Zadatak 061 (Ivan, Alen, studenti)

Odredite graničnu vrijednost (limes): $\lim\limits_{x\to\pi}\dfrac{\sqrt{1-tg\,x}-\sqrt{1+tg\,x}}{\sin 2x}$.

Rješenje 061

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)=a-b\quad,\quad\sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha.$$

$$\lim_{x\to\pi}\frac{\sqrt{1-tg\,x}-\sqrt{1+tg\,x}}{\sin 2x}=\left[\begin{array}{l}\text{racionaliziramo}\\ \text{brojnik}\end{array}\right]=\lim_{x\to\pi}\frac{\sqrt{1-tg\,x}-\sqrt{1+tg\,x}}{\sin 2x}\cdot\frac{\sqrt{1-tg\,x}+\sqrt{1+tg\,x}}{\sqrt{1-tg\,x}+\sqrt{1+tg\,x}}=$$

$$=\lim_{x\to\pi}\frac{\left(\sqrt{1-tg\,x}\right)^2-\left(\sqrt{1+tg\,x}\right)^2}{\sin 2x\cdot\left(\sqrt{1-tg\,x}+\sqrt{1+tg\,x}\right)}=\lim_{x\to\pi}\frac{1-tg\,x-(1+tg\,x)}{\sin 2x\cdot\left(\sqrt{1-tg\,x}+\sqrt{1+tg\,x}\right)}=$$

$$=\lim_{x\to\pi}\frac{1-tg\,x-1-tg\,x}{\sin 2x\cdot\left(\sqrt{1-tg\,x}+\sqrt{1+tg\,x}\right)}=\lim_{x\to\pi}\frac{-2\cdot tg\,x}{\sin 2x\cdot\left(\sqrt{1-tg\,x}+\sqrt{1+tg\,x}\right)}=$$

$$=\lim_{x\to\pi}\frac{-2\cdot tg\,x}{2\cdot\sin x\cdot\cos x\cdot\left(\sqrt{1-tg\,x}+\sqrt{1+tg\,x}\right)}=\lim_{x\to\pi}\frac{-\,tg\,x}{\sin x\cdot\cos x\cdot\left(\sqrt{1-tg\,x}+\sqrt{1+tg\,x}\right)}=$$

$$=\lim_{x\to\pi}\frac{-\dfrac{\sin x}{\cos x}}{\sin x\cdot\cos x\cdot\left(\sqrt{1-tg\,x}+\sqrt{1+tg\,x}\right)}=\lim_{x\to\pi}\frac{-1}{\cos^2 x\cdot\left(\sqrt{1-tg\,x}+\sqrt{1+tg\,x}\right)}=$$

$$=\frac{-1}{\cos^2\pi\cdot\left(\sqrt{1-tg\,\pi}+\sqrt{1+tg\,\pi}\right)}=\frac{-1}{(-1)^2\cdot\left(\sqrt{1-0}+\sqrt{1+0}\right)}=\frac{-1}{1\cdot\left(\sqrt{1}+\sqrt{1}\right)}=-\frac{1}{2}.$$

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Vježba 061

Odredite graničnu vrijednost (limes): $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x}$.

Rezultat: 5.

Zadatak 062 (Marina, studentica)

Odredite graničnu vrijednost (limes): $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}$.

Rješenje 062

Ponovimo!

$$1-\cos\alpha=2\cdot\sin^2\frac{\alpha}{2}\quad,\quad\lim_{x\to 0}\frac{\sin x}{x}=1\quad,\quad\lim_{x\to 0}\frac{\sin k\cdot x}{k\cdot x}=1\text{ za svako }k\in R\setminus\{0\}.$$

$$\lim_{x\to a}\left(f(x)\cdot g(x)\right)=\lim_{x\to a}f(x)\cdot\lim_{x\to a}g(x).$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=\lim_{x\to 0}\frac{2\cdot\sin^2\dfrac{x}{2}}{x}=\lim_{x\to 0}\frac{2\cdot\sin\dfrac{x}{2}\cdot\sin\dfrac{x}{2}}{x}=\lim_{x\to 0}\left(\sin\frac{x}{2}\cdot\frac{2\cdot\sin\dfrac{x}{2}}{x}\right)=$$

$$=\lim_{x\to 0}\left(\sin\frac{x}{2}\cdot\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)=\lim_{x\to 0}\sin\frac{x}{2}\cdot\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}=\left[\begin{array}{c}\text{supstitucija}\\ \frac{x}{2}=y\\ x\to 0\ \Rightarrow\ y\to 0\end{array}\right]=\lim_{x\to 0}\sin\frac{x}{2}\cdot\lim_{y\to 0}\frac{\sin y}{y}=$$

$$=\sin 0\cdot 1=0\cdot 1=0.$$

Vježba 062

Odredite graničnu vrijednost (limes): $\lim_{x\to 0}\frac{1-\cos x}{x^2}$.

