Zadatak 661 (Lucy, gimnazija)

Napiši u obliku umnoška algebarski izraz $x^2 - x \cdot y + y - 1$.

Rješenje 661

Ponovimo!

$$a^2 - b^2 = (a-b) \cdot (a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$x^2 - x \cdot y + y - 1 = \left(x^2 - 1\right) + (-x \cdot y + y) = (x-1) \cdot (x+1) - y \cdot (x-1) =$$

$$= (x-1) \cdot (x+1) - y \cdot (x-1) = (x-1) \cdot ((x+1) - y) = (x-1) \cdot (x+1-y) = (x-1) \cdot (x-y+1).$$

Vježba 661

Napiši u obliku umnoška algebarski izraz $x^2 - x \cdot y - 1 + y$.

Rezultat: $(x-1) \cdot (x-y+1)$.

Zadatak 662 (Lucy, gimnazija)

Napiši u obliku umnoška algebarski izraz $a^3 - 2 \cdot a^2 - 2 \cdot a + 1$.

Rješenje 662

Ponovimo!

$$a^3 + b^3 = (a+b) \cdot \left(a^2 - a \cdot b + b^2\right) \quad , \quad a^1 = a \quad , \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$a^3 - 2 \cdot a^2 - 2 \cdot a + 1 = \left(a^3 + 1\right) + \left(-2 \cdot a^2 - 2 \cdot a\right) = (a+1) \cdot \left(a^2 - a + 1\right) - 2 \cdot a \cdot (a+1) =$$

$$= (a+1) \cdot \left(a^2 - a + 1\right) - 2 \cdot a \cdot (a+1) = (a+1) \cdot \left(\left(a^2 - a + 1\right) - 2 \cdot a\right) =$$

$$= (a+1) \cdot \left(a^2 - a + 1 - 2 \cdot a\right) = (a+1) \cdot \left(a^2 - 3 \cdot a + 1\right).$$

Vježba 662

Napiši u obliku umnoška algebarski izraz $a^3 - 2 \cdot \left(a^2 + a\right) + 1$.

Rezultat: $(a+1) \cdot \left(a^2 - 3 \cdot a + 1\right)$.

Zadatak 663 (Lucy, gimnazija)

Napiši u obliku umnoška algebarski izraz $x \cdot (x+y)^2 - y \cdot \left(x^2 - y^2\right)$.

Rješenje 663

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad a^2 - b^2 = (a-b) \cdot (a+b) \quad , \quad a^1 = a.$$

$$a^n : a^m = a^{n-m} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^3 + b^3 = (a+b) \cdot \left(a^2 - a \cdot b + b^2\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$x\cdot(x+y)^2-y\cdot\left(x^2-y^2\right)=x\cdot(x+y)^2-y\cdot(x-y)\cdot(x+y)=$$

$$=x\cdot(x+y)^2-y\cdot(x-y)\cdot(x+y)=(x+y)\cdot(x\cdot(x+y)-y\cdot(x-y))=$$

$$=(x+y)\cdot\left(x^2+x\cdot y-y\cdot x+y^2\right)=(x+y)\cdot\left(x^2+x\cdot y-y\cdot x+y^2\right)=(x+y)\cdot\left(x^2+y^2\right).$$

2.inačica

$$x\cdot(x+y)^2-y\cdot\left(x^2-y^2\right)=x\cdot\left(x^2+2\cdot x\cdot y+y^2\right)-y\cdot x^2+y^3=$$

$$=x^3+2\cdot x^2\cdot y+x\cdot y^2-y\cdot x^2+y^3=x^3+x^2\cdot y+x\cdot y^2+y^3=$$

$$=\left(x^3+x^2\cdot y\right)+\left(x\cdot y^2+y^3\right)=x^2\cdot(x+y)+y^2\cdot(x+y)=$$

$$=x^2\cdot(x+y)+y^2\cdot(x+y)=(x+y)\cdot\left(x^2+y^2\right).$$

3.inačica

$$x\cdot(x+y)^2-y\cdot\left(x^2-y^2\right)=x\cdot\left(x^2+2\cdot x\cdot y+y^2\right)-y\cdot x^2+y^3=$$

$$=x^3+2\cdot x^2\cdot y+x\cdot y^2-y\cdot x^2+y^3=x^3+x^2\cdot y+x\cdot y^2+y^3=$$

$$=\left(x^3+x\cdot y^2\right)+\left(x^2\cdot y+y^3\right)=x\cdot\left(x^2+y^2\right)+y\cdot\left(x^2+y^2\right)=$$

$$=x\cdot\left(x^2+y^2\right)+y\cdot\left(x^2+y^2\right)=\left(x^2+y^2\right)\cdot(x+y)=(x+y)\cdot\left(x^2+y^2\right).$$

