Zadatak 781 (4B, TUPŠ)

Rastavi na faktore: $(3\cdot x-2\cdot y)^2+24\cdot x\cdot y.$

Rješenje 781

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$a^2=a\cdot a.$$

$$(3\cdot x-2\cdot y)^2+24\cdot x\cdot y=(3\cdot x)^2-2\cdot 3\cdot x\cdot 2\cdot y+(2\cdot y)^2+24\cdot x\cdot y=$$

$$=9\cdot x^2-12\cdot x\cdot y+4\cdot y^2+24\cdot x\cdot y=9\cdot x^2+12\cdot x\cdot y+4\cdot y^2=$$

$$=(3\cdot x+2\cdot y)^2=(3\cdot x+2\cdot y)\cdot(3\cdot x+2\cdot y).$$

Vježba 781

Rastavi na faktore: $24\cdot x\cdot y+(3\cdot x-2\cdot y)^2.$

Rezultat: $(3\cdot x+2\cdot y)\cdot(3\cdot x+2\cdot y).$

Zadatak 782 (4B, TUPŠ)

Rastavi na faktore: $40\cdot x\cdot y-(2\cdot x+5\cdot y)^2.$

Rješenje 782

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$a^2=a\cdot a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$40\cdot x\cdot y-(2\cdot x+5\cdot y)^2=40\cdot x\cdot y-\left((2\cdot x)^2+2\cdot 2\cdot x\cdot 5\cdot y+(5\cdot y)^2\right)=$$

$$=40\cdot x\cdot y-\left(4\cdot x^2+20\cdot x\cdot y+25\cdot y^2\right)=40\cdot x\cdot y-4\cdot x^2-20\cdot x\cdot y-25\cdot y^2=$$

$$=-4\cdot x^2+20\cdot x\cdot y-25\cdot y^2=-\left(4\cdot x^2-20\cdot x\cdot y+25\cdot y^2\right)=-(2\cdot x-5\cdot y)^2=$$

$$=-(2\cdot x-5\cdot y)\cdot(2\cdot x-5\cdot y).$$

Vježba 782

Rastavi na faktore: $24\cdot x\cdot y-(2\cdot x+3\cdot y)^2.$

Rezultat: $-(2\cdot x-3\cdot y)\cdot(2\cdot x-3\cdot y).$

Zadatak 783 (Miroslav, gimnazija)

Ako je $\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2}$, onda je $\frac{a}{b}-\frac{b}{a}$ jednako:

$A.\ \frac{1}{4} \qquad B.\ 2 \qquad C.\ \frac{1}{2} \qquad D.\ 8$

Rješenje 783

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad n=\frac{n}{1}.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

1.inačica

Preoblikujemo zadanu jednakost.

$$\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2} \Rightarrow \frac{(a+b)^2-(a-b)^2}{(a-b)\cdot(a+b)}=\frac{1}{2} \Rightarrow \frac{((a+b)-(a-b))\cdot((a+b)+(a-b))}{a^2-b^2}=\frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{(a+b-a+b)\cdot(a+b+a-b)}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{(a+b-a+b)\cdot(a+b+a-b)}{a^2-b^2}=\frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{2\cdot b\cdot 2\cdot a}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{4\cdot a\cdot b}{a^2-b^2}=\frac{1}{2} \Rightarrow \frac{4\cdot a\cdot b}{a^2-b^2}=\frac{1}{2} \ /\cdot\frac{1}{4} \Rightarrow \frac{a\cdot b}{a^2-b^2}=\frac{1}{8}.$$

Tada je

$$\frac{a}{b}-\frac{b}{a}=\frac{a^2-b^2}{a\cdot b}=\left(\frac{a\cdot b}{a^2-b^2}\right)^{-1}=\left[\frac{a\cdot b}{a^2-b^2}=\frac{1}{8}\right]=\left(\frac{1}{8}\right)^{-1}=8.$$

Odgovor je pod D.

2.inačica

Preoblikujemo zadanu jednakost.

$$\frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2} \Rightarrow \frac{a+b}{a-b}-\frac{a-b}{a+b}=\frac{1}{2} \ /\cdot 2\cdot(a-b)\cdot(a+b) \Rightarrow$$

$$\Rightarrow 2\cdot(a+b)^2-2\cdot(a-b)^2=(a-b)\cdot(a+b) \Rightarrow$$

$$\Rightarrow 2\cdot\left(a^2+2\cdot a\cdot b+b^2\right)-2\cdot\left(a^2-2\cdot a\cdot b+b^2\right)=a^2-b^2 \Rightarrow$$

$$\Rightarrow 2\cdot a^2+4\cdot a\cdot b+2\cdot b^2-2\cdot a^2+4\cdot a\cdot b-2\cdot b^2=a^2-b^2 \Rightarrow$$

$$\Rightarrow 2\cdot a^2+4\cdot a\cdot b+2\cdot b^2-2\cdot a^2+4\cdot a\cdot b-2\cdot b^2=a^2-b^2 \Rightarrow 4\cdot a\cdot b+4\cdot a\cdot b=a^2-b^2 \Rightarrow$$

$$\Rightarrow 8\cdot a\cdot b=a^2-b^2.$$

Tada je

$$\frac{a}{b}-\frac{b}{a}=\frac{a^2-b^2}{a\cdot b}=\left[a^2-b^2=8\cdot a\cdot b\right]=\frac{8\cdot a\cdot b}{a\cdot b}=\frac{8\cdot a\cdot b}{a\cdot b}=8.$$

Odgovor je pod D.

