Zadatak 561 (Tihomir, srednja škola)

Provjeri da je vrijednost razlomka $\frac{5^{n-1}\cdot 3^{n+1}-3^{n-1}\cdot 5^{n+1}}{15^{n}}$ uvijek isti broj, neovisan o prirodnom broju n.

Rješenje 561

Ponovimo!

$$a^{n}\cdot a^{m}=a^{n+m}\quad,\quad a^{1}=a\quad,\quad a^{-n}=\frac{1}{a^{n}}\quad,\quad a^{n}\cdot b^{n}=(a\cdot b)^{n}.$$

$$\frac{a}{b}\cdot c=\frac{a\cdot c}{b}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{5^{n-1}\cdot 3^{n+1}-3^{n-1}\cdot 5^{n+1}}{15^{n}}=\frac{5^{n}\cdot 5^{-1}\cdot 3^{n}\cdot 3^{1}-3^{n}\cdot 3^{-1}\cdot 5^{n}\cdot 5^{1}}{15^{n}}=$$

$$=\frac{5^{n}\cdot 3^{n}\cdot\frac{1}{5}\cdot 3-5^{n}\cdot 3^{n}\cdot\frac{1}{3}\cdot 5}{15^{n}}=\frac{\left(5^{n}\cdot 3^{n}\right)\cdot\frac{1}{5}\cdot 3-\left(5^{n}\cdot 3^{n}\right)\cdot\frac{1}{3}\cdot 5}{15^{n}}=$$

$$=\frac{(5\cdot 3)^{n}\cdot\frac{3}{5}-(5\cdot 3)^{n}\cdot\frac{5}{3}}{15^{n}}=\frac{15^{n}\cdot\frac{3}{5}-15^{n}\cdot\frac{5}{3}}{15^{n}}=\frac{15^{n}\cdot\frac{3}{5}-15^{n}\cdot\frac{5}{3}}{15^{n}}=\frac{15^{n}\cdot\left(\frac{3}{5}-\frac{5}{3}\right)}{15^{n}}=$$

$$=\frac{15^{n}\cdot\left(\frac{3}{5}-\frac{5}{3}\right)}{15^{n}}=\frac{3}{5}-\frac{5}{3}=\frac{9-25}{15}=-\frac{16}{25}.$$

Vježba 561

Provjeri da je vrijednost razlomka $\frac{3\cdot 5^{n-1}\cdot 3^{n}-5\cdot 3^{n-1}\cdot 5^{n}}{15^{n}}$ uvijek isti broj, neovisan o prirodnom broju n.

Rezultat: $-\frac{16}{25}.$

Zadatak 562 (Tihomir, srednja škola)

Izračunaj: $\left(4\cdot a-\frac{4\cdot a-1}{a}\right):\left(\frac{1}{a^{2}}-4\right).$

Rješenje 562

Ponovimo!

$$n=\frac{n}{1}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$a^{2}-2\cdot a\cdot b+b^{2}=(a-b)^{2}\quad,\quad (a-b)^{2}=(b-a)^{2}\quad,\quad a^{2}-b^{2}=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left(4\cdot a-\frac{4\cdot a-1}{a}\right):\left(\frac{1}{a^2}-4\right)=\left(\frac{4\cdot a}{1}-\frac{4\cdot a-1}{a}\right):\left(\frac{1}{a^2}-\frac{4}{1}\right)=\frac{4\cdot a^2-(4\cdot a-1)}{a}:\frac{1-4\cdot a^2}{a^2}=$$

$$=\frac{4\cdot a^2-4\cdot a+1}{a}\cdot\frac{a^2}{1-4\cdot a^2}=\frac{(2\cdot a-1)^2}{a}\cdot\frac{a^2}{(1-2\cdot a)\cdot(1+2\cdot a)}=\frac{(1-2\cdot a)^2}{a}\cdot\frac{a^2}{(1-2\cdot a)\cdot(1+2\cdot a)}=$$

$$=\frac{(1-2\cdot a)^2}{a}\cdot\frac{a^2}{(1-2\cdot a)\cdot(1+2\cdot a)}=\frac{1-2\cdot a}{1}\cdot\frac{a}{1+2\cdot a}=\frac{a\cdot(1-2\cdot a)}{1+2\cdot a}.$$

Vježba 562

Izračunaj: $\left(\frac{4\cdot a-1}{a}-4\cdot a\right):\left(4-\frac{1}{a^2}\right).$

Rezultat: $\frac{a\cdot(1-2\cdot a)}{1+2\cdot a}.$

Zadatak 563 (Nena, gimnazija)

Pojednostavni izraz: $\left(\frac{x}{x^2-4}-\frac{8}{x^2+2\cdot x}\right)\cdot\left(\frac{x-4}{2\cdot x-x^2}\right)^{-1}+\frac{x+8}{x+2}.$

Rješenje 563

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n} \quad , \quad a^1=a \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$a^2-2\cdot a\cdot b+b^2=(a-b)^2 \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left(\frac{x}{x^2-4}-\frac{8}{x^2+2\cdot x}\right)\cdot\left(\frac{x-4}{2\cdot x-x^2}\right)^{-1}+\frac{x+8}{x+2}=\left(\frac{x}{(x-2)\cdot(x+2)}-\frac{8}{x\cdot(x+2)}\right)\cdot\frac{2\cdot x-x^2}{x-4}+\frac{x+8}{x+2}=$$

$$=\frac{x^2-8\cdot(x-2)}{x\cdot(x-2)\cdot(x+2)}\cdot\frac{-x\cdot(x-2)}{x-4}+\frac{x+8}{x+2}=\frac{x^2-8\cdot x+16}{x\cdot(x-2)\cdot(x+2)}\cdot\frac{-x\cdot(x-2)}{x-4}+\frac{x+8}{x+2}=$$

$$=\frac{(x-4)^2}{x\cdot(x-2)\cdot(x+2)}\cdot\frac{-x\cdot(x-2)}{x-4}+\frac{x+8}{x+2}=\frac{(x-4)^2}{x\cdot(x-2)\cdot(x+2)}\cdot\frac{-x\cdot(x-2)}{x-4}+\frac{x+8}{x+2}=$$

$$=\frac{x-4}{x+2}\cdot\frac{-1}{1}+\frac{x+8}{x+2}=\frac{-(x-4)}{x+2}+\frac{x+8}{x+2}=\frac{-x+4}{x+2}+\frac{x+8}{x+2}=\frac{-x+4+x+8}{x+2}=\frac{-x+4+x+8}{x+2}=\frac{12}{x+2}.$$