4.inačica

$$x\cdot(x+y)^2-y\cdot\left(x^2-y^2\right)=x\cdot\left(x^2+2\cdot x\cdot y+y^2\right)-y\cdot x^2+y^3=$$

$$=x^3+2\cdot x^2\cdot y+x\cdot y^2-y\cdot x^2+y^3=x^3+x^2\cdot y+x\cdot y^2+y^3=$$

$$=\left(x^3+y^3\right)+\left(x^2\cdot y+x\cdot y^2\right)=(x+y)\cdot\left(x^2-x\cdot y+y^2\right)+x\cdot y\cdot(x+y)=$$

$$=(x+y)\cdot\left(x^2-x\cdot y+y^2\right)+x\cdot y\cdot(x+y)=(x+y)\cdot\left(\left(x^2-x\cdot y+y^2\right)+x\cdot y\right)=$$

$$=(x+y)\cdot\left(x^2-x\cdot y+y^2+x\cdot y\right)=(x+y)\cdot\left(x^2-x\cdot y+y^2+x\cdot y\right)=(x+y)\cdot\left(x^2+y^2\right).$$

Vježba 663

Napiši u obliku umnoška algebarski izraz $x\cdot(x+y)^2+y\cdot\left(y^2-x^2\right)$.

Rezultat: $(x+y)\cdot\left(x^2+y^2\right)$.

Zadatak 664 (Josip, gimnazija)

Vrijednost razlomka $\frac{27\cdot 3^{2\cdot n-1}}{9^{n+1}}$ ne ovisi o vrijednosti prirodnog broja n. Provjeri.

Rješenje 664

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad \left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^{-n} = \frac{1}{a^n}.$$

$$a^1 = a \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

$$\frac{27 \cdot 3^{2 \cdot n - 1}}{9^{n+1}} = \frac{27 \cdot 3^{2 \cdot n} \cdot 3^{-1}}{9^n \cdot 9^1} = \frac{27 \cdot \left(3^2\right)^n \cdot \frac{1}{3}}{9^n \cdot 9} = \frac{27 \cdot 9^n \cdot \frac{1}{3}}{9^n \cdot 9} = \frac{27 \cdot 9^n \cdot \frac{1}{3}}{9^n \cdot 9} = \frac{27 \cdot \frac{1}{3}}{9} =$$

$$= \frac{27 \cdot \frac{1}{3}}{9} = \frac{\frac{27}{1} \cdot \frac{1}{3}}{9} = \frac{\frac{27}{1} \cdot \frac{1}{3}}{9} = \frac{9}{9} = \frac{9}{9} = 1.$$

2.inačica

$$\frac{27 \cdot 3^{2 \cdot n - 1}}{9^{n+1}} = \frac{27 \cdot 3^{2 \cdot n} \cdot 3^{-1}}{\left(3^2\right)^{n+1}} = \frac{27 \cdot 3^{2 \cdot n} \cdot \frac{1}{3}}{3^{2 \cdot n + 2}} = \frac{27 \cdot 3^{2 \cdot n} \cdot \frac{1}{3}}{3^{2 \cdot n} \cdot 3^2} = \frac{27 \cdot 3^{2 \cdot n} \cdot \frac{1}{3}}{3^{2 \cdot n} \cdot 3^2} = \frac{27 \cdot \frac{1}{3}}{3^2} =$$

$$= \frac{27 \cdot \frac{1}{3}}{9} = \frac{\frac{27}{1} \cdot \frac{1}{3}}{9} = \frac{\frac{27}{1} \cdot \frac{1}{3}}{9} = \frac{9}{9} = \frac{9}{9} = 1.$$

3.inačica

$$\frac{27 \cdot 3^{2 \cdot n - 1}}{9^{n+1}} = \frac{3^3 \cdot 3^{2 \cdot n - 1}}{9^{n+1}} = \frac{3^{3 + 2 \cdot n - 1}}{9^{n+1}} = \frac{3^{2 \cdot n + 2}}{9^{n+1}} = \frac{3^{2 \cdot n + 2}}{\left(3^2\right)^{n+1}} = \frac{3^{2 \cdot n + 2}}{3^{2 \cdot n + 2}} = \frac{3^{2 \cdot n + 2}}{3^{2 \cdot n + 2}} = 1.$$

Vježba 664

Vrijednost razlomka $\frac{9^{n+1}}{27 \cdot 3^{2 \cdot n - 1}}$ ne ovisi o vrijednosti prirodnog broja n. Provjeri.

Rezultat: 1.

Zadatak 665 (Nataša, srednja škola)

Izračunajte $\left(2 \cdot x + 3\right)^2 + (3 - x) \cdot (3 + x).$

Rješenje 665

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad (a \cdot b)^n = a^n \cdot b^n \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$(2\cdot x+3)^2+(3-x)\cdot(3+x)=(2\cdot x)^2+2\cdot 2\cdot x\cdot 3+3^2+3^2-x^2=4\cdot x^2+12\cdot x+9+9-x^2=$$
$$=3\cdot x^2+12\cdot x+18=3\cdot\left(x^2+4\cdot x+6\right).$$

Vježba 665

Izračunajte $(2\cdot x-3)^2+(3-x)\cdot(3+x)$.

Rezultat: $3\cdot\left(x^2-4\cdot x+6\right)$.

Zadatak 666 (Veky, gimnazija)

Pojednostavnite: $\dfrac{1+a-\dfrac{1}{1-a}}{1-\dfrac{1}{1-a^2}}$.

