Zadatak 581 (Mica, gimnazija)

Pomnoži razlomke $\left(a-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}.$

Rješenje 581

Ponovimo!

$$n=\frac{n}{1}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad a^2-2\cdot a\cdot b+b^2=(a-b)^2.$$

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left(a-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}=\left(\frac{a}{1}-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}=\frac{4\cdot a-\left(a^2+4\right)}{4}\cdot\frac{8}{4-a^2}=$$

$$=\frac{4\cdot a-a^2-4}{4}\cdot\frac{8}{4-a^2}=\frac{-\left(a^2-4\cdot a+4\right)}{4}\cdot\frac{8}{-\left(a^2-4\right)}=\frac{a^2-4\cdot a+4}{4}\cdot\frac{8}{a^2-4}=$$

$$=\frac{(a-2)^2}{4}\cdot\frac{8}{(a-2)\cdot(a+2)}=\frac{(a-2)^2}{4}\cdot\frac{8}{(a-2)\cdot(a+2)}=\frac{a-2}{1}\cdot\frac{2}{a+2}=\frac{2\cdot(a-2)}{a+2}.$$

Vježba 581

Pomnoži razlomke $\left(\frac{a^2+4}{4}-a\right)\cdot\frac{8}{a^2-4}.$

Rezultat: $\frac{2\cdot(a-2)}{a+2}.$

Zadatak 582 (Hrvoje, gimnazija)

Pojednostavnite: $\frac{(x+y+z)^{2\cdot x-3\cdot y}}{(x+y+z)^{7\cdot x-3\cdot y+1}}\cdot\frac{(x+y+z)^3}{(x+y+z)^{5\cdot y+x}}.$

Rješenje 582

Ponovimo!

$$\frac{a^n}{a^m}=a^{n-m}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Uporabom pravila za dijeljenje i množenje potencija jednakih baza, imamo:

$$\frac{(x+y+z)^{2\cdot x-3\cdot y}}{(x+y+z)^{7\cdot x-3\cdot y+1}}\cdot\frac{(x+y+z)^{3}}{(x+y+z)^{5\cdot y+x}}=$$

$$=(x+y+z)^{2\cdot x-3\cdot y-(7\cdot x-3\cdot y+1)}\cdot(x+y+z)^{3-(5\cdot y+x)}=$$

$$=(x+y+z)^{2\cdot x-3\cdot y-7\cdot x+3\cdot y-1}\cdot(x+y+z)^{3-5\cdot y-x}=$$

$$=(x+y+z)^{2\cdot x-3\cdot y-7\cdot x+3\cdot y-1}\cdot(x+y+z)^{3-5\cdot y-x}=$$

$$=(x+y+z)^{2\cdot x-7\cdot x-1}\cdot(x+y+z)^{3-5\cdot y-x}=(x+y+z)^{-5\cdot x-1}\cdot(x+y+z)^{3-5\cdot y-x}=$$

$$=(x+y+z)^{-5\cdot x-1+3-5\cdot y-x}=(x+y+z)^{-6\cdot x-5\cdot y+2}.$$

2.inačica

Uporabom pravila za množenje i dijeljenje potencija jednakih baza, imamo:

$$\frac{(x+y+z)^{2\cdot x-3\cdot y}}{(x+y+z)^{7\cdot x-3\cdot y+1}}\cdot\frac{(x+y+z)^{3}}{(x+y+z)^{5\cdot y+x}}=$$

$$=\frac{(x+y+z)^{2\cdot x-3\cdot y}\cdot(x+y+z)^{3}}{(x+y+z)^{7\cdot x-3\cdot y+1}\cdot(x+y+z)^{5\cdot y+x}}=\frac{(x+y+z)^{2\cdot x-3\cdot y+3}}{(x+y+z)^{7\cdot x-3\cdot y+1+5\cdot y+x}}=$$

$$=\frac{(x+y+z)^{2\cdot x-3\cdot y+3}}{(x+y+z)^{8\cdot x+2\cdot y+1}}=(x+y+z)^{2\cdot x-3\cdot y+3-(8\cdot x+2\cdot y+1)}=$$

$$=(x+y+z)^{2\cdot x-3\cdot y+3-8\cdot x-2\cdot y-1}=(x+y+z)^{-6\cdot x-5\cdot y+2}.$$

Vježba 582

Pojednostavnite: $\frac{(x+y+z)^{3}}{(x+y+z)^{5\cdot y+x}}\cdot\frac{(x+y+z)^{2\cdot x-3\cdot y}}{(x+y+z)^{7\cdot x-3\cdot y+1}}.$

Rezultat: $(x+y+z)^{-6\cdot x-5\cdot y+2}.$

Zadatak 583 (Anita, gimnazija)

Pomnoži razlomke: $\left(\frac{a+b}{a^{2}-a\cdot b}-\frac{a+b}{a^{2}-b^{2}}\right)\cdot\frac{a^{2}-2\cdot a\cdot b+b^{2}}{4\cdot a\cdot b}.$

Rješenje 583

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$x^{2}-y^{2}=(x-y)\cdot(x+y)\quad,\quad x^{2}-2\cdot x\cdot y+y^{2}=(x-y)^{2}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

$$\left(\frac{a+b}{a^2 - a \cdot b} - \frac{a+b}{a^2 - b^2}\right) \cdot \frac{a^2 - 2 \cdot a \cdot b + b^2}{4 \cdot a \cdot b} = \left(\frac{a+b}{a \cdot (a-b)} - \frac{a+b}{(a-b) \cdot (a+b)}\right) \cdot \frac{(a-b)^2}{4 \cdot a \cdot b} =$$

$$= \left(\frac{a+b}{a \cdot (a-b)} - \frac{a+b}{(a-b) \cdot (a+b)}\right) \cdot \frac{(a-b)^2}{4 \cdot a \cdot b} = \left(\frac{a+b}{a \cdot (a-b)} - \frac{1}{a-b}\right) \cdot \frac{(a-b)^2}{4 \cdot a \cdot b} =$$

$$= \frac{a+b-a}{a \cdot (a-b)} \cdot \frac{(a-b)^2}{4 \cdot a \cdot b} = \frac{a+b-a}{a \cdot (a-b)} \cdot \frac{(a-b)^2}{4 \cdot a \cdot b} = \frac{b}{a \cdot (a-b)} \cdot \frac{(a-b)^2}{4 \cdot a \cdot b} = \frac{b}{a \cdot (a-b)} \cdot \frac{(a-b)^2}{4 \cdot a \cdot b} =$$

$$= \frac{1}{a} \cdot \frac{a-b}{4 \cdot a} = \frac{a-b}{4 \cdot a^2}.$$

Vježba 583

Pomnoži razlomke: $\left(\frac{a+b}{a^2 - a \cdot b} - \frac{a+b}{a^2 - b^2}\right) \cdot \frac{a^2 - 2 \cdot a \cdot b + b^2}{b}.$