Vježba 563

Izračunaj: $\left(\frac{x}{x^2-4}-\frac{8}{x^2+2\cdot x}\right)\cdot\left(\frac{4-x}{x^2-2\cdot x}\right)^{-1}+\left(\frac{x+2}{x+8}\right)^{-1}.$

Rezultat: $\frac{12}{x+2}.$

Zadatak 564 (Nena, gimnazija)

Pojednostavni izraz: $\left(\frac{x\cdot\sqrt{x}-y\cdot\sqrt{y}}{x-\sqrt{x}\cdot\sqrt{y}}-\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)\cdot\left(\frac{\sqrt{x}\cdot\sqrt{y}+2\cdot x}{3\cdot x}\right)^{-1}.$

Rješenje 564

Ponovimo!

$$a\cdot\sqrt[n]{b}=\sqrt[n]{a^n\cdot b}\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n\quad,\quad a^1=a.$$

$$\sqrt{a^n}=\left(\sqrt{a}\right)^n\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad n=\frac{n}{1}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left(\frac{x\cdot\sqrt{x}-y\cdot\sqrt{y}}{x-\sqrt{x}\cdot\sqrt{y}}-\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)\cdot\left(\frac{\sqrt{x}\cdot\sqrt{y}+2\cdot x}{3\cdot x}\right)^{-1}=$$

$$=\left(\frac{\sqrt{x^2\cdot x}-\sqrt{y^2\cdot y}}{\left(\sqrt{x}\right)^2-\sqrt{x}\cdot\sqrt{y}}-\frac{\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2}{\sqrt{x}+\sqrt{y}}\right)\cdot\frac{3\cdot x}{\sqrt{x}\cdot\sqrt{y}+2\cdot x}=$$

$$=\left(\frac{\sqrt{x^3}-\sqrt{y^3}}{\left(\sqrt{x}\right)\cdot\left(\sqrt{x}-\sqrt{y}\right)}-\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\sqrt{x}+\sqrt{y}\right)}{\sqrt{x}+\sqrt{y}}\right)\cdot\frac{3\cdot\left(\sqrt{x}\right)^2}{\sqrt{x}\cdot\sqrt{y}+2\cdot\left(\sqrt{x}\right)^2}=$$

$$=\left(\frac{\left(\sqrt{x}\right)^3-\left(\sqrt{y}\right)^3}{\left(\sqrt{x}\right)\cdot\left(\sqrt{x}-\sqrt{y}\right)}-\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\sqrt{x}+\sqrt{y}\right)}{\sqrt{x}+\sqrt{y}}\right)\cdot\frac{3\cdot\left(\sqrt{x}\right)^2}{\sqrt{x}\cdot\left(\sqrt{y}+2\cdot\sqrt{x}\right)}=$$

$$=\left(\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\left(\sqrt{x}\right)^2+\sqrt{x}\cdot\sqrt{y}+\left(\sqrt{y}\right)^2\right)}{\left(\sqrt{x}\right)\cdot\left(\sqrt{x}-\sqrt{y}\right)}-\frac{\sqrt{x}-\sqrt{y}}{1}\right)\cdot\frac{3\cdot\left(\sqrt{x}\right)^2}{\sqrt{x}\cdot\left(\sqrt{y}+2\cdot\sqrt{x}\right)}=$$

$$=\left(\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(x+\sqrt{x}\cdot\sqrt{y}+y\right)}{\left(\sqrt{x}\right)\cdot\left(\sqrt{x}-\sqrt{y}\right)}-\frac{\sqrt{x}-\sqrt{y}}{1}\right)\cdot\frac{3\cdot\sqrt{x}}{\sqrt{y}+2\cdot\sqrt{x}}=$$

$$=\left(\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(x+\sqrt{x}\cdot\sqrt{y}+y\right)}{\left(\sqrt{x}\right)\cdot\left(\sqrt{x}-\sqrt{y}\right)}-\frac{\sqrt{x}-\sqrt{y}}{1}\right)\cdot\frac{3\cdot\sqrt{x}}{\sqrt{y}+2\cdot\sqrt{x}}=$$

$$=\left(\frac{x+\sqrt{x}\cdot\sqrt{y}+y}{\sqrt{x}}-\frac{\sqrt{x}-\sqrt{y}}{1}\right)\cdot\frac{3\cdot\sqrt{x}}{\sqrt{y}+2\cdot\sqrt{x}}=$$

$$=\frac{x+\sqrt{x}\cdot\sqrt{y}+y-\sqrt{x}\cdot\left(\sqrt{x}-\sqrt{y}\right)}{\sqrt{x}}\cdot\frac{3\cdot\sqrt{x}}{\sqrt{y}+2\cdot\sqrt{x}}=$$

$$=\frac{x+\sqrt{x}\cdot\sqrt{y}+y-\left(\sqrt{x}\right)^2+\sqrt{x}\cdot\sqrt{y}}{\sqrt{x}}\cdot\frac{3\cdot\sqrt{x}}{\sqrt{y}+2\cdot\sqrt{x}}=\frac{x+\sqrt{x}\cdot\sqrt{y}+y-x+\sqrt{x}\cdot\sqrt{y}}{\sqrt{x}}\cdot\frac{3\cdot\sqrt{x}}{\sqrt{y}+2\cdot\sqrt{x}}=$$

$$=\frac{x+\sqrt{x}\cdot\sqrt{y}+y-x+\sqrt{x}\cdot\sqrt{y}}{1}\cdot\frac{3}{\sqrt{y}+2\cdot\sqrt{x}}=\frac{y+2\cdot\sqrt{x}\cdot\sqrt{y}}{1}\cdot\frac{3}{\sqrt{y}+2\cdot\sqrt{x}}=$$

$$=\frac{\left(\sqrt{y}\right)^2+2\cdot\sqrt{x}\cdot\sqrt{y}}{1}\cdot\frac{3}{\sqrt{y}+2\cdot\sqrt{x}}=\frac{\sqrt{y}\cdot\left(\sqrt{y}+2\cdot\sqrt{x}\right)}{1}\cdot\frac{3}{\sqrt{y}+2\cdot\sqrt{x}}=$$

$$=\frac{\sqrt{y}\cdot\left(\sqrt{y}+2\cdot\sqrt{x}\right)}{1}\cdot\frac{3}{\sqrt{y}+2\cdot\sqrt{x}}=3\cdot\sqrt{y}.$$

