Zadatak 781 (4B, TUPŠ)

Rastavi na faktore: $(3\cdot x-2\cdot y)^2+24\cdot x\cdot y$.

Rješenje 781

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$a^2=a\cdot a.$$

$$(3\cdot x-2\cdot y)^2+24\cdot x\cdot y=(3\cdot x)^2-2\cdot 3\cdot x\cdot 2\cdot y+(2\cdot y)^2+24\cdot x\cdot y=$$

$$=9\cdot x^2-12\cdot x\cdot y+4\cdot y^2+24\cdot x\cdot y=9\cdot x^2+12\cdot x\cdot y+4\cdot y^2=$$

$$=(3\cdot x+2\cdot y)^2=(3\cdot x+2\cdot y)\cdot(3\cdot x+2\cdot y).$$

Vježba 781

Rastavi na faktore: $24\cdot x\cdot y+(3\cdot x-2\cdot y)^2$.

Rezultat: $(3\cdot x+2\cdot y)\cdot(3\cdot x+2\cdot y)$.

Zadatak 782 (4B, TUPŠ)

Rastavi na faktore: $40\cdot x\cdot y-(2\cdot x+5\cdot y)^2$.

Rješenje 782

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$a^2=a\cdot a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$40\cdot x\cdot y-(2\cdot x+5\cdot y)^2=40\cdot x\cdot y-\left((2\cdot x)^2+2\cdot 2\cdot x\cdot 5\cdot y+(5\cdot y)^2\right)=$$

$$=40\cdot x\cdot y-\left(4\cdot x^2+20\cdot x\cdot y+25\cdot y^2\right)=40\cdot x\cdot y-4\cdot x^2-20\cdot x\cdot y-25\cdot y^2=$$

$$=-4\cdot x^2+20\cdot x\cdot y-25\cdot y^2=-\left(4\cdot x^2-20\cdot x\cdot y+25\cdot y^2\right)=-(2\cdot x-5\cdot y)^2=$$

$$=-(2\cdot x-5\cdot y)\cdot(2\cdot x-5\cdot y).$$

Vježba 782

Rastavi na faktore: $24\cdot x\cdot y-(2\cdot x+3\cdot y)^2$.

Rezultat: $-(2\cdot x-3\cdot y)\cdot(2\cdot x-3\cdot y)$.

Zadatak 783 (Miroslav, gimnazija)

Ako je $\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2}$, onda je $\frac{a}{b}-\frac{b}{a}$ jednako:

$$A.\ \frac{1}{4} \qquad B.\ 2 \qquad C.\ \frac{1}{2} \qquad D.\ 8$$

Rješenje 783

Ponovimo!

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad , \quad n = \frac{n}{1}.$$

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

1.inačica

Preoblikujemo zadanu jednakost.

$$\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{1}{2} \Rightarrow \frac{(a+b)^2 - (a-b)^2}{(a-b) \cdot (a+b)} = \frac{1}{2} \Rightarrow \frac{\big((a+b)-(a-b)\big) \cdot \big((a+b)+(a-b)\big)}{a^2 - b^2} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{(a+b-a+b) \cdot (a+b+a-b)}{a^2 - b^2} = \frac{1}{2} \Rightarrow \frac{(a+b-a+b) \cdot (a+b+a-b)}{a^2 - b^2} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{2 \cdot b \cdot 2 \cdot a}{a^2 - b^2} = \frac{1}{2} \Rightarrow \frac{4 \cdot a \cdot b}{a^2 - b^2} = \frac{1}{2} \Rightarrow \frac{4 \cdot a \cdot b}{a^2 - b^2} = \frac{1}{2} \ / \cdot \frac{1}{4} \Rightarrow \frac{a \cdot b}{a^2 - b^2} = \frac{1}{8}.$$

Tada je

$$\frac{a}{b} - \frac{b}{a} = \frac{a^2 - b^2}{a \cdot b} = \left(\frac{a \cdot b}{a^2 - b^2}\right)^{-1} = \left[\frac{a \cdot b}{a^2 - b^2} = \frac{1}{8}\right] = \left(\frac{1}{8}\right)^{-1} = 8.$$

Odgovor je pod D.