Rezultat: $\frac{a-b}{a}.$

Zadatak 584 (Dubravko, srednja škola)

Skrati razlomak: $\frac{a^3 \cdot b - a \cdot b^3}{a^3 \cdot b - 2 \cdot a^2 \cdot b^2 + a \cdot b^3}, \; a \neq 0, \; b \neq 0, \; a \neq b.$

Rješenje 584

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

$$a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$x^2 - y^2 = (x-y) \cdot (x+y) \quad , \quad x^2 - 2 \cdot x \cdot y + y^2 = (x-y)^2 \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \neq 0 \; , \; n \neq 1.$$

Brojnik i nazivnik razlomka rastavimo na faktore.

$$\frac{a^3 \cdot b - a \cdot b^3}{a^3 \cdot b - 2 \cdot a^2 \cdot b^2 + a \cdot b^3} = \frac{a \cdot b \cdot \left(a^2 - b^2\right)}{a \cdot b \cdot \left(a^2 - 2 \cdot a \cdot b + b^2\right)} = \frac{a \cdot b \cdot \left(a^2 - b^2\right)}{a \cdot b \cdot \left(a^2 - 2 \cdot a \cdot b + b^2\right)} =$$

$$= \frac{a^2 - b^2}{a^2 - 2 \cdot a \cdot b + b^2} = \frac{(a-b) \cdot (a+b)}{(a-b)^2} = \frac{(a-b) \cdot (a+b)}{(a-b)^2} = \frac{a+b}{a-b}.$$

Vježba 584

Skrati razlomak: $\frac{a^3\cdot b-2\cdot a^2\cdot b^2+a\cdot b^3}{a^3\cdot b-a\cdot b^3}$, $a\neq 0,\ b\neq 0,\ a\neq b.$

Rezultat: $\frac{a-b}{a+b}.$

Zadatak 585 (Dubravko, srednja škola)

Pomnoži razlomke $\frac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\frac{1+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}}{\frac{c+b-a}{a\cdot b\cdot c}}.$

Rješenje 585

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad n=\frac{n}{1}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad a^2+2\cdot a\cdot b+b^2=(a+b)^2\quad,\quad a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b},\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\frac{1+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}}{\frac{c+b-a}{a\cdot b\cdot c}}=\frac{\frac{b+c-a}{a\cdot(b+c)}}{\frac{b+c+a}{a\cdot(b+c)}}\cdot\frac{\frac{1}{1}+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}}{\frac{c+b-a}{a\cdot b\cdot c}}=\frac{\frac{b+c-a}{a\cdot(b+c)}}{\frac{b+c+a}{a\cdot(b+c)}}\cdot\frac{\frac{2\cdot b\cdot c+b^2+c^2-a^2}{2\cdot b\cdot c}}{\frac{c+b-a}{a\cdot b\cdot c}}=$$

$$=\frac{\frac{b+c-a}{1}}{\frac{b+c+a}{1}}\cdot\frac{\frac{2\cdot b\cdot c+b^2+c^2-a^2}{2\cdot b\cdot c}}{\frac{c+b-a}{a\cdot b\cdot c}}=\frac{b+c-a}{b+c+a}\cdot\frac{\frac{2\cdot b\cdot c+b^2+c^2-a^2}{2}}{\frac{c+b-a}{a}}=$$

$$=\frac{b+c-a}{b+c+a}\cdot\frac{\frac{b^2+2\cdot b\cdot c+c^2-a^2}{2}}{\frac{c+b-a}{a}}=\frac{b+c-a}{b+c+a}\cdot\frac{\frac{\left(b^2+2\cdot b\cdot c+c^2\right)-a^2}{2}}{\frac{c+b-a}{a}}=$$

$$=\frac{b+c-a}{b+c+a}\cdot\frac{\frac{(b+c)^2-a^2}{2}}{\frac{c+b-a}{a}}=\frac{b+c-a}{b+c+a}\cdot\frac{\frac{(b+c-a)\cdot(b+c+a)}{2}}{\frac{c+b-a}{a}}=\frac{b+c-a}{b+c+a}\cdot\frac{\frac{(b+c-a)\cdot(b+c+a)}{2}}{\frac{c+b-a}{a}}=$$

$$=\frac{b+c-a}{b+c+a}\cdot\frac{\frac{b+c+a}{2}}{\frac{1}{a}}=\frac{b+c-a}{b+c+a}\cdot\frac{a\cdot(b+c+a)}{2}=\frac{b+c-a}{b+c+a}\cdot\frac{a\cdot(b+c+a)}{2}=\frac{a\cdot(b+c-a)}{2}.$$

Vježba 585

Pomnoži razlomke $\frac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\frac{1-\frac{a^2-b^2-c^2}{2\cdot b\cdot c}}{\frac{c+b-a}{a\cdot b\cdot c}}$.

Rezultat: $\frac{a\cdot(b+c-a)}{2}$.

Zadatak 586 (Matea, gimnazija)

Racionaliziraj nazivnik $\frac{1}{2+\sqrt{5}+2\cdot\sqrt{2}+\sqrt{10}}$.