Vježba 564

Izračunaj: $\left(\frac{x\cdot\sqrt{x}-y\cdot\sqrt{y}}{x-\sqrt{x}\cdot\sqrt{y}}+\frac{y-x}{\sqrt{x}+\sqrt{y}}\right)\cdot\left(\frac{\sqrt{x}\cdot\sqrt{y}+2\cdot x}{3\cdot x}\right)^{-1}.$

Rezultat: $3\cdot\sqrt{y}.$

Zadatak 565 (Ante, gimnazija)

Pojednostavni dvojni razlomak: $\dfrac{\frac{2}{a-b}-\frac{1}{a}}{\frac{a^2+2\cdot a\cdot b+b^2}{a^2-b^2}}.$

Rješenje 565

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad a^2+2\cdot a\cdot b+b^2=\left(a+b\right)^2\quad,\quad a^1=a\quad,\quad a^n:a^m=a^{n-m}.$$

$$\frac{\frac{a}{b}}{\frac{a}{c}}=\frac{c}{b} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{b}}=\frac{a}{c} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$\frac{\frac{2}{a-b}-\frac{1}{a}}{\frac{a^2+2\cdot a\cdot b+b^2}{a^2-b^2}}=\frac{\frac{2\cdot a-(a-b)}{a\cdot(a-b)}}{\frac{(a+b)^2}{(a-b)\cdot(a+b)}}=\frac{\frac{2\cdot a-a+b}{a\cdot(a-b)}}{\frac{(a+b)^2}{(a-b)\cdot(a+b)}}=\frac{\frac{a+b}{a\cdot(a-b)}}{\frac{(a+b)^2}{(a-b)\cdot(a+b)}}=\frac{\frac{a+b}{a\cdot(a-b)}}{\frac{(a+b)^2}{(a-b)\cdot(a+b)}}=$$

$$=\frac{\frac{a+b}{a}}{\frac{(a+b)^2}{a+b}}=\frac{\frac{a+b}{a}}{\frac{(a+b)^2}{a+b}}=\frac{\frac{1}{a}}{\frac{a+b}{a+b}}=\frac{\frac{1}{a}}{\frac{a+b}{a+b}}=\frac{\frac{1}{a}}{\frac{1}{1}}=\frac{1}{a}.$$

Vježba 565

Pojednostavni dvojni razlomak: $\frac{\frac{1}{a}-\frac{2}{a-b}}{\frac{a^2+2\cdot a\cdot b+b^2}{b^2-a^2}}.$

Rezultat: $\frac{1}{a}.$

Zadatak 566 (Ante, gimnazija)

Ako je $a+b+c=0$, tada je $a\cdot(a+c)=b\cdot(b+c)$.

Rješenje 566

Ponovimo!

$$\left.\begin{array}{l} a=c \\ b=c \end{array}\right\} \Rightarrow a=b.$$

Iz

$$a+b+c=0$$

dobije se:

$$\left.\begin{array}{l} a+b+c=0 \Rightarrow a+c=-b \\ a+b+c=0 \Rightarrow b+c=-a \end{array}\right\}.$$

Sada provjeravamo jednakost

$$a\cdot(a+c)=b\cdot(b+c).$$

$$\left.\begin{array}{l} a\cdot(a+c)=[a+c=-b]=a\cdot(-b)=-a\cdot b \\ b\cdot(b+c)=[b+c=-a]=b\cdot(-a)=-a\cdot b \end{array}\right\} \Rightarrow a\cdot(a+c)=b\cdot(b+c).$$

Vježba 566

Ako je $a+b+c=0$, tada je $b\cdot(b+a)=c\cdot(c+a)$.

Rezultat: Dokaz analogan.

Zadatak 567 (Tonka, gimnazija)

Izračunajte: $(2+3\cdot x-y)^2$.

Rješenje 567

Ponovimo!

Kako glasi zakon asocijacije (združivanja) za zbrajanje?

Zbroj se ne mijenja združimo li pribrojnike na bilo koji način:

$$(a+b)+c=a+(b+c).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n.$$

$$(a+b+c)^2=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c.$$

1.inačica

$$(2+3\cdot x-y)^2=((2+3\cdot x)-y)^2=(2+3\cdot x)^2-2\cdot y\cdot(2+3\cdot x)+y^2=$$

$$=2^2+2\cdot 2\cdot 3\cdot x+(3\cdot x)^2-4\cdot y-6\cdot x\cdot y+y^2=4+12\cdot x+9\cdot x^2-4\cdot y-6\cdot x\cdot y+y^2=$$

$$=9\cdot x^2+y^2+12\cdot x-4\cdot y-6\cdot x\cdot y+4.$$

2.inačica

$$(2+3\cdot x-y)^2=(2+(3\cdot x-y))^2=2^2+2\cdot 2\cdot(3\cdot x-y)+(3\cdot x-y)^2=$$

$$=4+4\cdot(3\cdot x-y)+(3\cdot x)^2-2\cdot 3\cdot x\cdot y+y^2=4+12\cdot x-4\cdot y+9\cdot x^2-6\cdot x\cdot y+y^2=$$

$$=9\cdot x^2+y^2+12\cdot x-4\cdot y-6\cdot x\cdot y+4.$$

3.inačica

$$(2+3\cdot x-y)^2=2^2+(3\cdot x)^2+(-y)^2+2\cdot 2\cdot(3\cdot x)+2\cdot 2\cdot(-y)+2\cdot(3\cdot x)\cdot(-y)=$$

$$=4+9\cdot x^2+y^2+12\cdot x-4\cdot y-6\cdot x\cdot y=9\cdot x^2+y^2+12\cdot x-4\cdot y-6\cdot x\cdot y+4.$$

Vježba 567

Izračunajte: $(2+3\cdot x+y)^2$.