Vježba 783

Ako je $\frac{a+b}{a-b}+\frac{b-a}{a+b}=\frac{1}{2}$, onda je $\frac{a}{b}-\frac{b}{a}$ jednako:

$$A.\ \frac{1}{4} \qquad B.\ 2 \qquad C.\ \frac{1}{2} \qquad D.\ 8$$

Rezultat: D.

Zadatak 784 (Miroslav, gimnazija)

Izraz $\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}$ za $a\neq b\neq 0$ identičan je razlomku:

$$A.\ \frac{b^2}{b-a} \qquad B.\ \frac{a^2}{a-b} \qquad C.\ \frac{1}{a^2-b^2} \qquad D.\ \frac{1}{a\cdot b}$$

Rješenje 784

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{a}{n}-\frac{b}{n}=\frac{a-b}{n} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}=\left(\frac{b}{\frac{1}{1}-\frac{a}{b}}+\frac{a}{\frac{1}{1}-\frac{b}{a}}\right)\cdot\frac{1}{\frac{1}{1}-\frac{a^2}{b^2}}=\left(\frac{b}{\frac{b-a}{b}}+\frac{a}{\frac{a-b}{a}}\right)\cdot\frac{1}{\frac{b^2-a^2}{b^2}}=$$

$$=\left(\frac{\frac{b}{1}}{\frac{b-a}{b}}+\frac{\frac{a}{1}}{\frac{a-b}{a}}\right)\cdot\frac{\frac{1}{1}}{\frac{b^2-a^2}{b^2}}=\left(\frac{b^2}{b-a}+\frac{a^2}{a-b}\right)\cdot\frac{b^2}{b^2-a^2}=\left(\frac{b^2}{b-a}+\frac{a^2}{-(b-a)}\right)\cdot\frac{b^2}{b^2-a^2}=$$

$$=\left(\frac{b^2}{b-a}-\frac{a^2}{b-a}\right)\cdot\frac{b^2}{b^2-a^2}=\frac{b^2-a^2}{b-a}\cdot\frac{b^2}{b^2-a^2}=\frac{b^2-a^2}{b-a}\cdot\frac{b^2}{b^2-a^2}=$$

$$=\frac{1}{b-a}\cdot\frac{b^2}{1}=\frac{b^2}{b-a}.$$

Odgovor je pod A.

Vježba 784

Izraz $\left(\frac{b}{\frac{a}{b}-1}+\frac{a}{\frac{b}{a}-1}\right)\cdot\frac{1}{\frac{a^2}{b^2}-1}$ za $a \neq b \neq 0$ identičan je razlomku:

$A.\ \frac{b^2}{b-a}$ $\quad B.\ \frac{a^2}{a-b}$ $\quad C.\ \frac{1}{a^2-b^2}$ $\quad D.\ \frac{1}{a\cdot b}$

Rezultat: A.

Zadatak 785 (4B, TUPŠ)

Izračunajte: $\left(4\cdot x^2+1\right)\cdot(1-2\cdot x)\cdot(1+2\cdot x)-\left(x^2-1\right)\cdot\left(x^2+1\right).$

Rješenje 785

Ponovimo!

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \left(a^n\right)^m=a^{n\cdot m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(4\cdot x^2+1\right)\cdot(1-2\cdot x)\cdot(1+2\cdot x)-\left(x^2-1\right)\cdot\left(x^2+1\right)=$$

$$=\left(4\cdot x^2+1\right)\cdot\left(1^2-(2\cdot x)^2\right)-\left(\left(x^2\right)^2-1\right)=\left(4\cdot x^2+1\right)\cdot\left(1-4\cdot x^2\right)-\left(x^4-1\right)=$$

$$=\left(1+4\cdot x^2\right)\cdot\left(1-4\cdot x^2\right)-x^4+1=1^2-\left(4\cdot x^2\right)^2-x^4+1=1-16\cdot x^4-x^4+1=2-17\cdot x^4.$$

Vježba 785

Izračunajte: $\left(4\cdot x^2+1\right)\cdot(1-2\cdot x)\cdot(1+2\cdot x)+\left(1-x^2\right)\cdot\left(x^2+1\right).$

Rezultat: $2-17\cdot x^4.$

Zadatak 786 (Iva i Ivan, bivši srednjoškolci 😊)

Ako je $\frac{a+b}{c}=3$ i $\frac{a+1}{b}=2$, koliko je $b-c$?

$A.\ -3$ $\quad B.\ -\frac{1}{3}$ $\quad C.\ \frac{1}{3}$ $\quad D.\ 3$

Rješenje 786

Ponovimo!

$$\left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \ a+c=b+d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left.\begin{array}{l}\frac{a+b}{c}=3\\ \frac{a+1}{b}=2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\frac{a+b}{c}=3\ /\cdot c\\ \frac{a+1}{b}=2\ /\cdot b\end{array}\right\} \Rightarrow \left.\begin{array}{l}a+b=3\cdot c\\ a+1=2\cdot b\end{array}\right\} \Rightarrow \left.\begin{array}{l}a+b=3\cdot c\\ a=2\cdot b-1\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda}\\ \text{zamjene}\end{array}\right] \Rightarrow$$

$$\Rightarrow 2\cdot b-1+b=3\cdot c \Rightarrow 2\cdot b+b-3\cdot c=1 \Rightarrow 3\cdot b-3\cdot c=1 \Rightarrow 3\cdot(b-c)=1 \Rightarrow$$

$$\Rightarrow 3\cdot(b-c)=1\ /\cdot\frac{1}{3} \Rightarrow b-c=\frac{1}{3}.$$

Odgovor je pod C.