Rješenje 586

Ponovimo!

$$\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}\quad,\quad(a-b)\cdot(a+b)=a^2-b^2\quad,\quad\left(\sqrt{a}\right)^2=a.$$

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Najprije nazivnik rastavimo na faktore, a onda ga racionaliziramo.

$$\frac{1}{2+\sqrt{5}+2\cdot\sqrt{2}+\sqrt{10}}=\frac{1}{2+\sqrt{5}+2\cdot\sqrt{2}+\sqrt{2\cdot 5}}=\frac{1}{2+\sqrt{5}+2\cdot\sqrt{2}+\sqrt{2}\cdot\sqrt{5}}=$$

$$=\frac{1}{2+2\cdot\sqrt{2}+\sqrt{5}+\sqrt{2}\cdot\sqrt{5}}=\frac{1}{\left(2+2\cdot\sqrt{2}\right)+\left(\sqrt{5}+\sqrt{2}\cdot\sqrt{5}\right)}=$$

$$=\frac{1}{2\cdot\left(1+\sqrt{2}\right)+\sqrt{5}\cdot\left(1+\sqrt{2}\right)}=\frac{1}{2\cdot\left(1+\sqrt{2}\right)+\sqrt{5}\cdot\left(1+\sqrt{2}\right)}=\frac{1}{\left(1+\sqrt{2}\right)\cdot\left(2+\sqrt{5}\right)}=$$

$$=\frac{1}{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{5}+2\right)}=\frac{1}{\sqrt{2}+1}\cdot\frac{1}{\sqrt{5}+2}=\left[\begin{array}{l}\text{racionalizacija}\\\text{nazivnika}\end{array}\right]=$$

$$=\frac{1}{\sqrt{2}+1}\cdot\frac{\sqrt{2}-1}{\sqrt{2}-1}\cdot\frac{1}{\sqrt{5}+2}\cdot\frac{\sqrt{5}-2}{\sqrt{5}-2}=\frac{\sqrt{2}-1}{\left(\sqrt{2}\right)^2-1^2}\cdot\frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-2^2}=\frac{\sqrt{2}-1}{2-1}\cdot\frac{\sqrt{5}-2}{5-4}=$$

$$=\frac{\sqrt{2}-1}{1}\cdot\frac{\sqrt{5}-2}{1}=\left(\sqrt{2}-1\right)\cdot\left(\sqrt{5}-2\right).$$

Vježba 586

Racionaliziraj nazivnik $\frac{1}{\sqrt{10}+\sqrt{5}+2+2\cdot\sqrt{2}}$.

Rezultat: $\left(\sqrt{2}-1\right)\cdot\left(\sqrt{5}-2\right)$.

Zadatak 587 (Josip, tehnička škola)

Pojednostavnite izraz: $\frac{2^{3\cdot x}-2^{x}\cdot 9^{x}}{2^{x}+3^{x}}$.

Rješenje 587

Ponovimo!

$$a^{n}\cdot a^{m}=a^{n+m} \quad , \quad \left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{2}-b^{2}=(a-b)\cdot(a+b).$$

$$n=\frac{n}{1} \quad , \quad a^{n}\cdot b^{n}=(a\cdot b)^{n}$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Najprije brojnik rastavimo na faktore, a onda skratimo razlomak.

$$\frac{2^{3\cdot x}-2^{x}\cdot 9^{x}}{2^{x}+3^{x}}=\frac{2^{x}\cdot 2^{2\cdot x}-2^{x}\cdot 9^{x}}{2^{x}+3^{x}}=\frac{2^{x}\cdot 2^{2\cdot x}-2^{x}\cdot 9^{x}}{2^{x}+3^{x}}=\frac{2^{x}\cdot\left(2^{2\cdot x}-9^{x}\right)}{2^{x}+3^{x}}=$$

$$=\frac{2^{x}\cdot\left(\left(2^{x}\right)^{2}-\left(3^{2}\right)^{x}\right)}{2^{x}+3^{x}}=\frac{2^{x}\cdot\left(\left(2^{x}\right)^{2}-\left(3^{x}\right)^{2}\right)}{2^{x}+3^{x}}=\frac{2^{x}\cdot\left(2^{x}-3^{x}\right)\cdot\left(2^{x}+3^{x}\right)}{2^{x}+3^{x}}=$$

$$=\frac{2^{x}\cdot\left(2^{x}-3^{x}\right)\cdot\left(2^{x}+3^{x}\right)}{2^{x}+3^{x}}=\frac{2^{x}\cdot\left(2^{x}-3^{x}\right)}{1}=2^{x}\cdot\left(2^{x}-3^{x}\right)=$$

$$=2^{x}\cdot 2^{x}-2^{x}\cdot 3^{x}=(2\cdot 2)^{x}-(2\cdot 3)^{x}=4^{x}-6^{x}.$$

Vježba 587

Pojednostavnite izraz: $\frac{2^{3\cdot x}-2^{x}\cdot 9^{x}}{2^{x}-3^{x}}$.

Rezultat: $4^{x}+6^{x}$.

Zadatak 588 (Ema, srednja škola)

Izračunajte vrijednost izraza $(a-b)\cdot\sqrt{\frac{b+a}{b-a}+\frac{a-b}{b-a}}$ za $a=\sqrt{3}$, $b=2$.

Rješenje 588

Ponovimo!

$$a\cdot\sqrt{b}=\sqrt{a^{2}\cdot b} \quad , \quad (a-b)^{2}=(b-a)^{2} \quad , \quad a^{1}=a \quad , \quad a^{n}:a^{m}=a^{n-m}.$$

$$\left(\sqrt{a}\right)^2 = a \quad , \quad (a-b)\cdot(a+b) = a^2 - b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$(a-b)\cdot\sqrt{\frac{b+a}{b-a}} + \frac{a-b}{b-a} = \sqrt{(a-b)^2\cdot\frac{b+a}{b-a}} + \frac{-(b-a)}{b-a} = \sqrt{(b-a)^2\cdot\frac{b+a}{b-a}} - \frac{b-a}{b-a} =$$

$$= \sqrt{(b-a)^2\cdot\frac{b+a}{b-a}} - \frac{b-a}{b-a} = \sqrt{(b-a)\cdot(b+a)} - 1 = \sqrt{b^2 - a^2} - 1 = \left[\begin{matrix} a = \sqrt{3} \\ b = 2 \end{matrix}\right] =$$

$$= \sqrt{2^2 - \left(\sqrt{3}\right)^2} - 1 = \sqrt{4-3} - 1 = \sqrt{1} - 1 = 1 - 1 = 0.$$

Vježba 588

Izračunajte vrijednost izraza $(a-b)\cdot\sqrt{\frac{b+a}{b-a}} + \frac{a-b}{b-a}$ za $a = 2, \ b = \sqrt{5}$.

Rezultat: 0.

Zadatak 589 (Valerija, gimnazija)

Pojednostavnite $\left(\frac{x-1}{x+1} - 1\right) : \left(\frac{x+1}{x-1} - 1\right)$.

Rješenje 589

Ponovimo!