Rezultat: $9\cdot x^2+y^2+12\cdot x+4\cdot y+6\cdot x\cdot y+4.$

Zadatak 568 (Nena, gimnazija)

Pojednostavni: $\dfrac{\dfrac{1}{\sqrt{a-1}}+\sqrt{a+1}}{\dfrac{1}{\sqrt{a+1}}-\dfrac{1}{\sqrt{a-1}}}:\dfrac{\sqrt{a+1}}{(a-1)\cdot\sqrt{a+1}-(a+1)\cdot\sqrt{a-1}}$, za $a>1$.

Rješenje 568

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\sqrt{a^2} = a \ , \ a > 0 \quad , \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b} \quad , \quad \left(\sqrt{a}\right)^2 = a \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\frac{\frac{1}{\sqrt{a-1}} + \sqrt{a+1}}{\frac{1}{\sqrt{a+1}} - \frac{1}{\sqrt{a-1}}} : \frac{\sqrt{a+1}}{(a-1) \cdot \sqrt{a+1} - (a+1) \cdot \sqrt{a-1}} =$$

$$= \frac{\frac{1}{\sqrt{a-1}} + \frac{\sqrt{a+1}}{1}}{\frac{1}{\sqrt{a+1}} - \frac{1}{\sqrt{a-1}}} \cdot \frac{(a-1) \cdot \sqrt{a+1} - (a+1) \cdot \sqrt{a-1}}{\sqrt{a+1}} =$$

$$= \frac{\frac{1 + \sqrt{a+1} \cdot \sqrt{a-1}}{\sqrt{a-1}}}{\frac{\sqrt{a-1} - \sqrt{a+1}}{\sqrt{a+1} \cdot \sqrt{a-1}}} \cdot \frac{\left(\sqrt{a-1}\right)^2 \cdot \sqrt{a+1} - \left(\sqrt{a+1}\right)^2 \cdot \sqrt{a-1}}{\sqrt{a+1}} =$$

$$= \frac{\frac{1 + \sqrt{(a+1) \cdot (a-1)}}{\sqrt{a-1}}}{\frac{\sqrt{a-1} - \sqrt{a+1}}{\sqrt{a+1} \cdot \sqrt{a-1}}} \cdot \frac{\sqrt{a-1} \cdot \sqrt{a+1} \cdot \left(\sqrt{a-1} - \sqrt{a+1}\right)}{\sqrt{a+1}} =$$

$$= \frac{\frac{1 + \sqrt{a^2 - 1}}{\sqrt{a-1}}}{\frac{\sqrt{a-1} - \sqrt{a+1}}{\sqrt{a+1} \cdot \sqrt{a-1}}} \cdot \frac{\sqrt{a-1} \cdot \sqrt{a+1} \cdot \left(\sqrt{a-1} - \sqrt{a+1}\right)}{\sqrt{a+1}} =$$

$$= \frac{\frac{1 + \sqrt{a^2 - 1}}{\sqrt{a-1}}}{\frac{\sqrt{a-1} - \sqrt{a+1}}{\sqrt{a+1} \cdot \sqrt{a-1}}} \cdot \frac{\sqrt{a-1} \cdot \sqrt{a+1} \cdot \left(\sqrt{a-1} - \sqrt{a+1}\right)}{\sqrt{a+1}} =$$

$$=\frac{\frac{1+\sqrt{a^2-1}}{1}}{\frac{\sqrt{a-1}-\sqrt{a+1}}{\sqrt{a+1}}}\cdot\frac{\sqrt{a-1}\cdot\left(\sqrt{a-1}-\sqrt{a+1}\right)}{1}=$$

$$=\frac{\left(1+\sqrt{a^2-1}\right)\cdot\sqrt{a+1}}{\sqrt{a-1}-\sqrt{a+1}}\cdot\frac{\sqrt{a-1}\cdot\left(\sqrt{a-1}-\sqrt{a+1}\right)}{1}=$$

$$=\frac{\left(1+\sqrt{a^2-1}\right)\cdot\sqrt{a+1}}{\sqrt{a-1}-\sqrt{a+1}}\cdot\frac{\sqrt{a-1}\cdot\left(\sqrt{a-1}-\sqrt{a+1}\right)}{1}=\left(1+\sqrt{a^2-1}\right)\cdot\sqrt{a+1}\cdot\sqrt{a-1}=$$

$$=\left(1+\sqrt{a^2-1}\right)\cdot\sqrt{(a+1)\cdot(a-1)}=\left(1+\sqrt{a^2-1}\right)\cdot\sqrt{a^2-1}=\sqrt{a^2-1}+\left(\sqrt{a^2-1}\right)^2=$$

$$=\sqrt{a^2-1}+a^2-1=a^2-1+\sqrt{a^2-1}.$$

Vježba 568

Pojednostavni: $\dfrac{\frac{1}{\sqrt{a-1}}+\sqrt{a+1}}{\frac{1}{\sqrt{a-1}}-\frac{1}{\sqrt{a+1}}}:\dfrac{\sqrt{a+1}}{(a+1)\cdot\sqrt{a-1}-(a-1)\cdot\sqrt{a+1}}$, za $a>1$.

Rezultat: $a^2-1+\sqrt{a^2-1}.$

Zadatak 569 (Nena, gimnazija)

Pojednostavni: $\left(\sqrt{\frac{1}{x^2}-1}-\frac{1}{x}\right):\left(\frac{1-x}{\sqrt{1-x^2}-1+x}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right).$