2.inačica

$$\left.\begin{array}{l}\frac{a+b}{c}=3\\ \frac{a+1}{b}=2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\frac{a+b}{c}=3\ /\cdot c\\ \frac{a+1}{b}=2\ /\cdot b\end{array}\right\} \Rightarrow \left.\begin{array}{l}a+b=3\cdot c\\ a+1=2\cdot b\end{array}\right\} \Rightarrow \left.\begin{array}{l}a+b=3\cdot c\\ a-2\cdot b=-1\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda suprotnih}\\ \text{koeficijenata}\end{array}\right] \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}a+b=3\cdot c\\ a-2\cdot b=-1\ /\cdot(-1)\end{array}\right\} \Rightarrow \left.\begin{array}{l}a+b=3\cdot c\\ -a+2\cdot b=1\end{array}\right\} \Rightarrow 3\cdot b=3\cdot c+1 \Rightarrow 3\cdot b-3\cdot c=1 \Rightarrow$$

$$\Rightarrow 3\cdot(b-c)=1 \Rightarrow 3\cdot(b-c)=1\ /\cdot\frac{1}{3} \Rightarrow b-c=\frac{1}{3}.$$

Odgovor je pod C.

3.inačica

$$\left.\begin{array}{l}\frac{a+b}{c}=3\\ \frac{a+1}{b}=2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\frac{a+b}{c}=3\ /\cdot c\\ \frac{a+1}{b}=2\ /\cdot b\end{array}\right\} \Rightarrow \left.\begin{array}{l}a+b=3\cdot c\\ a+1=2\cdot b\end{array}\right\} \Rightarrow \left.\begin{array}{l}a=3\cdot c-b\\ a=2\cdot b-1\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda}\\ \text{komparacije}\end{array}\right] \Rightarrow$$

$$\Rightarrow 3\cdot c-b=2\cdot b-1 \Rightarrow 2\cdot b-1=3\cdot c-b \Rightarrow 2\cdot b-3\cdot c+b=1 \Rightarrow 3\cdot b-3\cdot c=1 \Rightarrow$$

$$\Rightarrow 3\cdot(b-c)=1 \Rightarrow 3\cdot(b-c)=1\ /\cdot\frac{1}{3} \Rightarrow b-c=\frac{1}{3}.$$

Odgovor je pod C.

Vježba 786

Ako je $\frac{a+b}{c}=3$ i $\frac{a+1}{b}=2$, koliko je $c-b$?

$$A.\ -3 \qquad B.\ -\frac{1}{3} \qquad C.\ \frac{1}{3} \qquad D.\ 3$$

Rezultat: B.

Zadatak 787 (Iva, maturantica)

Provedite računske operacije u izrazu $\left(\frac{1}{3\cdot a-b}-\frac{1}{3\cdot a+b}\right)\cdot\left(9\cdot a^2-b^2\right)$ i pojednostavnite ga do kraja za sve a, b za koje je taj izraz definiran.

Rješenje 787

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\frac{n}{1}=n.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

1.inačica

$$\left(\frac{1}{3\cdot a-b}-\frac{1}{3\cdot a+b}\right)\cdot\left(9\cdot a^2-b^2\right)=\frac{3\cdot a+b-(3\cdot a-b)}{(3\cdot a-b)\cdot(3\cdot a+b)}\cdot\frac{9\cdot a^2-b^2}{1}=$$

$$=\frac{3\cdot a+b-3\cdot a+b}{(3\cdot a-b)\cdot(3\cdot a+b)}\cdot\frac{9\cdot a^2-b^2}{1}=\frac{3\cdot a+b-3\cdot a+b}{(3\cdot a-b)\cdot(3\cdot a+b)}\cdot\frac{9\cdot a^2-b^2}{1}=\frac{2\cdot b}{(3\cdot a)^2-b^2}\cdot\frac{9\cdot a^2-b^2}{1}=$$

$$=\frac{2\cdot b}{9\cdot a^2-b^2}\cdot\frac{9\cdot a^2-b^2}{1}=\frac{2\cdot b}{9\cdot a^2-b^2}\cdot\frac{9\cdot a^2-b^2}{1}=\frac{2\cdot b}{1}=2\cdot b.$$

