cdot b^{m}+a^{2\cdot n}}{4\cdot a^{2\cdot n}}\cdot\frac{2\cdot b^{m}-2\cdot a^{n}}{a^{2\cdot n}-b^{2\cdot m}}=\frac{a^{n}\cdot b^{m}+\left(a^{n}\right)^{2}}{4\cdot a^{2\cdot n}}\cdot\frac{2\cdot\left(b^{m}-a^{n}\right)}{\left(a^{n}\right)^{2}-\left(b^{m}\right)^{2}}=$$

$$=\frac{a^{n}\cdot\left(b^{m}+a^{n}\right)}{4\cdot a^{2\cdot n}}\cdot\frac{2\cdot\left(b^{m}-a^{n}\right)}{\left(a^{n}-b^{m}\right)\cdot\left(a^{n}+b^{m}\right)}=\frac{a^{n}\cdot\left(a^{n}+b^{m}\right)}{4\cdot a^{2\cdot n}}\cdot\frac{2\cdot\left(b^{m}-a^{n}\right)}{\left(a^{n}-b^{m}\right)\cdot\left(a^{n}+b^{m}\right)}=$$

$$=\frac{a^{n}}{4\cdot a^{2\cdot n}}\cdot\frac{2\cdot\left(b^{m}-a^{n}\right)}{a^{n}-b^{m}}=\frac{a^{n}}{4\cdot a^{2\cdot n}}\cdot\frac{-2\cdot\left(a^{n}-b^{m}\right)}{a^{n}-b^{m}}=\frac{a^{n}}{4\cdot a^{2\cdot n}}\cdot\frac{-2\cdot\left(a^{n}-b^{m}\right)}{a^{n}-b^{m}}=$$

$$=\frac{-a^{n}}{2\cdot a^{2\cdot n}}=-\frac{1}{2}\cdot a^{n-2\cdot n}=-\frac{1}{2}\cdot a^{-n}=-\frac{1}{2}\cdot\frac{1}{a^{n}}=-\frac{1}{2\cdot a^{n}}.$$

Vježba 536

Pojednostavnite: $\dfrac{a^{n}\cdot b^{m}+a^{2\cdot n}}{4\cdot a^{2\cdot n}}:\dfrac{a^{2\cdot n}-b^{2\cdot m}}{2\cdot b^{m}-2\cdot a^{n}}$.

Rezultat: $-\dfrac{1}{2\cdot a^{n}}$.

Zadatak 537 (Ajax, gimnazija)

Nakon skraćivanja razlomka $\dfrac{3^{a+b}+3^{a}}{3^{b}+9^{b}}$ dobijemo potenciju:

$A.\ 3^{a}\qquad B.\ 3^{b}\qquad C.\ 3^{a+b}\qquad D.\ 3^{a-b}$

Rješenje 537

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad \left(a^n\right)^m = a^{n \cdot m} = \left(a^m\right)^n \quad , \quad a^n : a^m = \frac{a^n}{a^m} = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\frac{3^{a+b} + 3^a}{3^b + 9^b} = \frac{3^a \cdot 3^b + 3^a}{3^b + \left(3^2\right)^b} = \frac{3^a \cdot 3^b + 3^a}{3^b + \left(3^b\right)^2} = \begin{bmatrix} \text{u brojniku izlučimo } 3^a \\ \text{u nazivniku izlučimo } 3^b \end{bmatrix} =$$

$$= \frac{3^a \cdot 3^b + 3^a}{3^b + \left(3^b\right)^2} = \frac{3^a \cdot \left(3^b + 1\right)}{3^b \cdot \left(1 + 3^b\right)} = \frac{3^a \cdot \left(3^b + 1\right)}{3^b \cdot \left(3^b + 1\right)} = \frac{3^a \cdot \left(3^b + 1\right)}{3^b \cdot \left(3^b + 1\right)} = \frac{3^a}{3^b} = 3^{a-b}.$$

Odgovor je pod D.

Vježba 537

Nakon skraćivanja razlomka $\frac{3^{a+b} + 3^a}{3^b + 1}$ dobijemo potenciju:

$A.\ 3^a \qquad B.\ 3^b \qquad C.\ 3^{a+b} \qquad D.\ 3^{a-b}$

Rezultat: A.

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Zadatak 538 (Alen, gimnazija)

Pojednostavni: $\frac{25^4 - 25^3}{5^8 - 5^7 + 5^6}$.

Rješenje 538

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^1 = a.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\frac{25^4 - 25^3}{5^8 - 5^7 + 5^6} = \frac{\left(5^2\right)^4 - \left(5^2\right)^3}{5^8 - 5^7 + 5^6} = \frac{5^8 - 5^6}{5^8 - 5^7 + 5^6} = \frac{5^6 \cdot 5^2 - 5^6}{5^6 \cdot 5^2 - 5^6 \cdot 5^1 + 5^6} =$$

$$= \begin{bmatrix} \text{u brojniku izlučimo } 5^6 \\ \text{u nazivniku izlučimo } 5^6 \end{bmatrix} = \frac{5^6 \cdot 5^2 - 5^6}{5^6 \cdot 5^2 - 5^6 \cdot 5^1 + 5^6} = \frac{5^6 \cdot \left(5^2 - 1\right)}{5^6 \cdot \left(5^2 - 5^1 + 1\right)} =$$

$$=\frac{5^6\cdot 24}{5^6\cdot 21}=\frac{5^6\cdot 24}{5^6\cdot 21}=\frac{24}{21}=\frac{24}{21}=\frac{8}{7}.$$

Vježba 538

Pojednostavni: $\frac{3^{15}+3^{12}}{3^{16}+3^{14}+3^{12}}$.

Rezultat: $\frac{4}{13}$.

Zadatak 539 (AnaM, gimnazija)

Skrati razlomak: $\frac{a^2-(b+c)^2}{(a+b)^2-c^2}$.

Rješenje 539

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad ,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{a^2-(b+c)^2}{(a+b)^2-c^2}=\frac{(a-(b+c))\cdot(a+(b+c))}{((a+b)-c)\cdot((a+b)+c)}=\frac{(a-b-c)\cdot(a+b+c)}{(a+b-c)\cdot(a+b+c)}=$$

$$=\frac{(a-b-c)\cdot(a+b+c)}{(a+b-c)\cdot(a+b+c)}=\frac{a-b-c}{a+b-c}.$$

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Vježba 539

Skrati razlomak: $\frac{a^2-(b+c)^2}{(a+b)^2-c^2}$.

Rezultat: $\frac{4}{13}$.

Zadatak 540 (AnaM, gimnazija)

Izračunaj: $\frac{(x+2)^3}{x^2-4}\cdot\frac{1}{x^2+4\cdot x+4}$.

Rješenje 540

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)\quad ,\quad a^2+2\cdot a\cdot b+b^2=(a+b)^2\qquad a^1=a.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad ,\quad a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ \ ,\ \ n\neq 0\ \ ,\ \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{(x+2)^3}{x^2-4}\cdot\frac{1}{x^2+4\cdot x+4}=\frac{(x+2)^3}{(x-2)\cdot(x+2)}\cdot\frac{1}{(x+2)^2}=\frac{(x+2)^3}{(x-2)\cdot(x+2)^3}=$$

$$=\frac{(x+2)^3}{(x-2)\cdot(x+2)^3}=\frac{1}{x-2}.$$

Vježba 540

Izračunaj: $\frac{(x+2)^3}{x^2-4}:\left(x^2+4\cdot x+4\right).$

Rezultat: $\frac{1}{x-2}.$

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