Rješenje 569

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}\quad,\quad\left(\sqrt{a}\right)^2=a.$$

$$\sqrt{a^2}=a\ ,\ a>0\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$\frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}\quad,\quad\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq0\ ,\ n\neq1.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$\left(\sqrt{\frac{1}{x^2}-1}-\frac{1}{x}\right)\cdot\left(\frac{1-x}{\sqrt{1-x^2}-1+x}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=$$

$$=\left(\sqrt{\frac{1}{x^2}-\frac{1}{1}}-\frac{1}{x}\right)\cdot\left(\frac{1-x}{\sqrt{1-x^2}-(1-x)}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=$$

$$=\left(\sqrt{\frac{1-x^2}{x^2}}-\frac{1}{x}\right)\cdot\left(\frac{\left(\sqrt{1-x}\right)^2}{\sqrt{(1-x)\cdot(1+x)}-\left(\sqrt{1-x}\right)^2}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=$$

$$=\left(\frac{\sqrt{1-x^2}}{\sqrt{x^2}}-\frac{1}{x}\right)\cdot\left(\frac{\left(\sqrt{1-x}\right)^2}{\sqrt{1-x}\cdot\sqrt{1+x}-\left(\sqrt{1-x}\right)^2}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=$$

$$=\left(\frac{\sqrt{1-x^2}}{x}-\frac{1}{x}\right)\cdot\left(\frac{\left(\sqrt{1-x}\right)^2}{\sqrt{1-x}\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=$$

$$=\frac{\sqrt{1-x^2}-1}{x}\cdot\left(\frac{\left(\sqrt{1-x}\right)^2}{\sqrt{1-x}\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=$$

$$=\frac{\sqrt{1-x^2}-1}{x}\cdot\left(\frac{\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right)=\frac{\sqrt{1-x^2}-1}{x}\cdot\frac{\sqrt{1-x}+\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}=$$

$$=\frac{\sqrt{(1-x)\cdot(1+x)}-1}{x}\cdot\frac{\sqrt{1-x}+\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}=\frac{\sqrt{1-x}\cdot\sqrt{1+x}-1}{x}\cdot\frac{\sqrt{1-x}+\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}=$$

$$=\frac{\left(\sqrt{1-x}\cdot\sqrt{1+x}-1\right)\cdot\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}=$$

$$=\frac{\left(\sqrt{1-x}\right)^2\cdot\sqrt{1+x}-\sqrt{1-x}+\left(\sqrt{1+x}\right)^2\cdot\sqrt{1-x}-\sqrt{1+x}}{x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}=$$

$$=\frac{(1-x)\cdot\sqrt{1+x}-\sqrt{1-x}+(1+x)\cdot\sqrt{1-x}-\sqrt{1+x}}{x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}=$$

$$=\frac{\sqrt{1+x}-x\cdot\sqrt{1+x}-\sqrt{1-x}+\sqrt{1-x}+x\cdot\sqrt{1-x}-\sqrt{1+x}}{x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}=$$

$$=\frac{\sqrt{1+x}-x\cdot\sqrt{1+x}-\sqrt{1-x}+\sqrt{1-x}+x\cdot\sqrt{1-x}-\sqrt{1+x}}{x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}=\frac{-x\cdot\sqrt{1+x}+x\cdot\sqrt{1-x}}{x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}=$$

$$=\frac{-x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}{x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}=\frac{-x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}{x\cdot\left(\sqrt{1+x}-\sqrt{1-x}\right)}=-1.$$

Vježba 569

Pojednostavni: $\left(\sqrt{\frac{1}{x^2}-1}-\frac{1}{x}\right)\cdot\left(\frac{x-1}{1-x-\sqrt{1-x^2}}+\frac{\sqrt{1+x}}{\sqrt{1+x}-\sqrt{1-x}}\right).$

Rezultat: $-1.$

Zadatak 570 (Ivan, gimnazija)

Napiši u što jednostavnijem obliku $\left(\frac{1}{5^x+1}+\frac{1}{5^x-1}\right)\cdot\frac{1}{2\cdot 5^x}.$

Rješenje 570

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad (a-b)\cdot(a+b)=a^2-b^2\quad,\quad \left(a^n\right)^m=\left(a^m\right)^n=a^{n\cdot m}.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left(\frac{1}{5^x+1}+\frac{1}{5^x-1}\right)\cdot\frac{1}{2\cdot 5^x}=\frac{5^x-1+5^x+1}{\left(5^x+1\right)\cdot\left(5^x-1\right)}\cdot\frac{1}{2\cdot 5^x}=\frac{5^x-1+5^x+1}{\left(5^x+1\right)\cdot\left(5^x-1\right)}\cdot\frac{1}{2\cdot 5^x}=$$

$$=\frac{5^x+5^x}{\left(5^x+1\right)\cdot\left(5^x-1\right)}\cdot\frac{1}{2\cdot 5^x}=\frac{2\cdot 5^x}{\left(5^x+1\right)\cdot\left(5^x-1\right)}\cdot\frac{1}{2\cdot 5^x}=\frac{2\cdot 5^x}{\left(5^x+1\right)\cdot\left(5^x-1\right)}\cdot\frac{1}{2\cdot 5^x}=$$

$$=\frac{1}{\left(5^x+1\right)\cdot\left(5^x-1\right)}\cdot\frac{1}{1}=\frac{1}{\left(5^x+1\right)\cdot\left(5^x-1\right)}=\frac{1}{\left(5^x\right)^2-1}=\frac{1}{\left(5^2\right)^x-1}=\frac{1}{25^x-1}.$$

Vježba 570

Napiši u što jednostavnijem obliku $\frac{1}{2}\cdot\left(\frac{1}{5^x+1}+\frac{1}{5^x-1}\right).$

Rezultat: $\frac{5^x}{25^x-1}.$

Zadatak 571 (Ivan, gimnazija)

Pojednostavni: $\left(\frac{1}{a^{-1}+1}+\frac{1}{a^{-2}-1}\right)\cdot\frac{a^{-1}+1}{a^{-1}}.$

Rješenje 571

Ponovimo!

$$a^1=a\quad,\quad a^{-n}=\frac{1}{a^n}\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left(\frac{1}{a^{-1}+1}+\frac{1}{a^{-2}-1}\right)\cdot\frac{a^{-1}+1}{a^{-1}}=\left(\frac{1}{\frac{1}{a}+1}+\frac{1}{\frac{1}{a^2}-1}\right)\cdot\frac{\frac{1}{a}+1}{\frac{1}{a}}=\left(\frac{1}{\frac{1}{a}+\frac{1}{1}}+\frac{1}{\frac{1}{a^2}-\frac{1}{1}}\right)\cdot\frac{\frac{1}{a}+\frac{1}{1}}{\frac{1}{a}}=$$

$$=\left(\frac{1}{\frac{1+a}{a}}+\frac{1}{\frac{1-a^2}{a^2}}\right)\cdot\frac{\frac{1+a}{a}}{\frac{1}{a}}=\left(\frac{\frac{1}{1}}{\frac{1+a}{a}}+\frac{\frac{1}{1}}{\frac{1-a^2}{a^2}}\right)\cdot\frac{\frac{1+a}{a}}{\frac{1}{a}}=\left(\frac{\frac{1}{1}}{\frac{1+a}{a}}+\frac{\frac{1}{1}}{\frac{1-a^2}{a^2}}\right)\cdot\frac{\frac{1+a}{1}}{\frac{1}{1}}=$$