2.inačica

Preoblikujemo zadanu jednakost.

$$\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{1}{2} \Rightarrow \frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{1}{2} \ / \cdot 2 \cdot (a-b) \cdot (a+b) \Rightarrow$$

$$\Rightarrow 2 \cdot (a+b)^2 - 2 \cdot (a-b)^2 = (a-b) \cdot (a+b) \Rightarrow$$

$$\Rightarrow 2 \cdot \left(a^2 + 2 \cdot a \cdot b + b^2\right) - 2 \cdot \left(a^2 - 2 \cdot a \cdot b + b^2\right) = a^2 - b^2 \Rightarrow$$

$$\Rightarrow 2 \cdot a^2 + 4 \cdot a \cdot b + 2 \cdot b^2 - 2 \cdot a^2 + 4 \cdot a \cdot b - 2 \cdot b^2 = a^2 - b^2 \Rightarrow$$

$$\Rightarrow 2 \cdot a^2 + 4 \cdot a \cdot b + 2 \cdot b^2 - 2 \cdot a^2 + 4 \cdot a \cdot b - 2 \cdot b^2 = a^2 - b^2 \Rightarrow 4 \cdot a \cdot b + 4 \cdot a \cdot b = a^2 - b^2 \Rightarrow$$

$$\Rightarrow 8 \cdot a \cdot b = a^2 - b^2.$$

Tada je

$$\frac{a}{b} - \frac{b}{a} = \frac{a^2 - b^2}{a \cdot b} = \left[a^2 - b^2 = 8 \cdot a \cdot b\right] = \frac{8 \cdot a \cdot b}{a \cdot b} = \frac{8 \cdot a \cdot b}{a \cdot b} = 8.$$

Odgovor je pod D.

Vježba 783

Ako je $\frac{a+b}{a-b}+\frac{b-a}{a+b}=\frac{1}{2}$, onda je $\frac{a}{b}-\frac{b}{a}$ jednako:

$$A.\ \frac{1}{4} \qquad B.\ 2 \qquad C.\ \frac{1}{2} \qquad D.\ 8$$

Rezultat: D.

Zadatak 784 (Miroslav, gimnazija)

Izraz $\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}$ za $a\neq b\neq 0$ identičan je razlomku:

$$A.\ \frac{b^2}{b-a} \qquad B.\ \frac{a^2}{a-b} \qquad C.\ \frac{1}{a^2-b^2} \qquad D.\ \frac{1}{a\cdot b}$$

Rješenje 784

Ponovimo!

$$n=\frac{n}{1}\ ,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\ ,\quad a^1=a\ ,\quad a^n\cdot a^m=a^{n+m}\ ,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}\ ,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\ ,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}=\left(\frac{b}{\frac{1}{1}-\frac{a}{b}}+\frac{a}{\frac{1}{1}-\frac{b}{a}}\right)\cdot\frac{1}{\frac{1}{1}-\frac{a^2}{b^2}}=\left(\frac{b}{\frac{b-a}{b}}+\frac{a}{\frac{a-b}{a}}\right)\cdot\frac{1}{\frac{b^2-a^2}{b^2}}=$$

$$=\left(\frac{\frac{b}{1}}{\frac{b-a}{b}}+\frac{\frac{a}{1}}{\frac{a-b}{a}}\right)\cdot\frac{\frac{1}{1}}{\frac{b^2-a^2}{b^2}}=\left(\frac{b^2}{b-a}+\frac{a^2}{a-b}\right)\cdot\frac{b^2}{b^2-a^2}=\left(\frac{b^2}{b-a}+\frac{a^2}{-(b-a)}\right)\cdot\frac{b^2}{b^2-a^2}=$$

$$=\left(\frac{b^2}{b-a}-\frac{a^2}{b-a}\right)\cdot\frac{b^2}{b^2-a^2}=\frac{b^2-a^2}{b-a}\cdot\frac{b^2}{b^2-a^2}=\frac{b^2-a^2}{b-a}\cdot\frac{b^2}{b^2-a^2}=$$

$$=\frac{1}{b-a}\cdot\frac{b^2}{1}=\frac{b^2}{b-a}.$$

Odgovor je pod A.

Vježba 784

Izraz $\left(\dfrac{b}{\frac{a}{b}-1}+\dfrac{a}{\frac{b}{a}-1}\right)\cdot\dfrac{1}{\frac{a^2}{b^2}-1}$ za $a\neq b\neq 0$ identičan je razlomku:

$$A.\ \frac{b^2}{b-a} \qquad B.\ \frac{a^2}{a-b} \qquad C.\ \frac{1}{a^2-b^2} \qquad D.\ \frac{1}{a\cdot b}$$

Rezultat: A.

Zadatak 785 (4B, TUPŠ)

Izračunajte: $\left(4\cdot x^2+1\right)\cdot\left(1-2\cdot x\right)\cdot\left(1+2\cdot x\right)-\left(x^2-1\right)\cdot\left(x^2+1\right).$

Rješenje 785

Ponovimo!