Rješenje 666

Ponovimo!

$$n=\frac{n}{1}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad (a-b)\cdot(a+b)=a^2-b^2\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$
$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b},\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{1+a-\frac{1}{1-a}}{1-\frac{1}{1-a^2}}=\frac{\frac{1}{1}+\frac{a}{1}-\frac{1}{1-a}}{\frac{1}{1}-\frac{1}{1-a^2}}=\frac{\frac{1\cdot(1-a)+a\cdot(1-a)-1}{1-a}}{\frac{1\cdot\left(1-a^2\right)-1}{1-a^2}}=\frac{\frac{1-a+a-a^2-1}{1-a}}{\frac{1-a^2-1}{1-a^2}}=$$
$$=\frac{\frac{1-a+a-a^2-1}{1-a}}{\frac{1-a^2-1}{1-a^2}}=\frac{\frac{-a^2}{1-a}}{\frac{-a^2}{1-a^2}}=\frac{\frac{-a^2}{1-a}}{\frac{-a^2}{1-a^2}}=\frac{\frac{1}{1-a}}{\frac{1}{1-a^2}}=\frac{1-a^2}{1-a}=\frac{(1-a)\cdot(1+a)}{1-a}=\frac{(1-a)\cdot(1+a)}{1-a}=1+a.$$

2.inačica

$$\frac{1+a-\frac{1}{1-a}}{1-\frac{1}{1-a^2}}=\frac{\frac{1+a}{1}-\frac{1}{1-a}}{\frac{1}{1}-\frac{1}{1-a^2}}=\frac{\frac{(1+a)\cdot(1-a)-1}{1-a}}{\frac{1\cdot\left(1-a^2\right)-1}{1-a^2}}=\frac{\frac{1-a^2-1}{1-a}}{\frac{1-a^2-1}{1-a^2}}=\frac{\frac{1-a^2-1}{1-a}}{\frac{1-a^2-1}{1-a^2}}=\frac{\frac{-a^2}{1-a}}{\frac{-a^2}{1-a^2}}=$$

$$=\frac{\frac{-a^2}{1-a}}{\frac{-a^2}{1-a^2}}=\frac{\frac{1}{1-a}}{\frac{1}{1-a^2}}=\frac{1-a^2}{1-a}=\frac{(1-a)\cdot(1+a)}{1-a}=\frac{(1-a)\cdot(1+a)}{1-a}=1+a.$$

Vježba 666

Pojednostavnite: $\frac{1-\frac{1}{1-a^2}}{1+a-\frac{1}{1-a}}.$

Rezultat: $1+a.$

Zadatak 667 (Antonela, srednja škola)

Rastavi na faktore višečlani izraz: $2\cdot a\cdot b+4\cdot a+b^2+2\cdot b.$

Rješenje 667

Ponovimo!

$$a^1=a \quad , \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Metodom grupiranja preoblikujemo zadani izraz.

$$2\cdot a\cdot b+4\cdot a+b^2+2\cdot b=\left(2\cdot a\cdot b+b^2\right)+(4\cdot a+2\cdot b)=b\cdot(2\cdot a+b)+2\cdot(2\cdot a+b)=$$

$$=b\cdot(2\cdot a+b)+2\cdot(2\cdot a+b)=(2\cdot a+b)\cdot(b+2).$$

2.inačica

Metodom grupiranja preoblikujemo zadani izraz.

$$2\cdot a\cdot b+4\cdot a+b^2+2\cdot b=(2\cdot a\cdot b+4\cdot a)+\left(b^2+2\cdot b\right)=2\cdot a\cdot(b+2)+b\cdot(b+2)=$$

$$=2\cdot a\cdot(b+2)+b\cdot(b+2)=(b+2)\cdot(2\cdot a+b).$$

Vježba 667

Rastavi na faktore višečlani izraz: $2\cdot a\cdot b-4\cdot a+b^2-2\cdot b.$

Rezultat: $(b-2)\cdot(2\cdot a+b).$

Zadatak 668 (Antonela, srednja škola)

Rastavi na faktore višečlani izraz: $x^3-2\cdot x^2+x-2.$

Rješenje 668

Ponovimo!

$$a^1=a \quad , \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Metodom grupiranja preoblikujemo zadani izraz.

$$x^3-2\cdot x^2+x-2=\left(x^3-2\cdot x^2\right)+(x-2)=x^2\cdot(x-2)+(x-2)=x^2\cdot(x-2)+(x-2)=$$

$$=(x-2)\cdot\left(x^2+1\right).$$

2.inačica

Metodom grupiranja preoblikujemo zadani izraz.

$$x^3-2\cdot x^2+x-2=\left(x^3+x\right)+\left(-2\cdot x^2-2\right)=x\cdot\left(x^2+1\right)-2\cdot\left(x^2+1\right)=x\cdot\left(x^2+1\right)-2\cdot\left(x^2+1\right)=$$

$$=\left(x^2+1\right)\cdot(x-2).$$

Vježba 668

Rastavi na faktore višečlani izraz: $x^3-2\cdot x^2+x-2.$

Rezultat: $\left(x^2+1\right)\cdot(x+2).$

Zadatak 669 (Luka, srednja škola)

Ako je $(2\cdot x-y)\cdot(x-2\cdot y)=4,$ koliko je $-4\cdot x^2+10\cdot x\cdot y-4\cdot y^2$?