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a\cdot d - b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b} : \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} = \frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\left(\frac{x-1}{x+1} - 1\right) : \left(\frac{x+1}{x-1} - 1\right) = \left(\frac{x-1}{x+1} - \frac{1}{1}\right) : \left(\frac{x+1}{x-1} - \frac{1}{1}\right) = \frac{x-1-(x+1)}{x+1} : \frac{x+1-(x-1)}{x-1} =$$

$$= \frac{x-1-x-1}{x+1} : \frac{x+1-x+1}{x-1} = \frac{x-1-x-1}{x+1} : \frac{x+1-x+1}{x-1} = \frac{-2}{x+1} : \frac{2}{x-1} =$$

$$= \frac{-2}{x+1}\cdot\frac{x-1}{2} = \frac{-2}{x+1}\cdot\frac{x-1}{2} = \frac{-1}{x+1}\cdot\frac{x-1}{1} = \frac{-1\cdot(x-1)}{x+1} = \frac{-x+1}{x+1} = \frac{1-x}{1+x}.$$

Vježba 589

Pojednostavnite $\left(1 - \frac{1-x}{x+1}\right) : \left(\frac{x+1}{x-1} - 1\right)$.

Rezultat: $\frac{1-x}{1+x}.$

Zadatak 590 (Branko, srednja škola)

Pojednostavnite $\left(\left(1-\frac{a-4\cdot b}{a+b}\right):\left(4-\frac{4\cdot a+b}{a-b}\right)\right):\left(a^2-b^2\right).$

Rješenje 590

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left(\left(1-\frac{a-4\cdot b}{a+b}\right):\left(4-\frac{4\cdot a+b}{a-b}\right)\right):\left(a^2-b^2\right)=\left(\left(\frac{1}{1}-\frac{a-4\cdot b}{a+b}\right):\left(\frac{4}{1}-\frac{4\cdot a+b}{a-b}\right)\right):\frac{a^2-b^2}{1}=$$

$$=\left(\frac{a+b-(a-4\cdot b)}{a+b}:\frac{4\cdot(a-b)-(4\cdot a+b)}{a-b}\right):\frac{a^2-b^2}{1}=$$

$$=\left(\frac{a+b-a+4\cdot b}{a+b}:\frac{4\cdot a-4\cdot b-4\cdot a-b}{a-b}\right):\frac{a^2-b^2}{1}=$$

$$=\left(\frac{a+b-a+4\cdot b}{a+b}:\frac{4\cdot a-4\cdot b-4\cdot a-b}{a-b}\right):\frac{a^2-b^2}{1}=\left(\frac{b+4\cdot b}{a+b}:\frac{-4\cdot b-b}{a-b}\right):\frac{a^2-b^2}{1}=$$

$$=\left(\frac{5\cdot b}{a+b}:\frac{-5\cdot b}{a-b}\right):\frac{a^2-b^2}{1}=\frac{5\cdot b}{a+b}\cdot\frac{a-b}{-5\cdot b}\cdot\frac{1}{a^2-b^2}=\frac{5\cdot b}{a+b}\cdot\frac{a-b}{-5\cdot b}\cdot\frac{1}{(a-b)\cdot(a+b)}=$$

Vježba 590

Pojednostavnite $\left(\left(1+\frac{4\cdot b-a}{a+b}\right):\left(4+\frac{4\cdot a+b}{b-a}\right)\right):\left(a^2-b^2\right).$

Rezultat: $\frac{-1}{(a+b)^2}.$

Zadatak 591 (Branko, srednja škola)

Pojednostavnite $\frac{1+x}{1-x}-\frac{1-x}{1+x}+\frac{1+x^2}{1-x^2}.$

Rješenje 591

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{1+x}{1-x}-\frac{1-x}{1+x}+\frac{1+x^2}{1-x^2}=\frac{1+x}{1-x}-\frac{1-x}{1+x}+\frac{1+x^2}{(1-x)\cdot(1+x)}=\frac{(1+x)^2-(1-x)^2+1+x^2}{(1-x)\cdot(1+x)}=$$

$$=\frac{1+2\cdot x+x^2-\left(1-2\cdot x+x^2\right)+1+x^2}{(1-x)\cdot(1+x)}=\frac{1+2\cdot x+x^2-1+2\cdot x-x^2+1+x^2}{(1-x)\cdot(1+x)}=$$

$$=\frac{1+2\cdot x+x^2-1+2\cdot x-x^2+1+x^2}{(1-x)\cdot(1+x)}=\frac{2\cdot x+2\cdot x+1+x^2}{(1-x)\cdot(1+x)}=\frac{1+4\cdot x+x^2}{(1-x)\cdot(1+x)}=\frac{1+4\cdot x+x^2}{1-x^2}.$$

Vježba 591

Pojednostavnite $\frac{1+x}{1-x}+\frac{x-1}{1+x}+\frac{1+x^2}{1-x^2}$.

Rezultat: $\frac{1+4\cdot x+x^2}{1-x^2}$.

Zadatak 592 (Dominik, ekonomska škola)

Pomnožite $\left(\frac{2}{a-b}-\frac{1}{a}\right)\cdot\frac{a^3+a^2\cdot b-a\cdot b^2-b^3}{a^2+2\cdot a\cdot b+b^2}$.

Rješenje 592

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$a^1=a \quad , \quad a^n:a^m=a^{n-m} \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Najprije uočimo da se izraz u brojniku $a^3+a^2\cdot b-a\cdot b^2-b^3$ može rastaviti na faktore na više načina pomoću metode grupiranja i zakona distribucije množenja prema zbrajanju.