2.inačica

$$\left(\frac{1}{3\cdot a-b}-\frac{1}{3\cdot a+b}\right)\cdot\left(9\cdot a^2-b^2\right)=\left(\frac{1}{3\cdot a-b}-\frac{1}{3\cdot a+b}\right)\cdot\left((3\cdot a)^2-b^2\right)=$$

$$=\left(\frac{1}{3\cdot a-b}-\frac{1}{3\cdot a+b}\right)\cdot(3\cdot a-b)\cdot(3\cdot a+b)=$$

$$=\frac{1}{3\cdot a-b}\cdot(3\cdot a-b)\cdot(3\cdot a+b)-\frac{1}{3\cdot a+b}\cdot(3\cdot a-b)\cdot(3\cdot a+b)=$$

$$=\frac{1}{3\cdot a-b}\cdot(3\cdot a-b)\cdot(3\cdot a+b)-\frac{1}{3\cdot a+b}\cdot(3\cdot a-b)\cdot(3\cdot a+b)=3\cdot a+b-(3\cdot a-b)=$$

$$=3\cdot a+b-3\cdot a+b=3\cdot a+b-3\cdot a+b=2\cdot b.$$

Vježba 787

Provedite računske operacije u izrazu $\left(9\cdot a^2-b^2\right)\cdot\left(\frac{1}{3\cdot a-b}-\frac{1}{3\cdot a+b}\right)$ i pojednostavnite ga do kraja za sve a, b za koje je taj izraz definiran.

Rezultat: $2\cdot b.$

Zadatak 788 (Luka, građevinska škola)

Pojednostavnite: $\frac{e^{3\cdot x}-1}{e^{x}-1}.$

Rješenje 788

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}.$$

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right) \quad , \quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{e^{3\cdot x}-1}{e^{x}-1}=\frac{\left(e^{x}\right)^{3}-1}{e^{x}-1}=\frac{\left(e^{x}-1\right)\cdot\left(\left(e^{x}\right)^{2}+e^{x}\cdot 1+1\right)}{e^{x}-1}=\frac{\left(e^{x}-1\right)\cdot\left(e^{2\cdot x}+e^{x}+1\right)}{e^{x}-1}=$$

$$=\frac{\left(e^{x}-1\right)\cdot\left(e^{2\cdot x}+e^{x}+1\right)}{e^{x}-1}=e^{2\cdot x}+e^{x}+1.$$

Vježba 788

Pojednostavnite: $\frac{e^{3\cdot x}+1}{e^{x}+1}$.

Rezultat: $e^{2\cdot x}-e^{x}+1.$

Zadatak 789 (Đurđica, ekonomska škola)

Rastavi na faktore: $x^{3}+3\cdot x^{2}-4\cdot x.$

Rješenje 789

Ponovimo!

$$a^{1}=a\quad,\quad a^{n}:a^{m}=a^{n-m}\quad,\quad a^{2}-b^{2}=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$x^{3}+3\cdot x^{2}-4\cdot x=x\cdot\left(x^{2}+3\cdot x-4\right)=x\cdot\left(x^{2}-x+4\cdot x-4\right)=\left[\begin{array}{c}\text{metoda}\\ \text{grupiranja}\end{array}\right]=$$

$$=x\cdot\left(\left(x^{2}-x\right)+(4\cdot x-4)\right)=x\cdot(x\cdot(x-1)+4\cdot(x-1))=x\cdot(x-1)\cdot(x+4).$$

2.inačica

$$x^{3}+3\cdot x^{2}-4\cdot x=x\cdot\left(x^{2}+3\cdot x-4\right)=x\cdot\left(x^{2}+3\cdot x-1-3\right)=x\cdot\left(x^{2}-1+3\cdot x-3\right)=$$

$$=\left[\begin{array}{c}\text{metoda}\\ \text{grupiranja}\end{array}\right]=x\cdot\left(\left(x^{2}-1\right)+(3\cdot x-3)\right)=x\cdot((x-1)\cdot(x+1)+3\cdot(x-1))=$$

$$=x\cdot(x-1)\cdot(x+1+3)=x\cdot(x-1)\cdot(x+4).$$

Vježba 789

Rastavi na faktore: $x^{3}-3\cdot x^{2}-4\cdot x.$

Rezultat: $x\cdot(x+1)\cdot(x-4).$

Zadatak 790 (Ivan, tehnička škola)

Izračunajte vrijednost razlomka $\frac{x+2\cdot y}{x-2\cdot y}$, ako je $x^{2}+4\cdot y^{2}=5\cdot x\cdot y$ i $0<x<y.$

Rješenje 790

Ponovimo!

$$a^2+2\cdot a\cdot b+b^2=(a+b)^2 \quad , \quad a^2-2\cdot a\cdot b+b^2=(a-b)^2 \quad , \quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n.$$

$$\left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow \frac{a}{c}=\frac{b}{d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zadanu jednakost preoblikujemo na dva načina.

$$\left.\begin{array}{l} x^2+4\cdot y^2=5\cdot x\cdot y \\ x^2+4\cdot y^2=5\cdot x\cdot y \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2+4\cdot y^2=9\cdot x\cdot y-4\cdot x\cdot y \\ x^2+4\cdot y^2=x\cdot y+4\cdot x\cdot y \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x^2+4\cdot x\cdot y+4\cdot y^2=9\cdot x\cdot y \\ x^2-4\cdot x\cdot y+4\cdot y^2=x\cdot y \end{array}\right\} \Rightarrow \left.\begin{array}{l} (x+2\cdot y)^2=9\cdot x\cdot y \\ (x-2\cdot y)^2=x\cdot y \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{podijelimo} \\ \text{jednakosti} \end{array}\right] \Rightarrow$$

$$\Rightarrow \frac{(x+2\cdot y)^2}{(x-2\cdot y)^2}=\frac{9\cdot x\cdot y}{x\cdot y} \Rightarrow \left(\frac{x+2\cdot y}{x-2\cdot y}\right)^2=\frac{9\cdot x\cdot y}{x\cdot y} \Rightarrow \left(\frac{x+2\cdot y}{x-2\cdot y}\right)^2=9 \Rightarrow$$

$$\Rightarrow \left(\frac{x+2\cdot y}{x-2\cdot y}\right)^2=9 \ /\sqrt{\ } \Rightarrow \frac{x+2\cdot y}{x-2\cdot y}=\pm\sqrt{9} \Rightarrow \frac{x+2\cdot y}{x-2\cdot y}=\pm 3.$$