$$=\left(\frac{a}{1+a}+\frac{a^2}{1-a^2}\right)\cdot\frac{1+a}{1}=\left(\frac{a}{1+a}+\frac{a^2}{(1-a)\cdot(1+a)}\right)\cdot\frac{1+a}{1}=\frac{a\cdot(1-a)+a^2}{(1-a)\cdot(1+a)}\cdot\frac{1+a}{1}=$$

$$=\frac{a-a^2+a^2}{(1-a)\cdot(1+a)}\cdot\frac{1+a}{1}=\frac{a-a^2+a^2}{(1-a)\cdot(1+a)}\cdot\frac{1+a}{1}=\frac{a}{(1-a)\cdot(1+a)}\cdot\frac{1+a}{1}=\frac{a}{(1-a)\cdot(1+a)}\cdot\frac{1+a}{1}=$$

$$=\frac{a}{1-a}\cdot\frac{1}{1}=\frac{a}{1-a}.$$

2.inačica

$$\left(\frac{1}{a^{-1}+1}+\frac{1}{a^{-2}-1}\right)\cdot\frac{a^{-1}+1}{a^{-1}}=\left(\frac{1}{a^{-1}+1}+\frac{1}{\left(a^{-1}\right)^2-1}\right)\cdot\frac{a^{-1}+1}{a^{-1}}=$$

$$=\left(\frac{1}{a^{-1}+1}+\frac{1}{\left(a^{-1}-1\right)\cdot\left(a^{-1}+1\right)}\right)\cdot\frac{a^{-1}+1}{a^{-1}}=\frac{a^{-1}-1+1}{\left(a^{-1}-1\right)\cdot\left(a^{-1}+1\right)}\cdot\frac{a^{-1}+1}{a^{-1}}=$$

$$=\frac{a^{-1}-1+1}{\left(a^{-1}-1\right)\cdot\left(a^{-1}+1\right)}\cdot\frac{a^{-1}+1}{a^{-1}}=\frac{a^{-1}}{\left(a^{-1}-1\right)\cdot\left(a^{-1}+1\right)}\cdot\frac{a^{-1}+1}{a^{-1}}=\frac{a^{-1}}{\left(a^{-1}-1\right)\cdot\left(a^{-1}+1\right)}\cdot\frac{a^{-1}+1}{a^{-1}}=$$

$$=\frac{1}{a^{-1}-1}\cdot\frac{1}{1}=\frac{1}{a^{-1}-1}=\frac{1}{\frac{1}{a}-1}=\frac{1}{\frac{1}{a}-\frac{1}{1}}=\frac{1}{\frac{1-a}{a}}=\frac{\frac{1}{1}}{\frac{1-a}{a}}=\frac{a}{1-a}.$$

Vježba 571

Pojednostavni: $\left(\frac{1}{1+a^{-1}}-\frac{1}{1-a^{-2}}\right)\cdot\frac{a^{-1}+1}{a^{-1}}.$

Rezultat: $\frac{a}{1-a}.$

Zadatak 572 (Ivan, gimnazija)

Racionaliziraj nazivnik u razlomku : $\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}.$

Rješenje 572

Ponovimo!

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \sqrt[n\cdot p]{a^{m\cdot p}}=\sqrt[n]{a^m} \quad , \quad \left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}.$$

$$\left(\sqrt{a}\right)^2=a \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}\cdot\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt[4]{3}+\sqrt[4]{2}}=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\left(\sqrt[4]{3}-\sqrt[4]{2}\right)\cdot\left(\sqrt[4]{3}+\sqrt[4]{2}\right)}=$$

$$=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\left(\sqrt[4]{3}\right)^2-\left(\sqrt[4]{2}\right)^2}=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt{3}-\sqrt{2}}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=$$

$$=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{3-2}=$$

$$=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{1}=\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right).$$

Vježba 572

Racionaliziraj nazivnik u razlomku : $\frac{1}{\sqrt[4]{2}-1}.$

Rezultat: $\left(\sqrt[4]{2}+1\right)\cdot\left(\sqrt{2}+1\right).$

Zadatak 573 (Mirella, gimnazija)

Pojednostavni: $\left(\frac{1}{\sqrt{x}+1}+\frac{1}{x-1}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}.$

Rješenje 573

Ponovimo!

$$\left(\sqrt{a}\right)^2=a \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad ,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left(\frac{1}{\sqrt{x}+1}+\frac{1}{x-1}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\left(\frac{1}{\sqrt{x}+1}+\frac{1}{\left(\sqrt{x}\right)^2-1}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\left(\frac{1}{\sqrt{x}+1}+\frac{1}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{\sqrt{x}-1+1}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\frac{\sqrt{x}-1+1}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{\sqrt{x}}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{\sqrt{x}}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\frac{1}{\sqrt{x}-1}\cdot\frac{1}{1}=\frac{1}{\sqrt{x}-1}.$$

2.inačica

$$\left(\frac{1}{\sqrt{x}+1}+\frac{1}{x-1}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{1}{\sqrt{x}+1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}+\frac{1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\frac{1}{\sqrt{x}+1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}+\frac{\sqrt{x}+1}{\sqrt{x}\cdot(x-1)}=\frac{1}{\sqrt{x}}+\frac{\sqrt{x}+1}{\sqrt{x}\cdot\left(\left(\sqrt{x}\right)^2-1\right)}=$$

$$=\frac{1}{\sqrt{x}}+\frac{\sqrt{x}+1}{\sqrt{x}\cdot\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}=\frac{1}{\sqrt{x}}+\frac{\sqrt{x}+1}{\sqrt{x}\cdot\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}=$$

$$=\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{x}\cdot\left(\sqrt{x}-1\right)}=\frac{\sqrt{x}-1+1}{\sqrt{x}\cdot\left(\sqrt{x}-1\right)}=\frac{\sqrt{x}-1+1}{\sqrt{x}\cdot\left(\sqrt{x}-1\right)}=\frac{\sqrt{x}}{\sqrt{x}\cdot\left(\sqrt{x}-1\right)}=\frac{\sqrt{x}}{\sqrt{x}\cdot\left(\sqrt{x}-1\right)}=$$

$$=\frac{1}{\sqrt{x}-1}.$$

Vježba 573

Pojednostavni: $\left(\frac{1}{1+\sqrt{x}}-\frac{1}{1-x}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}.$