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \left(a^n\right)^m=a^{n\cdot m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(4\cdot x^2+1\right)\cdot\left(1-2\cdot x\right)\cdot\left(1+2\cdot x\right)-\left(x^2-1\right)\cdot\left(x^2+1\right)=$$

$$=\left(4\cdot x^2+1\right)\cdot\left(1^2-(2\cdot x)^2\right)-\left(\left(x^2\right)^2-1\right)=\left(4\cdot x^2+1\right)\cdot\left(1-4\cdot x^2\right)-\left(x^4-1\right)=$$

$$=\left(1+4\cdot x^2\right)\cdot\left(1-4\cdot x^2\right)-x^4+1=1^2-\left(4\cdot x^2\right)^2-x^4+1=1-16\cdot x^4-x^4+1=2-17\cdot x^4.$$

Vježba 785

Izračunajte: $\left(4\cdot x^2+1\right)\cdot\left(1-2\cdot x\right)\cdot\left(1+2\cdot x\right)+\left(1-x^2\right)\cdot\left(x^2+1\right).$

Rezultat: $2-17\cdot x^4.$

Zadatak 786 (Iva i Ivan, bivši srednjoškolci ☺)

Ako je $\dfrac{a+b}{c}=3$ i $\dfrac{a+1}{b}=2$, koliko je $b-c$?

$$A.\ -3 \qquad B.\ -\frac{1}{3} \qquad C.\ \frac{1}{3} \qquad D.\ 3$$

Rješenje 786

Ponovimo!

$$\left.\begin{matrix}a=b\\c=d\end{matrix}\right\}\ a+c=b+d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left.\begin{array}{l}\frac{a+b}{c}=3\\ \frac{a+1}{b}=2\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{a+b}{c}=3\ /\cdot c\\ \frac{a+1}{b}=2\ /\cdot b\end{array}\right\}\Rightarrow\left.\begin{array}{l}a+b=3\cdot c\\ a+1=2\cdot b\end{array}\right\}\Rightarrow\left.\begin{array}{l}a+b=3\cdot c\\ a=2\cdot b-1\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda}\\ \text{zamjene}\end{array}\right]\Rightarrow$$

$$\Rightarrow\ 2\cdot b-1+b=3\cdot c\ \Rightarrow\ 2\cdot b+b-3\cdot c=1\ \Rightarrow\ 3\cdot b-3\cdot c=1\ \Rightarrow\ 3\cdot(b-c)=1\ \Rightarrow$$

$$\Rightarrow\ 3\cdot(b-c)=1\ /\cdot\frac{1}{3}\ \Rightarrow\ b-c=\frac{1}{3}.$$

Odgovor je pod C.

2.inačica

$$\left.\begin{array}{l}\frac{a+b}{c}=3\\ \frac{a+1}{b}=2\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{a+b}{c}=3\ /\cdot c\\ \frac{a+1}{b}=2\ /\cdot b\end{array}\right\}\Rightarrow\left.\begin{array}{l}a+b=3\cdot c\\ a+1=2\cdot b\end{array}\right\}\Rightarrow\left.\begin{array}{l}a+b=3\cdot c\\ a-2\cdot b=-1\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda suprotnih}\\ \text{koeficijenata}\end{array}\right]\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}a+b=3\cdot c\\ a-2\cdot b=-1\ /\cdot(-1)\end{array}\right\}\Rightarrow\left.\begin{array}{l}a+b=3\cdot c\\ -a+2\cdot b=1\end{array}\right\}\Rightarrow\ 3\cdot b=3\cdot c+1\ \Rightarrow\ 3\cdot b-3\cdot c=1\ \Rightarrow$$

$$\Rightarrow\ 3\cdot(b-c)=1\ \Rightarrow\ 3\cdot(b-c)=1\ /\cdot\frac{1}{3}\ \Rightarrow\ b-c=\frac{1}{3}.$$

Odgovor je pod C.

3.inačica

$$\left.\begin{array}{l}\frac{a+b}{c}=3\\ \frac{a+1}{b}=2\end{array}\right\}\Rightarrow\left.\begin{array}{l}\frac{a+b}{c}=3\ /\cdot c\\ \frac{a+1}{b}=2\ /\cdot b\end{array}\right\}\Rightarrow\left.\begin{array}{l}a+b=3\cdot c\\ a+1=2\cdot b\end{array}\right\}\Rightarrow\left.\begin{array}{l}a=3\cdot c-b\\ a=2\cdot b-1\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda}\\ \text{komparacije}\end{array}\right]\Rightarrow$$

$$\Rightarrow\ 3\cdot c-b=2\cdot b-1\ \Rightarrow\ 2\cdot b-1=3\cdot c-b\ \Rightarrow\ 2\cdot b-3\cdot c+b=1\ \Rightarrow\ 3\cdot b-3\cdot c=1\ \Rightarrow$$

$$\Rightarrow\ 3\cdot(b-c)=1\ \Rightarrow\ 3\cdot(b-c)=1\ /\cdot\frac{1}{3}\ \Rightarrow\ b-c=\frac{1}{3}.$$

Odgovor je pod C.

Vježba 786

Ako je $\frac{a+b}{c}=3$ i $\frac{a+1}{b}=2$, koliko je $c-b$?

$A.\ -3$ $\quad B.\ -\frac{1}{3}$ $\quad C.\ \frac{1}{3}$ $\quad D.\ 3$

Rezultat: B.