Vježba 790

Izračunajte vrijednost razlomka $\frac{x-2\cdot y}{x+2\cdot y}$, ako je $x^2+4\cdot y^2=5\cdot x\cdot y$ i $0<x<y$.

Rezultat: $\pm\frac{1}{3}.$

Zadatak 791 (Neven, gimnazija)

Ako je $\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}=0$, tada je $\frac{a}{(b-c)^2}+\frac{b}{(c-a)^2}+\frac{c}{(a-b)^2}=0$. Dokazati!

Rješenje 791

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^n:a^m=a^{n-m} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\frac{0}{n}=0 \ , \ n\neq 0.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Iz

$$\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}=0$$

dobiju se tri jednakosti:

- $$\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}=0 \Rightarrow \frac{a}{b-c}=-\frac{b}{c-a}-\frac{c}{a-b} \Rightarrow \frac{a}{b-c}=\frac{-b\cdot(a-b)-c\cdot(c-a)}{(c-a)\cdot(a-b)} \Rightarrow$$

$$\Rightarrow \frac{a}{b-c}=\frac{-a\cdot b+b^2-c^2+a\cdot c}{(c-a)\cdot(a-b)} \Rightarrow \frac{a}{b-c}=\frac{-a\cdot b+b^2+b\cdot c+a\cdot c-b\cdot c-c^2}{(c-a)\cdot(a-b)} \Rightarrow$$

$$\Rightarrow \frac{a}{b-c}=\frac{-a\cdot b+b^2+b\cdot c+a\cdot c-b\cdot c-c^2}{(c-a)\cdot(a-b)} \Rightarrow \frac{a}{b-c}=\frac{b\cdot(-a+b+c)-c\cdot(-a+b+c)}{(c-a)\cdot(a-b)} \Rightarrow$$

$$\Rightarrow \frac{a}{b-c}=\frac{(-a+b+c)\cdot(b-c)}{(c-a)\cdot(a-b)} \Rightarrow \frac{a}{b-c}=\frac{(-a+b+c)\cdot(b-c)}{(c-a)\cdot(a-b)} \ /\cdot\frac{1}{b-c} \Rightarrow$$

$$\Rightarrow \frac{a}{(b-c)^2}=\frac{-a+b+c}{(c-a)\cdot(a-b)}$$

- $$\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}=0 \Rightarrow \frac{b}{c-a}=-\frac{a}{b-c}-\frac{c}{a-b} \Rightarrow \frac{b}{c-a}=\frac{-a\cdot(a-b)-c\cdot(b-c)}{(b-c)\cdot(a-b)} \Rightarrow$$

$$\Rightarrow \frac{b}{c-a}=\frac{-a^2+a\cdot b-b\cdot c+c^2}{(b-c)\cdot(a-b)} \Rightarrow \frac{b}{c-a}=\frac{a\cdot c-b\cdot c+c^2-a^2+a\cdot b-a\cdot c}{(b-c)\cdot(a-b)} \Rightarrow$$

$$\Rightarrow \frac{b}{c-a}=\frac{a\cdot c-b\cdot c+c^2-a^2+a\cdot b-a\cdot c}{(b-c)\cdot(a-b)} \Rightarrow \frac{b}{c-a}=\frac{c\cdot(a-b+c)-a\cdot(a-b+c)}{(b-c)\cdot(a-b)} \Rightarrow$$

$$\Rightarrow \frac{b}{c-a}=\frac{(a-b+c)\cdot(c-a)}{(b-c)\cdot(a-b)} \Rightarrow \frac{b}{c-a}=\frac{(a-b+c)\cdot(c-a)}{(b-c)\cdot(a-b)} \ /\cdot\frac{1}{c-a} \Rightarrow$$

$$\Rightarrow \frac{b}{(c-a)^2}=\frac{a-b+c}{(b-c)\cdot(a-b)}$$

- $$\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}=0 \Rightarrow \frac{c}{a-b}=-\frac{a}{b-c}-\frac{b}{c-a} \Rightarrow \frac{c}{a-b}=\frac{-a\cdot(c-a)-b\cdot(b-c)}{(b-c)\cdot(c-a)} \Rightarrow$$

$$\Rightarrow \frac{c}{a-b}=\frac{-a\cdot c+a^2-b^2+b\cdot c}{(b-c)\cdot(c-a)} \Rightarrow \frac{c}{a-b}=\frac{a^2+a\cdot b-a\cdot c-a\cdot b-b^2+b\cdot c}{(b-c)\cdot(c-a)} \Rightarrow$$

