Zadatak 761 (Ivana, srednja škola)

Izračunaj: $\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}+\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}.$

Rješenje 761

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \;, \; n\neq 0 \;, \; n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;, \; n\neq 0 \;, \; n\neq 1.$$

1.inačica

$$\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}+\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=$$

$$\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}\cdot\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}+\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}\cdot\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}=$$

$$=\frac{\left(\sqrt{x+y}+\sqrt{x-y}\right)^2}{\left(\sqrt{x+y}\right)^2-\left(\sqrt{x-y}\right)^2}+\frac{\left(\sqrt{x+y}-\sqrt{x-y}\right)^2}{\left(\sqrt{x+y}\right)^2-\left(\sqrt{x-y}\right)^2}=$$

$$=\frac{\left(\sqrt{x+y}\right)^2+2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2}{x+y-(x-y)}+\frac{\left(\sqrt{x+y}\right)^2-2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2}{x+y-(x-y)}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}+\frac{x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}+\frac{x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}=$$

$$=\frac{x+2\cdot\sqrt{(x+y)\cdot(x-y)}+x}{y+y}+\frac{x-2\cdot\sqrt{(x+y)\cdot(x-y)}+x}{y+y}=$$

$$=\frac{2\cdot x+2\cdot\sqrt{(x+y)\cdot(x-y)}}{2\cdot y}+\frac{2\cdot x-2\cdot\sqrt{(x+y)\cdot(x-y)}}{2\cdot y}=$$

$$=\frac{2\cdot x+2\cdot\sqrt{(x+y)\cdot(x-y)}+2\cdot x-2\cdot\sqrt{(x+y)\cdot(x-y)}}{2\cdot y}=$$

$$=\frac{2\cdot x+2\cdot\sqrt{(x+y)\cdot(x-y)}+2\cdot x-2\cdot\sqrt{(x+y)\cdot(x-y)}}{2\cdot y}=\frac{2\cdot x+2\cdot x}{2\cdot y}=\frac{4\cdot x}{2\cdot y}=\frac{4\cdot x}{2\cdot y}=\frac{2\cdot x}{y}.$$

2.inačica

$$\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}+\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}=$$

$$=\frac{\left(\sqrt{x+y}+\sqrt{x-y}\right)^2+\left(\sqrt{x+y}-\sqrt{x-y}\right)^2}{\left(\sqrt{x+y}-\sqrt{x-y}\right)\cdot\left(\sqrt{x+y}+\sqrt{x-y}\right)}=$$

$$=\frac{\left(\sqrt{x+y}\right)^2+2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2+\left(\sqrt{x+y}\right)^2-2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2}{\left(\sqrt{x+y}\right)^2-\left(\sqrt{x-y}\right)^2}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y+x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-(x-y)}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y+x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y+x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}=\frac{x+x+x+x}{y+y}=$$

$$=\frac{4\cdot x}{2\cdot y}=\frac{4\cdot x}{2\cdot y}=\frac{2\cdot x}{y}.$$

Vježba 761

Izračunaj: $\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}-\frac{\sqrt{x-y}-\sqrt{x+y}}{\sqrt{x+y}+\sqrt{x-y}}.$

Rezultat: $\frac{2\cdot x}{y}.$