Rezultat: $\frac{1}{\sqrt{x}-1}.$

Zadatak 574 (Lorena, srednja škola)

Pojednostavni: $\left(a-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}.$

Rješenje 574

Ponovimo!

$$n=\frac{n}{1}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad a^2-2\cdot a\cdot b+b^2=(a-b)^2\quad,\quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\frac{a}{-b}=\frac{-a}{b}\quad,\quad \frac{-a}{-b}=\frac{a}{b}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left(a-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}=\left(\frac{a}{1}-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}=\frac{4\cdot a-\left(a^2+4\right)}{4}\cdot\frac{8}{4-a^2}=$$

$$=\frac{4\cdot a-a^2-4}{4}\cdot\frac{8}{-\left(a^2-4\right)}=\frac{-a^2+4\cdot a-4}{4}\cdot\frac{-8}{a^2-4}=\frac{-\left(a^2-4\cdot a+4\right)}{4}\cdot\frac{-8}{a^2-4}=$$

$$=\frac{a^2-4\cdot a+4}{4}\cdot\frac{8}{a^2-4}=\frac{a^2-4\cdot a+4}{4}\cdot\frac{8}{a^2-4}=\frac{a^2-4\cdot a+4}{1}\cdot\frac{2}{a^2-4}=$$

$$=\frac{(a-2)^2}{1}\cdot\frac{2}{(a-2)\cdot(a+2)}=\frac{(a-2)^2}{1}\cdot\frac{2}{(a-2)\cdot(a+2)}=\frac{a-2}{1}\cdot\frac{2}{a+2}=\frac{2\cdot(a-2)}{a+2}.$$

2.inačica

$$\left(a-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}=\left(\frac{a}{1}-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}=\frac{a}{1}\cdot\frac{8}{4-a^2}-\frac{a^2+4}{4}\cdot\frac{8}{4-a^2}=$$

$$=\frac{8\cdot a}{4-a^2}-\frac{a^2+4}{4}\cdot\frac{8}{4-a^2}=\frac{8\cdot a}{4-a^2}-\frac{a^2+4}{1}\cdot\frac{2}{4-a^2}=\frac{8\cdot a}{4-a^2}-\frac{2\cdot\left(a^2+4\right)}{4-a^2}=$$

$$=\frac{8\cdot a}{4-a^2}-\frac{2\cdot a^2+8}{4-a^2}=\frac{8\cdot a-\left(2\cdot a^2+8\right)}{4-a^2}=\frac{8\cdot a-2\cdot a^2-8}{4-a^2}=$$

$$=\frac{-2\cdot a^2+8\cdot a-8}{4-a^2}=\frac{-2\cdot\left(a^2-4\cdot a+4\right)}{-\left(a^2-4\right)}=\frac{2\cdot\left(a^2-4\cdot a+4\right)}{a^2-4}=\frac{2\cdot(a-2)^2}{(a-2)\cdot(a+2)}=$$

$$=\frac{2\cdot(a-2)^2}{(a-2)\cdot(a+2)}=\frac{2\cdot(a-2)}{a+2}.$$

Vježba 574

Pojednostavni: $\left(\frac{a^2+4}{4}-a\right)\cdot\frac{8}{a^2-4}.$

Rezultat: $\frac{2\cdot(a-2)}{a+2}.$

Zadatak 575 (Tomislav, gimnazija)

Izračunaj: $(a-1)^2\cdot\left(a^2+1\right)^2\cdot(a+1)^2.$

Rješenje 575

Ponovimo!

$$a^n\cdot b^n=(a\cdot b)^n \quad , \quad (a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left(a^n\right)^m=a^{n\cdot m}.$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$(a-1)^2\cdot\left(a^2+1\right)^2\cdot(a+1)^2=(a-1)^2\cdot(a+1)^2\cdot\left(a^2+1\right)^2=((a-1)\cdot(a+1))^2\cdot\left(a^2+1\right)^2=$$

$$=((a-1)\cdot(a+1))^2\cdot\left(a^2+1\right)^2=\left(a^2-1\right)^2\cdot\left(a^2+1\right)^2=\left(\left(a^2\right)^2-1\right)^2=\left(a^4-1\right)^2=$$

$$=\left(a^4\right)^2-2\cdot a^4+1=a^8-2\cdot a^4+1.$$

Vježba 575

Izračunaj: $(a+1)^2\cdot\left(a^2+1\right)^2\cdot(a-1)^2.$

Rezultat: $a^8-2\cdot a^4+1.$

Zadatak 576 (Tomislav, gimnazija)

Izračunaj: $\left(a^2-1\right)^3\cdot\left(a^2+1\right)^3\cdot\left(a^2+1\right)^3.$

Rješenje 576

Ponovimo!

$$a^n\cdot b^n=(a\cdot b)^n \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2.$$

$$(a-b)^3=a^3-3\cdot a^2\cdot b+3\cdot a\cdot b^2-b^3.$$

$$\left(a^2-1\right)^3\cdot\left(a^2+1\right)^3\cdot\left(a^4+1\right)^3=\left(\left(a^2-1\right)\cdot\left(a^2+1\right)\right)^3\cdot\left(a^4+1\right)^3=\left(\left(a^2\right)^2-1\right)^3\cdot\left(a^4+1\right)^3=$$

$$=\left(a^4-1\right)^3\cdot\left(a^4+1\right)^3=\left(\left(a^4-1\right)\cdot\left(a^4+1\right)\right)^3=\left(\left(a^4\right)^2-1\right)^3=\left(a^8-1\right)^3=$$

$$=\left(a^8\right)^3-3\cdot\left(a^8\right)^2\cdot1+3\cdot a^8\cdot1^2-1^3=a^{24}-3\cdot a^{16}+3\cdot a^8-1.$$

Vježba 576

Izračunaj: $\left(a^2-1\right)^3\cdot\left(a^2+1\right)^3\cdot\left(a^2+1\right)^3.$