$$\Rightarrow \frac{c}{a-b}=\frac{a^2+a\cdot b-a\cdot c-a\cdot b-b^2+b\cdot c}{(b-c)\cdot(c-a)} \Rightarrow \frac{c}{a-b}=\frac{a\cdot(a+b-c)-b\cdot(a+b-c)}{(b-c)\cdot(c-a)} \Rightarrow$$

$$\Rightarrow \frac{c}{a-b}=\frac{(a+b-c)\cdot(a-b)}{(b-c)\cdot(c-a)} \Rightarrow \frac{c}{a-b}=\frac{(a+b-c)\cdot(a-b)}{(b-c)\cdot(c-a)} \ /\cdot\frac{1}{a-b} \Rightarrow$$

$$\Rightarrow \frac{c}{(a-b)^2}=\frac{a+b-c}{(b-c)\cdot(c-a)}.$$

Sada je:

$$\frac{a}{(b-c)^2}+\frac{b}{(c-a)^2}+\frac{c}{(a-b)^2}=\left[\begin{array}{c}\frac{a}{(b-c)^2}=\frac{-a+b+c}{(c-a)\cdot(a-b)}\\ \frac{b}{(c-a)^2}=\frac{a-b+c}{(b-c)\cdot(a-b)}\\ \frac{c}{(a-b)^2}=\frac{a+b-c}{(b-c)\cdot(c-a)}\end{array}\right]=$$

$$=\frac{-a+b+c}{(c-a)\cdot(a-b)}+\frac{a-b+c}{(b-c)\cdot(a-b)}+\frac{a+b-c}{(b-c)\cdot(c-a)}=$$

$$=\frac{(b-c)\cdot(-a+b+c)+(c-a)\cdot(a-b+c)+(a-b)\cdot(a+b-c)}{(c-a)\cdot(a-b)\cdot(b-c)}=$$

$$=\frac{-a\cdot b+b^2+b\cdot c+a\cdot c-b\cdot c-c^2+a\cdot c-b\cdot c+c^2-a^2+a\cdot b-a\cdot c+a^2+a\cdot b-a\cdot c-a\cdot b-b^2+b\cdot c}{(c-a)\cdot(a-b)\cdot(b-c)}=$$

$$=\frac{-a\cdot b+b^2+b\cdot c+a\cdot c-b\cdot c-c^2+a\cdot c-b\cdot c+c^2-a^2+a\cdot b-a\cdot c+a^2+a\cdot b-a\cdot c-a\cdot b-b^2+b\cdot c}{(c-a)\cdot(a-b)\cdot(b-c)}=$$

$$=\frac{0}{(c-a)\cdot(a-b)\cdot(b-c)}=0.$$

Dokaz gotov.

Vježba 791

Ako je $\frac{a}{b-c}-\frac{b}{a-c}-\frac{c}{b-a}=0,$ tada je $\frac{a}{(b-c)^2}+\frac{b}{(c-a)^2}+\frac{c}{(a-b)^2}=0.$ Dokazati!

Rezultat: Dokaz analogan.

Zadatak 792 (Zlatko, srednja škola)

Izračunati: $\left(\frac{1}{2+2\cdot\sqrt{a}}+\frac{1}{2-2\cdot\sqrt{a}}-\frac{a^2+1}{1-a^2}\right)\cdot\left(1+\frac{1}{a}\right).$

Rješenje 792

Ponovimo!

$$a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^n:a^m=a^{n-m}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad\frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad n=\frac{n}{1}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left(\frac{1}{2+2\cdot\sqrt{a}}+\frac{1}{2-2\cdot\sqrt{a}}-\frac{a^2+1}{1-a^2}\right)\cdot\left(1+\frac{1}{a}\right)=\left(\frac{1}{2\cdot\left(1+\sqrt{a}\right)}+\frac{1}{2\cdot\left(1-\sqrt{a}\right)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\left(\frac{1}{1}+\frac{1}{a}\right)=$$

$$=\left(\frac{1-\sqrt{a}+1+\sqrt{a}}{2\cdot\left(1+\sqrt{a}\right)\cdot\left(1-\sqrt{a}\right)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=\left(\frac{1-\sqrt{a}+1+\sqrt{a}}{2\cdot\left(1-\left(\sqrt{a}\right)^2\right)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=$$

$$=\left(\frac{2}{2\cdot(1-a)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=\left(\frac{2}{2\cdot(1-a)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=$$

$$=\left(\frac{1}{1-a}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=\frac{1+a-\left(a^2+1\right)}{(1-a)\cdot(1+a)}\cdot\frac{a+1}{a}=\frac{1+a-a^2-1}{(1-a)\cdot(1+a)}\cdot\frac{a+1}{a}=$$

$$=\frac{1+a-a^2-1}{1-a}\cdot\frac{1}{a}=\frac{a-a^2}{a\cdot(1-a)}=\frac{a\cdot(1-a)}{a\cdot(1-a)}=\frac{a\cdot(1-a)}{a\cdot(1-a)}=1.$$