$A.\ -4 \qquad B.\ -8 \qquad C.\ 4 \qquad D.\ 8$

Rješenje 669

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^n:a^m=a^{n-m}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Preoblikujemo postavljenu jednakost.

$$(2\cdot x-y)\cdot(x-2\cdot y)=4 \Rightarrow 2\cdot x^2-4\cdot x\cdot y-x\cdot y+2\cdot y^2=4 \Rightarrow 2\cdot x^2-5\cdot x\cdot y+2\cdot y^2=4.$$

Sada je:

$$-4\cdot x^2+10\cdot x\cdot y-4\cdot y^2=-2\cdot\left(2\cdot x^2-5\cdot x\cdot y+2\cdot y^2\right)=\left[\begin{array}{c}\text{uvjet}\\ 2\cdot x^2-5\cdot x\cdot y+2\cdot y^2=4\end{array}\right]=$$

$$=-2\cdot 4=-8.$$

Odgovor je pod B.

2.inačica

Transformiramo zadani trinom.

$$-4\cdot x^2+10\cdot x\cdot y-4\cdot y^2=-2\cdot\left(2\cdot x^2-5\cdot x\cdot y+2\cdot y^2\right)=\left[\begin{array}{l}\text{metoda}\\ \text{grupiranja}\end{array}\right]=$$

$$=-2\cdot\left(2\cdot x^2-4\cdot x\cdot y-x\cdot y+2\cdot y^2\right)=-2\cdot\left(\left(2\cdot x^2-4\cdot x\cdot y\right)+\left(-x\cdot y+2\cdot y^2\right)\right)=$$

$$=-2\cdot(2\cdot x\cdot(x-2\cdot y)-y\cdot(x-2\cdot y))=-2\cdot(2\cdot x\cdot(x-2\cdot y)-y\cdot(x-2\cdot y))=$$

$$=-2\cdot(x-2\cdot y)\cdot(2\cdot x-y)=-2\cdot(2\cdot x-y)\cdot(x-2\cdot y)=\left[\begin{array}{c}\text{uvjet}\\ 2\cdot x^2-5\cdot x\cdot y+2\cdot y^2=4\end{array}\right]=$$

$$=-2\cdot 4=-8.$$

Odgovor je pod B.

Vježba 669

Ako je $(2\cdot x-y)\cdot(x-2\cdot y)=-4,$ koliko je $-4\cdot x^2+10\cdot x\cdot y-4\cdot y^2$?

$A.\ -4 \qquad B.\ -8 \qquad C.\ 4 \qquad D.\ 8$

Rezultat: D.

Zadatak 670 (Luka, srednja škola)

Pomnoži razlomke: $\left(a-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}.$

$A.\ \frac{a+2}{a-2} \qquad B.\ \frac{2\cdot(a-2)}{a+2} \qquad C.\ \frac{2\cdot(a+2)}{a-2} \qquad D.\ \frac{a-2}{a+2}$

Rješenje 670

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad a^2-2\cdot a\cdot b+b^2=(a-b)^2\quad,\quad a^1=a.$$

$$a^{n+m}=a^n\cdot a^m\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left(a-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}=\left(\frac{a}{1}-\frac{a^2+4}{4}\right)\cdot\frac{8}{4-a^2}=\frac{4\cdot a-\left(a^2+4\right)}{4}\cdot\frac{8}{4-a^2}=$$

$$=\frac{4\cdot a-a^2-4}{4}\cdot\frac{8}{4-a^2}=\frac{-\left(a^2-4\cdot a+4\right)}{4}\cdot\frac{8}{4-a^2}=\frac{a^2-4\cdot a+4}{4}\cdot\frac{8}{-\left(4-a^2\right)}=$$

$$=\frac{(a-2)^2}{4}\cdot\frac{8}{a^2-4}=\frac{(a-2)^2}{4}\cdot\frac{8}{(a-2)\cdot(a+2)}=\frac{(a-2)^2}{4}\cdot\frac{8}{(a-2)\cdot(a+2)}=$$

$$=\frac{a-2}{1}\cdot\frac{2}{a+2}=\frac{2\cdot(a-2)}{a+2}.$$

Odgovor je pod B.

Vježba 670

Pomnoži razlomke: $\left(\frac{a^2+4}{4}-a\right)\cdot\frac{8}{a^2-4}.$

$A.\ \frac{a+2}{a-2} \qquad B.\ \frac{2\cdot(a-2)}{a+2} \qquad C.\ \frac{2\cdot(a+2)}{a-2} \qquad D.\ \frac{a-2}{a+2}$

Rezultat: B.