Rezultat: $\frac{1}{2}$.

Zadatak 063 (Ante, student)

Odredite graničnu vrijednost (limes): $\lim_{x\to 1}\frac{\sin(x-1)}{1-x^3}$.

Rješenje 063

Ponovimo!

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right)\quad,\quad \lim_{x\to 0}\frac{\sin x}{x}=1.$$

$$\lim_{x\to h}\frac{\sin(x-h)}{x-h}=\left[\begin{array}{c}\text{supstitucija}\\ x-h=y\\ x\to h\ \Rightarrow\ y\to 0\end{array}\right]=\lim_{y\to 0}\frac{\sin y}{y}=1.$$

$$\lim_{x\to a}\left(f(x)\cdot g(x)\right)=\lim_{x\to a}f(x)\cdot\lim_{x\to a}g(x).$$

$$\lim_{x\to 1}\frac{\sin(x-1)}{1-x^3}=\lim_{x\to 1}\frac{\sin(x-1)}{-\left(x^3-1\right)}=-\lim_{x\to 1}\frac{\sin(x-1)}{x^3-1}=-\lim_{x\to 1}\frac{\sin(x-1)}{(x-1)\cdot\left(x^2+x+1\right)}=$$

$$=-\lim_{x\to 1}\left(\frac{\sin(x-1)}{x-1}\cdot\frac{1}{x^2+x+1}\right)=-\lim_{x\to 1}\frac{\sin(x-1)}{x-1}\cdot\lim_{x\to 1}\frac{1}{x^2+x+1}=$$

$$=\left[\begin{array}{c}\text{supstitucija}\\ x-1=y\\ x\to 1\ \Rightarrow\ y\to 0\end{array}\right]=-\lim_{y\to 0}\frac{\sin y}{y}\cdot\lim_{x\to 1}\frac{1}{x^2+x+1}=-1\cdot\frac{1}{1^2+1+1}=-1\cdot\frac{1}{3}=-\frac{1}{3}.$$

Vježba 063

Odredite graničnu vrijednost (limes): $\lim_{x\to 1}\frac{\sin(x-1)}{x^2-1}$.

Rezultat: $\frac{1}{2}$.

Zadatak 064 (Dado, gimnazija)

Odredite graničnu vrijednost (limes): $\lim_{x\to 1}(1-x)\cdot tg\frac{\pi\cdot x}{2}$.

Rješenje 064

Ponovimo!

$$tg\left(\frac{\pi}{2}-\alpha\right)=ctg\,\alpha.$$

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$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}\ ,\ \lim_{x\to a}g(x)\neq 0.$$

$$\lim_{x\to 1}(1-x)\cdot tg\frac{\pi\cdot x}{2}=\left[\begin{matrix}\text{supstitucija}\\ 1-x=y\ \Rightarrow\ x=1-y\\ x\to 1\ \Rightarrow\ y\to 0\end{matrix}\right]=\lim_{y\to 0}y\cdot tg\frac{\pi\cdot(1-y)}{2}=\lim_{y\to 0}y\cdot tg\frac{\pi-\pi\cdot y}{2}=$$

$$=\lim_{y\to 0}y\cdot tg\left(\frac{\pi}{2}-\frac{\pi}{2}\cdot y\right)=\lim_{y\to 0}y\cdot ctg\frac{\pi}{2}\cdot y=\lim_{y\to 0}y\cdot\frac{\cos\frac{\pi}{2}\cdot y}{\sin\frac{\pi}{2}\cdot y}=\lim_{y\to 0}\frac{\cos\frac{\pi}{2}\cdot y}{\frac{\sin\frac{\pi}{2}\cdot y}{y}}=\frac{\lim_{y\to 0}\cos\frac{\pi}{2}\cdot y}{\lim_{y\to 0}\frac{\sin\frac{\pi}{2}\cdot y}{y}}=$$

$$=\left[\lim_{y\to 0}\cos\frac{\pi}{2}\cdot y=1\right]=\frac{1}{\lim_{y\to 0}\frac{\sin\frac{\pi}{2}\cdot y}{\frac{\pi}{2}\cdot y}\cdot\frac{\pi}{2}}=\left[\begin{matrix}\text{supstitucija}\\ \frac{\pi}{2}\cdot y=t\\ y\to 0\ \Rightarrow\ t\to 0\end{matrix}\right]=\frac{1}{\lim_{t\to 0}\frac{\sin t}{t}\cdot\frac{\pi}{2}}=\frac{1}{1\cdot\frac{\pi}{2}}=\frac{2}{\pi}.$$