2.inačica

$$\left(\frac{1}{2+2\cdot\sqrt{a}}+\frac{1}{2-2\cdot\sqrt{a}}-\frac{a^2+1}{1-a^2}\right)\cdot\left(1+\frac{1}{a}\right)=\left(\frac{1}{2\cdot\left(1+\sqrt{a}\right)}+\frac{1}{2\cdot\left(1-\sqrt{a}\right)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\left(\frac{1}{1}+\frac{1}{a}\right)=$$

$$=\left(\frac{1-\sqrt{a}+1+\sqrt{a}}{2\cdot\left(1+\sqrt{a}\right)\cdot\left(1-\sqrt{a}\right)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=\left(\frac{1-\sqrt{a}+1+\sqrt{a}}{2\cdot\left(1-\left(\sqrt{a}\right)^2\right)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=$$

$$=\left(\frac{2}{2\cdot(1-a)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=\left(\frac{2}{2\cdot(1-a)}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=$$

$$=\left(\frac{1}{1-a}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=\frac{a+1}{a\cdot(1-a)}-\frac{\left(a^2+1\right)\cdot(a+1)}{a\cdot(1-a)\cdot(1+a)}=$$

$$=\left(\frac{1}{1-a}-\frac{a^2+1}{(1-a)\cdot(1+a)}\right)\cdot\frac{a+1}{a}=\frac{a+1}{a\cdot(1-a)}-\frac{\left(a^2+1\right)\cdot(a+1)}{a\cdot(1-a)\cdot(1+a)}=\frac{a+1}{a\cdot(1-a)}-\frac{a^2+1}{a\cdot(1-a)}=$$

$$=\frac{a+1}{a\cdot(1-a)}-\frac{a^2+1}{a\cdot(1-a)}=\frac{a+1-\left(a^2+1\right)}{a\cdot(1-a)}=\frac{a+1-a^2-1}{a\cdot(1-a)}=\frac{a+1-a^2-1}{a\cdot(1-a)}=\frac{a-a^2}{a\cdot(1-a)}=$$

$$=\frac{a\cdot(1-a)}{a\cdot(1-a)}=\frac{a\cdot(1-a)}{a\cdot(1-a)}=1.$$

Vježba 792

Izračunati: $\left(\frac{1}{2+2\cdot\sqrt{a}}+\frac{1}{2-2\cdot\sqrt{a}}+\frac{a^{2}+1}{a^{2}-1}\right)\cdot\left(1+\frac{1}{a}\right).$

Rezultat: 1.

Zadatak 793 (Igor, srednja škola)

Izraz $\frac{1}{1-b}:\frac{a}{b}-\frac{1}{a-a\cdot b}$ napišite kao jedan do kraja skraćen razlomak za sve a, b za koje je taj izraz definiran.

Rješenje 793

Ponovimo!

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{1}{1-b}:\frac{a}{b}-\frac{1}{a-a\cdot b}=\frac{1}{1-b}\cdot\frac{b}{a}-\frac{1}{a\cdot(1-b)}=\frac{b}{a\cdot(1-b)}-\frac{1}{a\cdot(1-b)}=\frac{b-1}{a\cdot(1-b)}=$$

$$=\frac{-(1-b)}{a\cdot(1-b)}=-\frac{1-b}{a\cdot(1-b)}=-\frac{1-b}{a\cdot(1-b)}=-\frac{1}{a}.$$

2.inačica

$$\frac{1}{1-b}:\frac{a}{b}-\frac{1}{a-a\cdot b}=\frac{1}{1-b}\cdot\frac{b}{a}-\frac{1}{a\cdot(1-b)}=\frac{1}{1-b}\cdot\left(\frac{b}{a}-\frac{1}{a}\right)=\frac{1}{1-b}\cdot\frac{b-1}{a}=$$

$$=\frac{1}{1-b}\cdot\frac{-(1-b)}{a}=\frac{1}{1-b}\cdot\frac{-(1-b)}{a}=-\frac{1}{a}.$$

Vježba 793

Izraz $\frac{1}{1-b}:\frac{a}{b}+\frac{1}{a\cdot b-a}$ napišite kao jedan do kraja skraćen razlomak za sve a, b za koje je taj izraz definiran.

Rezultat: $-\frac{1}{a}.$

Zadatak 794 (Blek, srednja škola)

Skrati razlomak: $\frac{a^{2}-1}{5+5\cdot a}$ $(a\neq -1).$

Rješenje 794

Ponovimo!

$$a^{2}-b^{2}=(a-b)\cdot(a+b).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad ,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{a^2-1}{5+5\cdot a}=\frac{(a-1)\cdot(a+1)}{5\cdot(1+a)}=\frac{(a-1)\cdot(a+1)}{5\cdot(a+1)}=\frac{(a-1)\cdot(a+1)}{5\cdot(a+1)}=\frac{a-1}{5}.$$

Vježba 794

Skrati razlomak: $\dfrac{a^2-1}{3+3\cdot a}\ (a\neq -1).$

Rezultat: $\dfrac{a-1}{3}.$

Zadatak 795 (TNT, gimnazija)

Izvedi operacije: $\left[\dfrac{(a+b)^3}{3\cdot a\cdot b}-a-b\right]^n:\left[\dfrac{(a-b)^2}{a\cdot b}+1\right]^n.$

Rješenje 795

Ponovimo!