Rezultat: $a^{24}-3\cdot a^{16}+3\cdot a^8-1.$

Zadatak 577 (Petra, gimnazija)

Rastavi na faktore: $x^2 - x - 6.$

Rješenje 577

Ponovimo!

$$a^2 - b^2 = (a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

1.inačica

Trinom preoblikujemo tako da srednji član – x rastavimo na dva člana – 3 · x i 2 · x i uporabimo metodu grupiranja.

$$x^2 - x - 6 = x^2 - 3\cdot x + 2\cdot x - 6 = \left(x^2 - 3\cdot x\right) + (2\cdot x - 6) = x\cdot(x-3) + 2\cdot(x-3) =$$

$$= x\cdot(x-3) + 2\cdot(x-3) = (x-3)\cdot(x+2).$$

2.inačica

Trinom preoblikujemo tako da zadnji član – 6 rastavimo na dva člana – 4 i – 2 i uporabimo metodu grupiranja.

$$x^2 - x - 6 = x^2 - x - 4 - 2 = x^2 - 4 - x - 2 = \left(x^2 - 4\right) - (x+2) = (x-2)\cdot(x+2) - (x+2) =$$

$$= (x-2)\cdot(x+2) - (x+2) = (x+2)\cdot(x-2-1) = (x+2)\cdot(x-3).$$

Vježba 577

Rastavi na faktore: $x^2 + x - 6.$

Rezultat: $(x-2)\cdot(x+3).$

Zadatak 578 (Mica, gimnazija)

Izračunaj $\left(\frac{4}{5}\right)^4 \cdot \left(-\frac{5}{4}\right)^3.$

Rješenje 578

Ponovimo!

$$(-a)^{2\cdot n-1} = -a^{2\cdot n-1} \quad , \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad , \quad \left(a^n\right)^m = a^{n\cdot m}.$$

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^1 = a \quad , \quad a^n \cdot b^n = (a\cdot b)^n.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \;,\; n \ne 0 \;,\; n \ne 1.$$

1.inačica

$$\left(\frac{4}{5}\right)^4 \cdot \left(-\frac{5}{4}\right)^3 = \left(\frac{4}{5}\right)^4 \cdot \left(-\left(\frac{5}{4}\right)^3\right) = -\left(\frac{4}{5}\right)^4 \cdot \left(\frac{5}{4}\right)^3 = -\left(\frac{4}{5}\right)^4 \cdot \left(\left(\frac{4}{5}\right)^{-1}\right)^3 = -\left(\frac{4}{5}\right)^4 \cdot \left(\frac{4}{5}\right)^{-3} =$$

$$= -\left(\frac{4}{5}\right)^1 = -\frac{4}{5}.$$

2.inačica

$$\left(\frac{4}{5}\right)^4\cdot\left(-\frac{5}{4}\right)^3=\left(\frac{4}{5}\right)^4\cdot\left(-\left(\frac{5}{4}\right)^3\right)=-\left(\frac{4}{5}\right)^4\cdot\left(\frac{5}{4}\right)^3=-\left(\frac{4}{5}\right)^1\cdot\left(\frac{4}{5}\right)^3\cdot\left(\frac{5}{4}\right)^3=$$

$$=-\frac{4}{5}\cdot\left(\frac{4}{5}\cdot\frac{5}{4}\right)^3=-\frac{4}{5}\cdot\left(\frac{4}{5}\cdot\frac{5}{4}\right)^3=-\frac{4}{5}\cdot 1^3=-\frac{4}{5}\cdot 1=-\frac{4}{5}.$$

Vježba 578

Izračunaj $\left(\frac{4}{5}\right)^4\cdot\left(\frac{5}{4}\right)^3$.

Rezultat: $\frac{4}{5}$.

Zadatak 579 (Mica, gimnazija)

Izračunaj $\left(\frac{2}{3}\right)^{11}\cdot\left(\frac{3}{2}\right)^{12}$.

Rješenje 579

Ponovimo!

$$\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$a^n\cdot b^n=(a\cdot b)^n.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\left(\frac{2}{3}\right)^{11}\cdot\left(\frac{3}{2}\right)^{12}=\left(\frac{2}{3}\right)^{11}\cdot\left(\left(\frac{2}{3}\right)^{-1}\right)^{11}=\left(\frac{2}{3}\right)^{11}\cdot\left(\frac{2}{3}\right)^{-12}=\left(\frac{2}{3}\right)^{-1}=\left(\frac{3}{2}\right)^1=\frac{3}{2}.$$

2.inačica

$$\left(\frac{2}{3}\right)^{11}\cdot\left(\frac{3}{2}\right)^{12}=\left(\frac{2}{3}\right)^{11}\cdot\left(\frac{3}{2}\right)^{11}\cdot\left(\frac{3}{2}\right)^{1}=\left(\frac{2}{3}\cdot\frac{3}{2}\right)^{11}\cdot\frac{3}{2}=\left(\frac{2}{3}\cdot\frac{3}{2}\right)^{11}\cdot\frac{3}{2}=1^{11}\cdot\frac{3}{2}=1\cdot\frac{3}{2}=\frac{3}{2}.$$

Vježba 579

Izračunaj $\left(\frac{3}{2}\right)^{12}\cdot\left(\frac{2}{3}\right)^{11}$.

Rezultat: $\frac{3}{2}$.

Zadatak 580 (Mica, gimnazija)

Nađi n, ako je $3^n+3^n+3^n=3^4$.

Rješenje 580

Ponovimo!

$$\underbrace{a+a+a+\ldots+a}_{\text{n - pribrojnika}}=n\cdot a \quad , \quad a^{1}=a \quad , \quad a^{n}\cdot a^{m}=a^{n+m}.$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$3^{n}+3^{n}+3^{n}=3^{4} \Rightarrow 3\cdot 3^{n}=3^{4} \Rightarrow 3^{1}\cdot 3^{n}=3^{4} \Rightarrow 3^{1+n}=3^{4} \Rightarrow$$

$$\Rightarrow 1+n=4 \Rightarrow n=4-1 \Rightarrow n=3.$$

Vježba 580

Nađi n, ako je $2^{n}+2^{n}+2^{n}+2^{n}=2^{5}$.

Rezultat: n = 3.