Zadatak 671 (Gordana, srednja škola)

Pojednostavni: $\left(\frac{a^{-3}}{3\cdot b^{-2}}\right)^{-3}\cdot\left(9\cdot a^{4}\cdot b^{-1}\right)^{-2}.$

Rješenje 671

Ponovimo!

$$a^{1}=a\quad,\quad\left(\frac{a}{b}\right)^{n}=\frac{a^{n}}{b^{n}}\quad,\quad(a\cdot b)^{n}=a^{n}\cdot b^{n}\quad,\quad\left(a^{n}\right)^{m}=a^{n\cdot m}.$$

$$a^{-n}=\frac{1}{a^{n}}\quad,\quad\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}\quad,\quad\frac{a^{n}}{a^{m}}=a^{n-m}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\left(\frac{a^{-3}}{3\cdot b^{-2}}\right)^{-3}\cdot\left(9\cdot a^{4}\cdot b^{-1}\right)^{-2}=\left(\frac{a^{-3}}{3^{1}\cdot b^{-2}}\right)^{-3}\cdot\left(3^{2}\cdot a^{4}\cdot b^{-1}\right)^{-2}=$$

$$=\frac{\left(a^{-3}\right)^{-3}}{\left(3^{1}\cdot b^{-2}\right)^{-3}}\cdot\left(3^{2}\right)^{-2}\cdot\left(a^{4}\right)^{-2}\cdot\left(b^{-1}\right)^{-2}=\frac{a^{9}}{3^{-3}\cdot b^{6}}\cdot 3^{-4}\cdot a^{-8}\cdot b^{2}=$$

$$=\frac{3^{3}\cdot a^{9}}{b^{6}}\cdot\frac{b^{2}}{3^{4}\cdot a^{8}}=\frac{3^{3}\cdot a^{9}}{b^{6}}\cdot\frac{b^{2}}{3^{4}\cdot a^{8}}=\frac{1\cdot a^{1}}{b^{4}}\cdot\frac{1}{3^{1}\cdot 1}=\frac{a}{b^{4}}\cdot\frac{1}{3}=\frac{a}{3\cdot b^{4}}.$$

2.inačica

$$\left(\frac{a^{-3}}{3\cdot b^{-2}}\right)^{-3}\cdot\left(9\cdot a^{4}\cdot b^{-1}\right)^{-2}=\left(\frac{b^{2}}{3\cdot a^{3}}\right)^{-3}\cdot\left(\frac{9\cdot a^{4}}{b^{1}}\right)^{-2}=\left(\frac{3\cdot a^{3}}{b^{2}}\right)^{3}\cdot\left(\frac{b^{1}}{9\cdot a^{4}}\right)^{2}=$$

$$=\frac{\left(3\cdot a^{3}\right)^{3}}{\left(b^{2}\right)^{3}}\cdot\frac{\left(b^{1}\right)^{2}}{\left(9\cdot a^{4}\right)^{2}}=\frac{\left(3^{1}\cdot a^{3}\right)^{3}}{\left(b^{2}\right)^{3}}\cdot\frac{\left(b^{1}\right)^{2}}{\left(3^{2}\cdot a^{4}\right)^{2}}=\frac{\left(3^{1}\right)^{3}\cdot\left(a^{3}\right)^{3}}{\left(b^{2}\right)^{3}}\cdot\frac{\left(b^{1}\right)^{2}}{\left(3^{2}\right)^{2}\cdot\left(a^{4}\right)^{2}}=$$

$$=\frac{3^{3}\cdot a^{9}}{b^{6}}\cdot\frac{b^{2}}{3^{4}\cdot a^{8}}=\frac{3^{3}\cdot a^{9}}{b^{6}}\cdot\frac{b^{2}}{3^{4}\cdot a^{8}}=\frac{1\cdot a^{1}}{b^{4}}\cdot\frac{1}{3^{1}\cdot 1}=\frac{a}{b^{4}}\cdot\frac{1}{3}=\frac{a}{3\cdot b^{4}}.$$

Vježba 671

Pojednostavni: $\left(\frac{a^{-3}}{3\cdot b^{-2}}\right)^{-3}\cdot\frac{1}{\left(9\cdot a^{4}\cdot b^{-1}\right)^{2}}.$

Zadatak 672 (Veky, gimnazija)

Ako je $a \cdot b = 3$ i $a^2 \cdot b + a \cdot b^2 = (a+b) - 2$, koliko je $a^2 + b^2$?

Rješenje 672

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Preoblikujemo zadanu jednakost.

$$a^2 \cdot b + a \cdot b^2 = (a+b) - 2 \Rightarrow a^2 \cdot b + a \cdot b^2 - (a+b) = -2 \Rightarrow$$

$$\Rightarrow a \cdot b \cdot (a+b) - (a+b) = -2 \Rightarrow a \cdot b \cdot (a+b) - (a+b) = -2 \Rightarrow (a+b) \cdot (a \cdot b - 1) = -2 \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{uvjet} \\ a \cdot b = 3 \end{bmatrix} \Rightarrow (a+b) \cdot (3-1) = -2 \Rightarrow 2 \cdot (a+b) = -2 \Rightarrow 2 \cdot (a+b) = -2 \ /:2 \Rightarrow a+b = -1.$$