1.inačica

$$a^3+a^2\cdot b-a\cdot b^2-b^3=\left(a^3+a^2\cdot b\right)+\left(-a\cdot b^2-b^3\right)=a^2\cdot(a+b)-b^2\cdot(a+b)=$$

$$=a^2\cdot(a+b)-b^2\cdot(a+b)=(a+b)\cdot\left(a^2-b^2\right)=(a+b)\cdot(a-b)\cdot(a+b)=(a+b)^2\cdot(a-b).$$

2.inačica

$$a^3+a^2\cdot b-a\cdot b^2-b^3=\left(a^3-a\cdot b^2\right)+\left(a^2\cdot b-b^3\right)=a\cdot\left(a^2-b^2\right)+b\cdot\left(a^2-b^2\right)=$$

$$=a\cdot\left(a^2-b^2\right)+b\cdot\left(a^2-b^2\right)=\left(a^2-b^2\right)\cdot(a+b)=(a-b)\cdot(a+b)\cdot(a+b)=(a-b)\cdot(a+b)^2.$$

3.inačica

$$a^3+a^2\cdot b-a\cdot b^2-b^3=\left(a^3-b^3\right)+\left(a^2\cdot b-a\cdot b^2\right)=$$

$$=(a-b)\cdot\left(a^2+a\cdot b+b^2\right)+a\cdot b\cdot(a-b)=(a-b)\cdot\left(a^2+a\cdot b+b^2\right)+a\cdot b\cdot(a-b)=$$

$$=(a-b)\cdot\left(a^2+a\cdot b+b^2+a\cdot b\right)=(a-b)\cdot\left(a^2+2\cdot a\cdot b+b^2\right)=(a-b)\cdot(a+b)^2.$$

Sada množimo razlomke.

$$\left(\frac{2}{a-b}-\frac{1}{a}\right)\cdot\frac{a^3+a^2\cdot b-a\cdot b^2-b^3}{a^2+2\cdot a\cdot b+b^2}=\frac{2\cdot a-(a-b)}{a\cdot(a-b)}\cdot\frac{a^3+a^2\cdot b-a\cdot b^2-b^3}{a^2+2\cdot a\cdot b+b^2}=$$

$$=\left[a^3+a^2\cdot b-a\cdot b^2-b^3=(a-b)\cdot(a+b)^2\right]=\frac{2\cdot a-a+b}{a\cdot(a-b)}\cdot\frac{(a-b)\cdot(a+b)^2}{(a+b)^2}=$$

$$=\frac{a+b}{a\cdot(a-b)}\cdot\frac{(a-b)\cdot(a+b)^2}{(a+b)^2}=\frac{a+b}{a\cdot(a-b)}\cdot\frac{(a-b)\cdot(a+b)^2}{(a+b)^2}=\frac{a+b}{a}\cdot\frac{1}{1}=\frac{a+b}{a}.$$

Vježba 592

Pomnožite $\left(\frac{-2}{b-a}-\frac{1}{a}\right)\cdot\frac{a^3+a^2\cdot b-a\cdot b^2-b^3}{a^2+2\cdot a\cdot b+b^2}$.

Rezultat: $\frac{a+b}{a}$.

Zadatak 593 (Vlado, gimnazija)

Pojednostavnite $\left[\left(\frac{\sqrt{a}}{\sqrt{b}}-1\right)^2+\frac{\sqrt{a}}{\sqrt{b}}\right]:\frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}$.

Rješenje 593

Ponovimo!

$$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}\quad,\quad\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}\quad,\quad(a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad\left(\sqrt{a}\right)^2=a.$$

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right)\quad,\quad n=\frac{n}{1}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}.$$

$$\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\left[\left(\frac{\sqrt{a}}{\sqrt{b}}-1\right)^2+\frac{\sqrt{a}}{\sqrt{b}}\right]:\frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\left[\left(\sqrt{\frac{a}{b}}-1\right)^2+\sqrt{\frac{a}{b}}\right]\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}}=$$

$$=\left[\left(\sqrt{\frac{a}{b}}\right)^2-2\cdot\sqrt{\frac{a}{b}}+1+\sqrt{\frac{a}{b}}\right]\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}}=\left[\left(\sqrt{\frac{a}{b}}\right)^2-\sqrt{\frac{a}{b}}+1\right]\cdot\left(\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{b}}\right)=$$

$$=\left[\left(\sqrt{\frac{a}{b}}\right)^2-\sqrt{\frac{a}{b}}+1\right]\cdot\left(\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{b}}\right)=\left[\left(\sqrt{\frac{a}{b}}\right)^2-\sqrt{\frac{a}{b}}+1\right]\cdot\left(\frac{\sqrt{a}}{\sqrt{b}}+1\right)=$$

$$=\left[\left(\sqrt{\frac{a}{b}}\right)^2-\sqrt{\frac{a}{b}}+1\right]\cdot\left(\sqrt{\frac{a}{b}}+1\right)=\left(\sqrt{\frac{a}{b}}\right)^3+1=\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^3+1=\frac{\left(\sqrt{a}\right)^3}{\left(\sqrt{b}\right)^3}+1=$$

$$=\frac{\left(\sqrt{a}\right)^3}{\left(\sqrt{b}\right)^3}+\frac{1}{1}=\frac{\left(\sqrt{a}\right)^3+\left(\sqrt{b}\right)^3}{\left(\sqrt{a}\right)^3}.$$

2.inačica

$$\left[\left(\frac{\sqrt{a}}{\sqrt{b}}-1\right)^2+\frac{\sqrt{a}}{\sqrt{b}}\right]:\frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\left[\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2-2\cdot\frac{\sqrt{a}}{\sqrt{b}}+1+\frac{\sqrt{a}}{\sqrt{b}}\right]\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}}=$$

$$=\left[\left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2-\frac{\sqrt{a}}{\sqrt{b}}+1\right]\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}}=\left[\frac{\left(\sqrt{a}\right)^2}{\left(\sqrt{b}\right)^2}-\frac{\sqrt{a}}{\sqrt{b}}+1\right]\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}}=$$

$$=\left[\frac{\left(\sqrt{a}\right)^2}{\left(\sqrt{b}\right)^2}-\frac{\sqrt{a}}{\sqrt{b}}+\frac{1}{1}\right]\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}}=\frac{\left(\sqrt{a}\right)^2-\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2}{\left(\sqrt{b}\right)^2}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{b}}=$$

$$=\frac{\left(\left(\sqrt{a}\right)^2-\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{b}\right)^3}=\frac{\left(\sqrt{a}\right)^3+\left(\sqrt{b}\right)^3}{\left(\sqrt{b}\right)^3}.$$

Vježba 593

Pojednostavnite $\left[\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)^2+\frac{\sqrt{a}}{\sqrt{b}}\right]:\frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}.$

Rezultat: $\frac{\left(\sqrt{a}\right)^3+\left(\sqrt{b}\right)^3}{\left(\sqrt{b}\right)^3}.$

Zadatak 594 (Martin, srednja škola)

Pojednostavnite $\dfrac{\frac{1}{x}+\frac{1}{y}-\frac{a}{x\cdot y}}{\frac{1}{x^2}+\frac{1}{y^2}+\frac{2}{x\cdot y}-\frac{a^2}{x^2\cdot y^2}}\cdot(a+x+y)$.