$$n=\frac{n}{1}\quad ,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad ,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$(a+b)^3=a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3\quad ,\quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$a^n:b^n=(a:b)^n\quad ,\quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}\quad ,\quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left[\frac{(a+b)^3}{3\cdot a\cdot b}-a-b\right]^n:\left[\frac{(a-b)^2}{a\cdot b}+1\right]^n=\left[\frac{(a+b)^3}{3\cdot a\cdot b}-\frac{a}{1}-\frac{b}{1}\right]^n:\left[\frac{(a-b)^2}{a\cdot b}+\frac{1}{1}\right]^n=$$

$$=\left[\frac{(a+b)^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2}{3\cdot a\cdot b}\right]^n:\left[\frac{(a-b)^2+a\cdot b}{a\cdot b}\right]^n=$$

$$=\left[\frac{a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2}{3\cdot a\cdot b}\right]^n:\left[\frac{a^2-2\cdot a\cdot b+b^2+a\cdot b}{a\cdot b}\right]^n=$$

$$=\left[\frac{a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2}{3\cdot a\cdot b}\right]^n:\left[\frac{a^2-a\cdot b+b^2}{a\cdot b}\right]^n=$$

$$=\left[\frac{a^3+b^3}{3\cdot a\cdot b}\right]^n:\left[\frac{a^2-a\cdot b+b^2}{a\cdot b}\right]^n=\left[\frac{a^3+b^3}{3\cdot a\cdot b}:\frac{a^2-a\cdot b+b^2}{a\cdot b}\right]^n=$$

$$=\left[\frac{a^3+b^3}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-a\cdot b+b^2}\right]^n=\left[\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-a\cdot b+b^2}\right]^n=$$

$$=\left[\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-a\cdot b+b^2}\right]^n=\left[\frac{a+b}{3}\cdot\frac{1}{1}\right]^n=\left[\frac{a+b}{3}\right]^n.$$

Vježba 795

Izvedi operacije: $\left[\frac{(a+b)^3}{3\cdot a\cdot b}-a-b\right]^n:\left[1+\frac{(a-b)^2}{a\cdot b}\right]^n.$

Rezultat: $\left[\frac{a+b}{3}\right]^n.$

Zadatak 796 (Ivan, gimnazija)

Neka su x, y i z realni brojevi takvi da je $x+y+z=x\cdot y\cdot z.$ Dokažite da vrijedi identitet
$x\cdot\left(1-y^2\right)\cdot\left(1-z^2\right)+y\cdot\left(1-z^2\right)\cdot\left(1-x^2\right)+z\cdot\left(1-x^2\right)\cdot\left(1-y^2\right)=4\cdot x\cdot y\cdot z.$

Rješenje 796

Ponovimo!

$$a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^n:a^m=a^{n-m}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Preoblikujemo lijevu stranu jednakosti.

$$x\cdot\left(1-y^2\right)\cdot\left(1-z^2\right)+y\cdot\left(1-z^2\right)\cdot\left(1-x^2\right)+z\cdot\left(1-x^2\right)\cdot\left(1-y^2\right)=$$

$$=x\cdot\left(1-z^2-y^2+y^2\cdot z^2\right)+y\cdot\left(1-x^2-z^2+z^2\cdot x^2\right)+z\cdot\left(1-y^2-x^2+x^2\cdot y^2\right)=$$

$$=x-x\cdot z^2-x\cdot y^2+x\cdot y^2\cdot z^2+y-y\cdot x^2-y\cdot z^2+y\cdot z^2\cdot x^2+z-z\cdot y^2-z\cdot x^2+z\cdot x^2\cdot y^2=$$

$$=\left[\begin{array}{l}\text{metoda}\\\text{grupiranja}\end{array}\right]=(x+y+z)+\left(z\cdot x^2\cdot y^2-x^2\cdot y-x\cdot y^2\right)+\left(x\cdot y^2\cdot z^2-z^2\cdot y-z\cdot y^2\right)+$$

$$+\left(y\cdot z^2\cdot x^2-x\cdot z^2-z\cdot x^2\right)=$$

$$=(x+y+z)+x\cdot y\cdot(z\cdot x\cdot y-x-y)+y\cdot z\cdot(x\cdot y\cdot z-z-y)+x\cdot z\cdot(y\cdot z\cdot x-z-x)=$$

$$=(x+y+z)+x\cdot y\cdot(x\cdot y\cdot z-x-y)+y\cdot z\cdot(x\cdot y\cdot z-y-z)+z\cdot x\cdot(x\cdot y\cdot z-x-z)=$$

$$=\left[\begin{array}{c}\text{uvjet}\\ x+y+z=x\cdot y\cdot z\\ z=x\cdot y\cdot z-x-y\\ x=x\cdot y\cdot z-y-z\\ y=x\cdot y\cdot z-x-z\end{array}\right]=x\cdot y\cdot z+x\cdot y\cdot z+y\cdot z\cdot x+z\cdot x\cdot y=$$

$$=x\cdot y\cdot z+x\cdot y\cdot z+x\cdot y\cdot z+x\cdot y\cdot z=4\cdot x\cdot y\cdot z.$$

Vježba 796

Neka su x, y i z realni brojevi takvi da je $x+y+z=x\cdot y\cdot z.$ Dokažite da vrijedi identitet $x\cdot\left(y^2-1\right)\cdot\left(z^2-1\right)+y\cdot\left(z^2-1\right)\cdot\left(x^2-1\right)+z\cdot\left(x^2-1\right)\cdot\left(y^2-1\right)=4\cdot x\cdot y\cdot z.$

Rezultat: Dokaz analogan.