Sada je:

$$a^2 + b^2 = a^2 + b^2 + 2 \cdot a \cdot b - 2 \cdot a \cdot b = a^2 + 2 \cdot a \cdot b + b^2 - 2 \cdot a \cdot b =$$

$$= \left(a^2 + 2 \cdot a \cdot b + b^2\right) - 2 \cdot a \cdot b = (a+b)^2 - 2 \cdot a \cdot b = \begin{bmatrix} a+b=-1 \\ a \cdot b = 3 \end{bmatrix} = (-1)^2 - 2 \cdot 3 = 1 - 6 = -5.$$

Vježba 672

Ako je $a \cdot b - 3 = 0$ i $a^2 \cdot b + a \cdot b^2 - a - b + 2 = 0$, koliko je $a^2 + b^2$?

Rezultat: -5.

Zadatak 673 (Bruno, gimnazija)

Skrati razlomak $\dfrac{x^2 - 1}{\sqrt{x^3} - x + \sqrt{x} - 1}$.

Rješenje 673

Ponovimo!

$$a^2 - b^2 = (a-b) \cdot (a+b) \quad , \quad a^1 = a \quad , \quad \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \quad , \quad a^n \cdot a^m = a^{n+m}.$$

$$\sqrt{a^2} = a \ , \ a > 0 \quad , \quad \left(\sqrt{a}\right)^2 = a.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\frac{x^2 - 1}{\sqrt{x^3} - x + \sqrt{x} - 1} = \frac{(x-1) \cdot (x+1)}{\sqrt{x^2 \cdot x} - x + \sqrt{x} - 1} = \frac{(x-1) \cdot (x+1)}{\sqrt{x^2} \cdot \sqrt{x} - x + \sqrt{x} - 1} = \frac{(x-1) \cdot (x+1)}{x \cdot \sqrt{x} - x + \sqrt{x} - 1} =$$

$$=\begin{bmatrix}\text{metoda grupiranja}\\ \text{u nazivniku}\end{bmatrix}=\frac{(x-1)\cdot(x+1)}{\left(x\cdot\sqrt{x}-x\right)+\left(\sqrt{x}-1\right)}=\frac{(x-1)\cdot(x+1)}{\left(x\cdot\sqrt{x}-x\right)+\left(\sqrt{x}-1\right)}=\frac{(x-1)\cdot(x+1)}{x\cdot\left(\sqrt{x}-1\right)+\left(\sqrt{x}-1\right)}=$$

$$=\frac{(x-1)\cdot(x+1)}{x\cdot\left(\sqrt{x}-1\right)+\left(\sqrt{x}-1\right)}=\frac{(x-1)\cdot(x+1)}{\left(\sqrt{x}-1\right)\cdot(x+1)}=\frac{(x-1)\cdot(x+1)}{\left(\sqrt{x}-1\right)\cdot(x+1)}=\frac{x-1}{\sqrt{x}-1}=$$

$$=\frac{\left(\sqrt{x}\right)^2-1}{\sqrt{x}-1}=\frac{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}{\sqrt{x}-1}=\frac{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}{\sqrt{x}-1}=\sqrt{x}+1.$$

Vježba 673

Skrati razlomak $\dfrac{x^2-1}{\sqrt{x^3}-x+\sqrt{x}-1}$.

Rezultat: $\dfrac{1}{\sqrt{x}+1}$.

Zadatak 674 (Branko, gimnazija)

Ako je $(2\cdot a-1)^3=m$, onda je $(3-6\cdot a)^3$ jednako

$A.\ -3\cdot m \qquad B.\ -9\cdot m \qquad C.\ -27\cdot m \qquad D.\ -m^2$

Rješenje 674

Ponovimo!

$$(a\cdot b)^3=a^3\cdot b^3 \quad , \quad (-a)^3=-a^3.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$(3-6\cdot a)^3=\left(-3\cdot(2\cdot a-1)\right)^3=(-3)^3\cdot(2\cdot a-1)^3=-27\cdot(2\cdot a-1)^3=$$

$$=\left[(2\cdot a-1)^3=m\right]=-27\cdot m.$$

Odgovor je pod C.

Vježba 674

Ako je $(2\cdot a-1)^3=m$, onda je $(2-4\cdot a)^3$ jednako

$A.\ -2\cdot m \qquad B.\ -4\cdot m \qquad C.\ -8\cdot m \qquad D.\ -m^2$

Rezultat: C.