A. a B. a^2 C. $x\cdot y$ D. $x^2\cdot y^2$

Rješenje 594

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad a^1=a \quad , \quad a^n:a^m=a^{n-m} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{\frac{1}{x}+\frac{1}{y}-\frac{a}{x\cdot y}}{\frac{1}{x^2}+\frac{1}{y^2}+\frac{2}{x\cdot y}-\frac{a^2}{x^2\cdot y^2}}\cdot(a+x+y)=\frac{\frac{y+x-a}{x\cdot y}}{\frac{y^2+x^2+2\cdot x\cdot y-a^2}{x^2\cdot y^2}}\cdot(a+x+y)=$$

$$=\frac{\frac{y+x-a}{x\cdot y}}{\frac{x^2+2\cdot x\cdot y+y^2-a^2}{x^2\cdot y^2}}\cdot(a+x+y)=\frac{\frac{y+x-a}{x\cdot y}}{\frac{\left(x^2+2\cdot x\cdot y+y^2\right)-a^2}{x^2\cdot y^2}}\cdot(a+x+y)=$$

$$=\frac{\frac{y+x-a}{x\cdot y}}{\frac{(x+y)^2-a^2}{x^2\cdot y^2}}\cdot(a+x+y)=\frac{\frac{y+x-a}{x\cdot y}}{\frac{(x+y)^2-a^2}{x^2\cdot y^2}}\cdot(a+x+y)=\frac{\frac{y+x-a}{1}}{\frac{(x+y)^2-a^2}{x\cdot y}}\cdot(a+x+y)=$$

$$=\frac{x\cdot y\cdot(y+x-a)}{(x+y)^2-a^2}\cdot(a+x+y)=\frac{x\cdot y\cdot(y+x-a)}{(x+y-a)\cdot(x+y+a)}\cdot(a+x+y)=$$

$$=\frac{x\cdot y\cdot(y+x-a)}{(x+y-a)\cdot(x+y+a)}\cdot(a+x+y)=x\cdot y.$$

Odgovor je pod C.

Vježba 594

Pojednostavnite $\dfrac{\dfrac{a}{x\cdot y}-\dfrac{1}{x}-\dfrac{1}{y}}{\dfrac{a^2}{x^2\cdot y^2}-\dfrac{1}{x^2}-\dfrac{1}{y^2}-\dfrac{2}{x\cdot y}}\cdot(a+x+y).$

A. a B. a^2 C. $x\cdot y$ D. $x^2\cdot y^2$

Rezultat: C.

Zadatak 595 (Martin, srednja škola)

Oduzmi razlomke $\dfrac{a^3+b^3}{a-b+\dfrac{a\cdot b}{a-b}}-\dfrac{a^3-b^3}{a+b-\dfrac{a\cdot b}{a+b}}.$

Rješenje 595

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right)\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{a^3+b^3}{a-b+\frac{a\cdot b}{a-b}}-\frac{a^3-b^3}{a+b-\frac{a\cdot b}{a+b}}=\frac{a^3+b^3}{\frac{a-b}{1}+\frac{a\cdot b}{a-b}}-\frac{a^3-b^3}{\frac{a+b}{1}-\frac{a\cdot b}{a+b}}=\frac{\frac{a^3+b^3}{1}}{\frac{(a-b)^2+a\cdot b}{a-b}}-\frac{\frac{a^3-b^3}{1}}{\frac{(a+b)^2-a\cdot b}{a+b}}=$$

$$=\frac{\frac{a^3+b^3}{1}}{\frac{a^2-2\cdot a\cdot b+b^2+a\cdot b}{a-b}}-\frac{\frac{a^3-b^3}{1}}{\frac{a^2+2\cdot a\cdot b+b^2-a\cdot b}{a+b}}=\frac{\frac{a^3+b^3}{1}}{\frac{a^2-a\cdot b+b^2}{a-b}}-\frac{\frac{a^3-b^3}{1}}{\frac{a^2+a\cdot b+b^2}{a+b}}=$$

$$=\frac{\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{1}}{\frac{a^2-a\cdot b+b^2}{a-b}}-\frac{\frac{(a-b)\cdot\left(a^2+a\cdot b+b^2\right)}{1}}{\frac{a^2+a\cdot b+b^2}{a+b}}=$$

$$=\frac{\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{1}}{\frac{a^2-a\cdot b+b^2}{a-b}}-\frac{\frac{(a-b)\cdot\left(a^2+a\cdot b+b^2\right)}{1}}{\frac{a^2+a\cdot b+b^2}{a+b}}=\frac{\frac{a+b}{1}}{\frac{1}{a-b}}-\frac{\frac{a-b}{1}}{\frac{1}{a+b}}=$$

$$=(a+b)\cdot(a-b)-(a+b)\cdot(a-b)=(a+b)\cdot(a-b)-(a+b)\cdot(a-b)=0.$$

Vježba 595

Oduzmi razlomke $\frac{a^3+b^3}{a-b+\frac{a\cdot b}{a-b}}+\frac{b^3-a^3}{a+b-\frac{a\cdot b}{a+b}}.$

Rezultat: 0.

Zadatak 596 (Silvija, gimnazija)

Pomnoži razlomke $\frac{\frac{1}{a^2}-\frac{1}{a-1}}{\frac{1}{a^2}+\frac{1}{a+1}}\cdot\frac{1-\frac{1}{a^3}}{1+\frac{1}{a^3}}.$