Vježba 064

Odredite graničnu vrijednost (limes): $\lim_{x\to 0}\frac{tg\,x-\sin x}{x^3}.$

Rezultat: $\frac{1}{2}.$

Zadatak 065 (Sanja, gimnazija)

Odredite graničnu vrijednost (limes): $\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^x.$

Rješenje 065

Ponovimo!

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\ ,\ \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}\ ,\ \lim_{x\to a}g(x)\neq 0.$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e\ ,\ \lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x=\frac{1}{e}.$$

$$\lim_{x\to\infty}\left(\frac{x+1}{x-1}\right)^x=\left[\begin{matrix}\text{u brojniku i nazivniku}\\ \text{izlučimo } x\end{matrix}\right]=\lim_{x\to\infty}\left(\frac{x\cdot\left(1+\frac{1}{x}\right)}{x\cdot\left(1-\frac{1}{x}\right)}\right)^x=\lim_{x\to\infty}\left(\frac{x\cdot\left(1+\frac{1}{x}\right)}{x\cdot\left(1-\frac{1}{x}\right)}\right)^x=$$

$$=\lim_{x\to\infty}\left(\frac{1+\frac{1}{x}}{1-\frac{1}{x}}\right)^x=\lim_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)^x}{\left(1-\frac{1}{x}\right)^x}=\frac{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x}{\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x}=\frac{e}{\frac{1}{e}}=e^2.$$

Vježba 065

Odredite graničnu vrijednost (limes): $\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^x.$

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Rezultat: $\frac{1}{e^2}.$

Zadatak 066 (Roby, gimnazija)

Odredite graničnu vrijednost (limes): $\lim_{x\to 0}\frac{\ln(1+x)}{x}.$

Rješenje 066

Ponovimo!

$$\lim_{x\to a}\ln f(x)=\ln\lim_{x\to a}f(x)\quad,\quad \ln a^n=n\cdot\ln a\quad,\quad \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e.$$

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}=\lim_{x\to 0}\frac{1}{x}\cdot\ln(1+x)=\lim_{x\to 0}\ln(1+x)^{\frac{1}{x}}=\left[\begin{array}{c}\text{supstitucija}\\ \frac{1}{x}=y\ \Rightarrow\ x=\frac{1}{y}\\ x\to 0\ \Rightarrow\ y\to\infty\end{array}\right]=$$

$$=\lim_{y\to\infty}\ln\left(1+\frac{1}{y}\right)^y=\ln\lim_{y\to\infty}\left(1+\frac{1}{y}\right)^y=\ln e=1.$$

Vježba 066

Odredite graničnu vrijednost (limes): $\lim_{x\to 0}\frac{e^x-1}{x}.$

Rezultat: 1. Naputak: $\lim_{x\to 0}\frac{e^x-1}{x}=\left[\begin{array}{c}\text{supstitucija}\\ e^x-1=y\ \Rightarrow\ e^x=1+y\ \Rightarrow\ x=\ln(1+y)\\ x\to 0\ \Rightarrow\ y\to 0\end{array}\right]=\ldots=1.$

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Zadatak 067 (Marijana, gimnazija)

Odredite graničnu vrijednost (limes): $\lim_{x\to 0}\frac{x}{\sqrt{1-\cos x}}.$

Rješenje 067

Ponovimo!

$$1-\cos\alpha=2\cdot\sin^2\frac{\alpha}{2}\quad,\quad\lim_{x\to 0}\frac{\sin x}{x}=1\quad,\quad\lim_{x\to 0}\frac{\sin k\cdot x}{k\cdot x}=1\ \text{za svako}\ k\in R\setminus\{0\}.$$