Zadatak 675 (Klementina, gimnazija)

Ako je $a\cdot(a-b)=11$, a $b\cdot(a-b)=13$, tada je:

$A.\ a^2-b^2=143 \qquad B.\ a^2-b^2=2 \qquad C.\ a^2-b^2=24 \qquad D.\ a^2-b^2=48$

Rješenje 675

Ponovimo!

$$\left.\begin{matrix}a=b\\ c=d\end{matrix}\right\}\Rightarrow a+c=b+d \quad , \quad (a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left.\begin{matrix}a=b\\ c=d\end{matrix}\right\}\Rightarrow \frac{a}{c}=\frac{b}{d}.$$

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

$$\frac{a}{b} \cdot \frac{b}{a} = 1 \quad , \quad (a \cdot b)^2 = a^2 \cdot b^2 \quad , \quad \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2} \quad , \quad \frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \;,\; n \neq 0 \;,\; n \neq 1.$$

1.inačica

$$\left.\begin{array}{l} a \cdot (a-b) = 11 \\ b \cdot (a-b) = 13 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow a \cdot (a-b) + b \cdot (a-b) = 11 + 13 \Rightarrow$$

$$\Rightarrow a \cdot (a-b) + b \cdot (a-b) = 24 \Rightarrow (a-b) \cdot (a+b) = 24 \Rightarrow a^2 - b^2 = 24.$$

Odgovor je pod C.

2.inačica

$$\left.\begin{array}{l} a \cdot (a-b) = 11 \\ b \cdot (a-b) = 13 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{podijelimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow \frac{a \cdot (a-b)}{b \cdot (a-b)} = \frac{11}{13} \Rightarrow \frac{a \cdot (a-b)}{b \cdot (a-b)} = \frac{11}{13} \Rightarrow$$

$$\Rightarrow \frac{a}{b} = \frac{11}{13} \Rightarrow \frac{a}{b} = \frac{11}{13} \;/ \cdot b \Rightarrow a = \frac{11}{13} \cdot b.$$

Izračunamo b^2.

$$\left.\begin{array}{l} a \cdot (a-b) = 11 \\ a = \frac{11}{13} \cdot b \end{array}\right\} \Rightarrow \frac{11}{13} \cdot b \cdot \left(\frac{11}{13} \cdot b - b\right) = 11 \Rightarrow \frac{11}{13} \cdot b \cdot b \cdot \left(\frac{11}{13} - 1\right) = 11 \Rightarrow$$

$$\Rightarrow \frac{11}{13} \cdot b^2 \cdot \left(\frac{11}{13} - \frac{1}{1}\right) = 11 \Rightarrow \frac{11}{13} \cdot b^2 \cdot \frac{11-13}{13} = 11 \Rightarrow \frac{11}{13} \cdot b^2 \cdot \left(-\frac{2}{13}\right) = 11 \Rightarrow$$

$$\Rightarrow -\frac{22}{169} \cdot b^2 = 11 \Rightarrow -\frac{22}{169} \cdot b^2 = 11 \;/ \cdot \left(-\frac{169}{2}\right) \Rightarrow b^2 = -\frac{169}{2}.$$

Sada a^2 iznosi:

$$\left.\begin{array}{l} a = \frac{11}{13} \cdot b \\ b^2 = -\frac{169}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = \frac{11}{13} \cdot b \;/\, ^2 \\ b^2 = -\frac{169}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a^2 = \left(\frac{11}{13} \cdot b\right)^2 \\ b^2 = -\frac{169}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a^2 = \left(\frac{11}{13}\right)^2 \cdot b^2 \\ b^2 = -\frac{169}{2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a^2 = \frac{121}{169} \cdot b^2 \\ b^2 = -\frac{169}{2} \end{array}\right\} \Rightarrow a^2 = \frac{121}{169} \cdot \left(-\frac{169}{2}\right) \Rightarrow a^2 = \frac{121}{169} \cdot \left(-\frac{169}{2}\right) \Rightarrow a^2 = -\frac{121}{2}.$$

Konačno je:

$$a^2-b^2=\begin{bmatrix} a^2=-\frac{121}{2} \\ b^2=-\frac{169}{2} \end{bmatrix}=-\frac{121}{2}-\left(-\frac{169}{2}\right)=-\frac{121}{2}+\frac{169}{2}=\frac{-121+169}{2}=\frac{48}{2}=\frac{48}{2}=24.$$

Odgovor je pod C.

Vježba 675

Ako je $a\cdot(a-b)=9$, a $b\cdot(a-b)=11$, tada je:

$A.\ a^2-b^2=99$ $B.\ a^2-b^2=2$ $C.\ a^2-b^2=20$ $D.\ a^2-b^2=40$

Rezultat: C.

Zadatak 676 (Josip, gimnazija)

Ako je $\frac{a}{a-b}=\frac{2}{3}$, koliko je $\frac{a}{b}$?

Rješenje 676

Ponovimo!

$$\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c} \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a-b}{n}=\frac{a}{n}-\frac{b}{n} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}, \quad n\neq 0 \ , \ n\neq 1.$$

1.inačica

$$\frac{a}{a-b}=\frac{2}{3} \Rightarrow \frac{a}{a-b}=\frac{2}{3} \ /\cdot 3\cdot(a-b) \Rightarrow 3\cdot a=2\cdot(a-b) \Rightarrow 3\cdot a=2\cdot a-2\cdot b \Rightarrow$$

$$\Rightarrow 3\cdot a-2\cdot a=-2\cdot b \Rightarrow a=-2\cdot b \Rightarrow a=-2\cdot b \ /\cdot\frac{1}{b} \Rightarrow \frac{a}{b}=-2.$$