Rješenje 596

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right)\quad,\quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\quad,\quad n\neq 0\quad,\quad n\neq 1.$$

$$\frac{\frac{1}{a^2}-\frac{1}{a-1}}{\frac{1}{a^2}+\frac{1}{a+1}}\cdot\frac{1-\frac{1}{a^3}}{1+\frac{1}{a^3}}=\frac{\frac{1}{a^2}-\frac{1}{a-1}}{\frac{1}{a^2}+\frac{1}{a+1}}\cdot\frac{\frac{1}{1}-\frac{1}{a^3}}{\frac{1}{1}+\frac{1}{a^3}}=\frac{\frac{a-1-a^2}{a^2\cdot(a-1)}}{\frac{a+1+a^2}{a^2\cdot(a+1)}}\cdot\frac{\frac{a^3-1}{a^3}}{\frac{a^3+1}{a^3}}=$$

$$=\frac{\frac{a-1-a^2}{a^2\cdot(a-1)}}{\frac{a+1+a^2}{a^2\cdot(a+1)}}\cdot\frac{\frac{a^3-1}{a^3}}{\frac{a^3+1}{a^3}}=\frac{\frac{a-1-a^2}{a-1}}{\frac{a+1+a^2}{a+1}}\cdot\frac{\frac{a^3-1}{1}}{\frac{a^3+1}{1}}=\frac{\left(a-1-a^2\right)\cdot(a+1)}{(a-1)\cdot\left(a+1+a^2\right)}\cdot\frac{a^3-1}{a^3+1}=$$

$$=\frac{\left(a-1-a^{2}\right)\cdot(a+1)}{(a-1)\cdot\left(a+1+a^{2}\right)}\cdot\frac{(a-1)\cdot\left(a^{2}+a+1\right)}{(a+1)\cdot\left(a^{2}-a+1\right)}=\frac{-\left(a^{2}-a+1\right)\cdot(a+1)}{(a-1)\cdot\left(a^{2}+a+1\right)}\cdot\frac{(a-1)\cdot\left(a^{2}+a+1\right)}{(a+1)\cdot\left(a^{2}-a+1\right)}=$$

$$=\frac{-\left(a^{2}-a+1\right)\cdot(a+1)}{(a-1)\cdot\left(a^{2}+a+1\right)}\cdot\frac{(a-1)\cdot\left(a^{2}+a+1\right)}{(a+1)\cdot\left(a^{2}-a+1\right)}=\frac{-1}{1}\cdot\frac{1}{1}=-1.$$

Vježba 596

Pomnoži razlomke $\dfrac{\dfrac{1}{a^{2}}+\dfrac{1}{1-a}}{\dfrac{1}{a^{2}}+\dfrac{1}{a+1}}\cdot\dfrac{1-\dfrac{1}{a^{3}}}{1+\dfrac{1}{a^{3}}}.$

Rezultat: – 1.

Zadatak 597 (Anchy, gimnazija)

Pojednostavni izraz: $\left(\dfrac{a+1}{a+2}+\dfrac{1}{a}\right):\left(\dfrac{a+1}{a}-\dfrac{1}{a+2}\right).$

Rješenje 597

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b},\ n\neq 0\ ,\ n\neq 1.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$\left(\frac{a+1}{a+2}+\frac{1}{a}\right):\left(\frac{a+1}{a}-\frac{1}{a+2}\right)=\frac{a\cdot(a+1)+a+2}{a\cdot(a+2)}:\frac{(a+1)\cdot(a+2)-a}{a\cdot(a+2)}=$$

$$=\frac{a\cdot(a+1)+a+2}{a\cdot(a+2)}\cdot\frac{a\cdot(a+2)}{(a+1)\cdot(a+2)-a}=\frac{a\cdot(a+1)+a+2}{a\cdot(a+2)}\cdot\frac{a\cdot(a+2)}{(a+1)\cdot(a+2)-a}=$$

$$=\frac{a\cdot(a+1)+a+2}{1}\cdot\frac{1}{(a+1)\cdot(a+2)-a}=\frac{a\cdot(a+1)+a+2}{(a+1)\cdot(a+2)-a}=\frac{a^{2}+a+a+2}{a^{2}+2\cdot a+a+2-2}=$$

$$=\frac{a^{2}+a+a+2}{a^{2}+2\cdot a+a+2-2}=\frac{a^{2}+a+a+2}{a^{2}+2\cdot a+a}=\frac{a^{2}+2\cdot a+2}{a^{2}+2\cdot a+a}=\frac{a^{2}+2\cdot a+2}{a^{2}+2\cdot a+a}=1.$$

Vježba 597

Pojednostavni izraz: $\left(\dfrac{1}{a}+\dfrac{a+1}{a+2}\right):\left(\dfrac{a+1}{a}-\dfrac{1}{a+2}\right).$

Rezultat: 1.

Zadatak 598 (Ivana, gimnazija)

Pojednostavni izraz: $\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^{-2}\cdot\left(x^{-1}+y^{-1}\right)+2\cdot\left(x^{-\frac{1}{2}}+y^{-\frac{1}{2}}\right)\cdot\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^{-3}$.

Rješenje 598

Ponovimo!

$$a^{1}=a \quad , \quad a^{\frac{m}{n}}=\sqrt[n]{a^{m}} \quad , \quad a^{-n}=\frac{1}{a^{n}} \quad , \quad a^{-\frac{m}{n}}=\frac{1}{\sqrt[n]{a^{m}}} \quad , \quad \left(\sqrt{a}\right)^{2}=a.$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad a^{n}:a^{m}=a^{n-m} \quad , \quad \left(\frac{a}{b}\right)^{n}=\frac{a^{n}}{b^{n}}.$$

$$\frac{a\cdot b}{c\cdot d}=\frac{a}{c}\cdot\frac{b}{d} \quad , \quad (a+b)^{2}=a^{2}+2\cdot a\cdot b+b^{2}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^{-2}\cdot\left(x^{-1}+y^{-1}\right)+2\cdot\left(x^{-\frac{1}{2}}+y^{-\frac{1}{2}}\right)\cdot\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^{-3}=$$