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}\quad,\quad\lim_{x\to a}g(x)\neq 0.$$

$$\lim_{x\to 0}\frac{x}{\sqrt{1-\cos x}}=\lim_{x\to 0}\frac{x}{\sqrt{2\cdot\sin^2\frac{x}{2}}}=\lim_{x\to 0}\frac{x}{\sin\frac{x}{2}\cdot\sqrt{2}}=\frac{1}{\sqrt{2}}\cdot\lim_{x\to 0}\frac{x}{\sin\frac{x}{2}}=\frac{1}{\sqrt{2}}\cdot\lim_{x\to 0}\frac{1}{\frac{\sin\frac{x}{2}}{x}}=$$

$$=\frac{1}{\sqrt{2}}\cdot\lim_{x\to 0}\frac{1}{\frac{\sin\frac{x}{2}}{\frac{x}{2}\cdot 2}}=\frac{1}{\sqrt{2}}\cdot\lim_{x\to 0}\frac{1}{\frac{\sin\frac{x}{2}}{\frac{x}{2}\cdot 2}}=\frac{1}{\sqrt{2}}\cdot\lim_{x\to 0}\frac{2}{\frac{\sin\frac{x}{2}}{\frac{x}{2}}}=\frac{1}{\sqrt{2}}\cdot\frac{2}{\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}}=\left[\begin{array}{c}\text{supstitucija}\\ \frac{x}{2}=y\\ x\to 0\ \to\ y\to 0\end{array}\right]=$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{2}{\lim\limits_{y \to 0} \frac{\sin y}{y}} = \frac{1}{\sqrt{2}} \cdot \frac{2}{1} = \frac{2}{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^2}{\sqrt{2}} = \sqrt{2}.$$

Vježba 067

Odredite graničnu vrijednost (limes): $\lim\limits_{x \to 0} \frac{\sqrt{1 - \cos x}}{x}$.

Rezultat: $\frac{\sqrt{2}}{2}$.

Zadatak 068 (Goran, gimnazija)

Odredite graničnu vrijednost (limes): $\lim\limits_{x \to 0} \frac{tg\, x - \sin x}{\sin^3 x}$.

Rješenje 068

Ponovimo!

$$1 - \cos \alpha = 2 \cdot \sin^2 \frac{\alpha}{2} \quad , \quad \sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha.$$

$$\lim_{x \to 0} \frac{tg\, x - \sin x}{\sin^3 x} = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x} = \lim_{x \to 0} \frac{\frac{\sin x - \sin x \cdot \cos x}{\cos x}}{\sin^3 x} = \lim_{x \to 0} \frac{\sin x - \sin x \cdot \cos x}{\cos x \cdot \sin^3 x} =$$

$$= \lim_{x \to 0} \frac{\sin x \cdot (1 - \cos x)}{\cos x \cdot \sin^3 x} = \lim_{x \to 0} \frac{1 - \cos x}{\cos x \cdot \sin^2 x} = \lim_{x \to 0} \frac{2 \cdot \sin^2 \frac{x}{2}}{\cos x \cdot \sin x \cdot \sin x} = \lim_{x \to 0} \frac{2 \cdot \sin \frac{x}{2} \cdot \sin \frac{x}{2}}{\cos x \cdot \sin x \cdot \sin x} =$$

$$= \lim_{x \to 0} \frac{2 \cdot \sin \frac{x}{2} \cdot \sin \frac{x}{2}}{\cos x \cdot 2 \cdot \sin \frac{x}{2} \cdot \cos \frac{x}{2} \cdot 2 \cdot \sin \frac{x}{2} \cdot \cos \frac{x}{2}} = \lim_{x \to 0} \frac{2 \cdot \sin \frac{x}{2} \cdot \sin \frac{x}{2}}{\cos x \cdot 2 \cdot \sin \frac{x}{2} \cdot \cos \frac{x}{2} \cdot 2 \cdot \sin \frac{x}{2} \cdot \cos \frac{x}{2}} =$$

$$= \lim_{x \to 0} \frac{1}{2 \cdot \cos x \cdot \cos \frac{x}{2} \cdot \cos \frac{x}{2}} = \frac{1}{2} \cdot \lim_{x \to 0} \frac{1}{\cos x \cdot \cos^2 \frac{x}{2}} = \frac{1}{2} \cdot \frac{1}{\cos 0 \cdot \cos^2 0} = \frac{1}{2} \cdot \frac{1}{1 \cdot 1} = \frac{1}{2}.$$

Vježba 068

Odredite graničnu vrijednost (limes): $\lim\limits_{x \to 0} \frac{\sin^3 x}{tg\, x - \sin x}$.

Rezultat: 2.

Zadatak 069 (Tony, gimnazija)

Odredite graničnu vrijednost (limes): $\lim\limits_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{tg\, x} \right)$.