2.inačica

$$\frac{a}{a-b}=\frac{2}{3} \Rightarrow \frac{a-b}{a}=\frac{3}{2} \Rightarrow \frac{a}{a}-\frac{b}{a}=\frac{3}{2} \Rightarrow \frac{a}{a}-\frac{b}{a}=\frac{3}{2} \Rightarrow 1-\frac{b}{a}=\frac{3}{2} \Rightarrow -\frac{b}{a}=\frac{3}{2}-1 \Rightarrow$$

$$\Rightarrow -\frac{b}{a}=\frac{3}{2}-\frac{1}{1} \Rightarrow -\frac{b}{a}=\frac{3-2}{2} \Rightarrow -\frac{b}{a}=\frac{1}{2} \Rightarrow -\frac{b}{a}=\frac{1}{2} \ /\cdot(-1) \Rightarrow \frac{b}{a}=-\frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{a}{b}=-\frac{2}{1} \Rightarrow \frac{a}{b}=-2.$$

Vježba 676

Ako je $\frac{a}{a-b}=\frac{1}{3}$, koliko je $\frac{a}{b}$?

Rezultat: $-\frac{1}{2}.$

Zadatak 677 (Pero, pomorska škola)

Rastavi na faktore: $\left(a^2+b^2\right)^3-a^2\cdot\left(a^2+b^2\right).$

Rješenje 677

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(a^2+b^2\right)^3-a^2\cdot\left(a^2+b^2\right)=\left(a^2+b^2\right)\cdot\left(\left(a^2+b^2\right)^2-a^2\right)=$$

$$=\left(a^2+b^2\right)\cdot\left(\left(a^2+b^2\right)-a\right)\cdot\left(\left(a^2+b^2\right)+a\right)=\left(a^2+b^2\right)\cdot\left(a^2+b^2-a\right)\cdot\left(a^2+b^2+a\right).$$

Vježba 677

Rastavi na faktore: $\left(a^2+b^2\right)^3-b^2\cdot\left(a^2+b^2\right).$

Rezultat: $\left(a^2+b^2\right)\cdot\left(a^2+b^2-b\right)\cdot\left(a^2+b^2+b\right).$

Zadatak 678 (Pero, pomorska škola)

Rastavi na faktore: $3\cdot\left(a^2-b^2\right)-6\cdot(a-b).$

Rješenje 678

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$3\cdot\left(a^2-b^2\right)-6\cdot(a-b)=3\cdot(a-b)\cdot(a+b)-6\cdot(a-b)=$$

$$=3\cdot(a-b)\cdot((a+b)-2)=3\cdot(a-b)\cdot(a+b-2).$$

Vježba 678

Rastavi na faktore: $3\cdot\left(a^2-b^2\right)-9\cdot(a-b).$

Rezultat: $3\cdot(a-b)\cdot(a+b-3).$

Zadatak 679 (Pero, pomorska škola)

Rastavi na faktore: $x^2-16\cdot(x-1)^2.$

Rješenje 679

Ponovimo!

$$a^2\cdot b^2=(a\cdot b)^2 \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$x^2-16\cdot(x-1)^2=x^2-4^2\cdot(x-1)^2=x^2-(4\cdot(x-1))^2=$$

$$= \left(x - 4 \cdot (x-1)\right) \cdot \left(x + 4 \cdot (x-1)\right) = (x - 4 \cdot x + 4) \cdot (x + 4 \cdot x - 4) = (-3 \cdot x + 4) \cdot (5 \cdot x - 4).$$

Vježba 679

Rastavi na faktore: $x^2 - 9 \cdot (x-1)^2$.

Rezultat: $(-2 \cdot x + 3) \cdot (4 \cdot x - 3)$.

Zadatak 680 (Pero, pomorska škola)

Rastavi na faktore: $49 \cdot (3 \cdot a - b)^3 - (3 \cdot a - b)$.

Rješenje 680

Ponovimo!

$$a^2 \cdot b^2 = (a \cdot b)^2 \quad , \quad a^2 - b^2 = (a - b) \cdot (a + b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$49 \cdot (3 \cdot a - b)^3 - (3 \cdot a - b) = (3 \cdot a - b) \cdot \left(49 \cdot (3 \cdot a - b)^2 - 1\right) = (3 \cdot a - b) \cdot \left(7^2 \cdot (3 \cdot a - b)^2 - 1\right) =$$

$$= (3 \cdot a - b) \cdot \left(\left(7 \cdot (3 \cdot a - b)\right)^2 - 1\right) = (3 \cdot a - b) \cdot \left(7 \cdot (3 \cdot a - b) - 1\right) \cdot \left(7 \cdot (3 \cdot a - b) + 1\right) =$$

$$= (3 \cdot a - b) \cdot (21 \cdot a - 7 \cdot b - 1) \cdot (21 \cdot a - 7 \cdot b + 1).$$

Vježba 680

Rastavi na faktore: $49 \cdot (a - b)^3 - (a - b)$.

Rezultat: $(a - b) \cdot (7 \cdot a - 7 \cdot b - 1) \cdot (7 \cdot a - 7 \cdot b + 1)$.