$$=\left(\sqrt{x}+\sqrt{y}\right)^{-2}\cdot\left(\frac{1}{x}+\frac{1}{y}\right)+2\cdot\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}\right)\cdot\left(\sqrt{x}+\sqrt{y}\right)^{-3}=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left(\frac{1}{x}+\frac{1}{y}\right)+2\cdot\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}\right)\cdot\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{3}}=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left[\left(\frac{1}{x}+\frac{1}{y}\right)+2\cdot\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}\right)\cdot\frac{1}{\sqrt{x}+\sqrt{y}}\right]=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left[\frac{1}{x}+\frac{1}{y}+2\cdot\frac{\sqrt{y}+\sqrt{x}}{\sqrt{x}\cdot\sqrt{y}}\cdot\frac{1}{\sqrt{x}+\sqrt{y}}\right]=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left[\frac{1}{x}+\frac{1}{y}+2\cdot\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}\cdot\sqrt{y}}\cdot\frac{1}{\sqrt{x}+\sqrt{y}}\right]=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left[\frac{1}{x}+\frac{1}{y}+2\cdot\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}\cdot\sqrt{y}}\cdot\frac{1}{\sqrt{x}+\sqrt{y}}\right]=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left[\frac{1}{x}+\frac{1}{y}+2\cdot\frac{1}{\sqrt{x}\cdot\sqrt{y}}\cdot\frac{1}{1}\right]=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left[\frac{1}{x}+\frac{1}{y}+2\cdot\frac{1}{\sqrt{x}\cdot\sqrt{y}}\right]=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left[\frac{1}{x}+2\cdot\frac{1}{\sqrt{x}\cdot\sqrt{y}}+\frac{1}{y}\right]=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left[\frac{1}{\left(\sqrt{x}\right)^{2}}+2\cdot\frac{1}{\sqrt{x}}\cdot\frac{1}{\sqrt{y}}+\frac{1}{\left(\sqrt{y}\right)^{2}}\right]=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}\right)^{2}=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left(\frac{\sqrt{y}+\sqrt{x}}{\sqrt{x}\cdot\sqrt{y}}\right)^{2}=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}\cdot\sqrt{y}}\right)^{2}=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\frac{\left(\sqrt{x}+\sqrt{y}\right)^{2}}{\left(\sqrt{x}\cdot\sqrt{y}\right)^{2}}=$$

$$=\frac{1}{\left(\sqrt{x}+\sqrt{y}\right)^{2}}\cdot\frac{\left(\sqrt{x}+\sqrt{y}\right)^{2}}{\left(\sqrt{x}\cdot\sqrt{y}\right)^{2}}=\frac{1}{1}\cdot\frac{1}{\left(\sqrt{x}\cdot\sqrt{y}\right)^{2}}=\frac{1}{\left(\sqrt{x\cdot y}\right)^{2}}=\frac{1}{x\cdot y}.$$

Vježba 598

Pojednostavni izraz: $\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^{-2}\cdot\frac{x+y}{x\cdot y}+2\cdot\left(x^{-\frac{1}{2}}+y^{-\frac{1}{2}}\right)\cdot\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^{-3}.$

Rezultat: $\frac{1}{x\cdot y}.$

Zadatak 599 (Tom, gimnazija)

Pojednostavnite izraz: $\left[\frac{(a+b)^{3}}{3\cdot a\cdot b}-a-b\right]:\left[\frac{(a-b)^{2}}{a\cdot b}+1\right].$

Rješenje 599

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad a^{1}=a\quad,\quad a^{n}\cdot a^{m}=a^{n+m}.$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad(a+b)^{3}=a^{3}+3\cdot a^{2}\cdot b+3\cdot a\cdot b^{2}+b^{3}\quad,\quad(a-b)^{2}=a^{2}-2\cdot a\cdot b+b^{2}.$$

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^{3}+b^{3}=(a+b)\cdot\left(a^{2}-a\cdot b+b^{2}\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left[\frac{(a+b)^{3}}{3\cdot a\cdot b}-a-b\right]:\left[\frac{(a-b)^{2}}{a\cdot b}+1\right]=\left[\frac{(a+b)^{3}}{3\cdot a\cdot b}-\frac{a}{1}-\frac{b}{1}\right]:\left[\frac{(a-b)^{2}}{a\cdot b}+\frac{1}{1}\right]=$$

$$=\frac{(a+b)^{3}-3\cdot a^{2}\cdot b-3\cdot a\cdot b^{2}}{3\cdot a\cdot b}:\frac{(a-b)^{2}+a\cdot b}{a\cdot b}=\frac{(a+b)^{3}-3\cdot a^{2}\cdot b-3\cdot a\cdot b^{2}}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{(a-b)^{2}+a\cdot b}=$$

$$=\frac{(a+b)^{3}-3\cdot a^{2}\cdot b-3\cdot a\cdot b^{2}}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{(a-b)^{2}+a\cdot b}=\frac{(a+b)^{3}-3\cdot a^{2}\cdot b-3\cdot a\cdot b^{2}}{3}\cdot\frac{1}{(a-b)^{2}+a\cdot b}=$$

$$=\frac{a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2}{3}\cdot\frac{1}{a^2-2\cdot a\cdot b+b^2+a\cdot b}=$$

$$=\frac{a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2}{3}\cdot\frac{1}{a^2-a\cdot b+b^2}=$$

$$=\frac{a^3+b^3}{3}\cdot\frac{1}{a^2-a\cdot b+b^2}=\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{3}\cdot\frac{1}{a^2-a\cdot b+b^2}=$$

$$=\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{3}\cdot\frac{1}{a^2-a\cdot b+b^2}=\frac{a+b}{3}\cdot\frac{1}{1}=\frac{a+b}{3}.$$

Vježba 599

Pojednostavnite izraz: $\left[\frac{(a+b)^3}{3\cdot a\cdot b}-(a+b)\right]:\left[\frac{(b-a)^2}{a\cdot b}+1\right].$

Rezultat: $\frac{a+b}{3}.$

Zadatak 600 (Domagoj, srednja škola)

Izračunaj $\left(\frac{a-1}{a+1}+\frac{2\cdot a}{a^2-1}\right):\frac{a^2+1}{a-1}.$

Rješenje 600

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left(\frac{a-1}{a+1}+\frac{2\cdot a}{a^2-1}\right):\frac{a^2+1}{a-1}=\left(\frac{a-1}{a+1}+\frac{2\cdot a}{(a-1)\cdot(a+1)}\right)\cdot\frac{a-1}{a^2+1}=\frac{(a-1)^2+2\cdot a}{(a-1)\cdot(a+1)}\cdot\frac{a-1}{a^2+1}=$$

$$=\frac{a^2-2\cdot a+1+2\cdot a}{(a-1)\cdot(a+1)}\cdot\frac{a-1}{a^2+1}=\frac{a^2-2\cdot a+1+2\cdot a}{(a-1)\cdot(a+1)}\cdot\frac{a-1}{a^2+1}=\frac{a^2+1}{(a-1)\cdot(a+1)}\cdot\frac{a-1}{a^2+1}=$$

$$=\frac{a^2+1}{(a-1)\cdot(a+1)}\cdot\frac{a-1}{a^2+1}=\frac{1}{a+1}\cdot\frac{1}{1}=\frac{1}{a+1}.$$

Vježba 600

Izračunaj $\frac{a^2+1}{a-1}:\left(\frac{a-1}{a+1}+\frac{2\cdot a}{a^2-1}\right).$

Rezultat: a + 1.