Rješenje 069

Ponovimo!

$$1 - \cos \alpha = 2 \cdot \sin^2 \frac{\alpha}{2} \quad , \quad \sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha.$$

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$$\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{tg\ x}\right)=\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{\frac{\sin x}{\cos x}}\right)=\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)=\lim_{x\to 0}\frac{1-\cos x}{\sin x}=$$

$$=\lim_{x\to 0}\frac{2\cdot\sin^2\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\lim_{x\to 0}\frac{2\cdot\sin\frac{x}{2}\cdot\sin\frac{x}{2}}{2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}}=\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\lim_{x\to 0} tg\frac{x}{2}=tg\ 0=0$$

Vježba 069

Odredite graničnu vrijednost (limes): $\lim_{x\to 0}\left(\frac{1}{tg\ x}-\frac{1}{\sin x}\right)$.

Rezultat: 0.

Zadatak 070 (Vedrana, studentica)

Odredite graničnu vrijednost (limes): $\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\cdot tg\ x$.

Rješenje 070

Ponovimo!

$$\lim_{x\to 0}\frac{\sin x}{x}=1\quad,\quad \lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}\ ,\ \lim_{x\to a} g(x)\neq 0.$$

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$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\cdot tg\ x=\left[\begin{array}{c}\text{supstitucija}\\ \frac{\pi}{2}-x=y\ \Rightarrow\ x=\frac{\pi}{2}-y\\ x\to\frac{\pi}{2}\ \Rightarrow\ y\to 0\end{array}\right]=\lim_{y\to 0} y\cdot tg\left(\frac{\pi}{2}-y\right)=\lim_{y\to 0} y\cdot ctg\ y=$$

$$=\lim_{y\to 0} y\cdot\frac{\cos y}{\sin y}=\lim_{y\to 0}\frac{\cos y}{\frac{\sin y}{y}}=\frac{\lim_{y\to 0}\cos y}{\lim_{y\to 0}\frac{\sin y}{y}}=\frac{\cos 0}{1}=\frac{1}{1}=1.$$

Vježba 070

Odredite graničnu vrijednost (limes): $\lim_{x\to\frac{\pi}{2}}\left(x-\frac{\pi}{2}\right)\cdot tg\ x$.

Rezultat: – 1.

Zadatak 071 (Vedrana, studentica)

Odredite graničnu vrijednost (limes): $\lim_{x\to\infty}\frac{3^x-2^x}{5^x-2^x}$.

Rješenje 071

Ponovimo!

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n\quad,\quad \lim_{x\to\infty}\left(\frac{a}{b}\right)^x=0\ \text{za}\ a<b.$$

$$\lim_{x\to a}\left(f(x)\cdot g(x)\right)=\lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x).$$

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}\ ,\ \lim_{x\to a} g(x)\neq 0.$$

$$\lim_{x\to\infty}\frac{3^x-2^x}{5^x-2^x}=\begin{bmatrix}\text{u brojniku izlučimo } 3^x\\ \text{u nazivniku izlučimo } 5^x\end{bmatrix}=\lim_{x\to\infty}\frac{3^x\cdot\left(1-\frac{2^x}{3^x}\right)}{5^x\cdot\left(1-\frac{2^x}{5^x}\right)}=\lim_{x\to\infty}\frac{3^x\cdot\left(1-\left(\frac{2}{3}\right)^x\right)}{5^x\cdot\left(1-\left(\frac{2}{5}\right)^x\right)}=$$

$$=\lim_{x\to\infty}\left(\frac{3}{5}\right)^x\cdot\frac{1-\left(\frac{2}{3}\right)^x}{1-\left(\frac{2}{5}\right)^x}=\lim_{x\to\infty}\left(\frac{3}{5}\right)^x\cdot\lim_{x\to\infty}\frac{1-\left(\frac{2}{3}\right)^x}{1-\left(\frac{2}{5}\right)^x}=\lim_{x\to\infty}\left(\frac{3}{5}\right)^x\cdot\frac{\lim_{x\to\infty}\left(1-\left(\frac{2}{3}\right)^x\right)}{\lim_{x\to\infty}\left(1-\left(\frac{2}{5}\right)^x\right)}=$$

$$=0\cdot\frac{1-0}{1-0}=0.$$

Vježba 071

Odredite graničnu vrijednost (limes): $\lim_{x\to\infty}\frac{2^x-3^x}{2^x-5^x}$.

Rezultat: 0.

Zadatak 072 (Vedrana, studentica)

Odredite graničnu vrijednost (limes): $\lim_{x\to-\infty}\frac{3^x-2^x}{5^x-2^x}$.

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Rješenje 072

Ponovimo!

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n\quad,\quad\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n\quad,\quad\lim_{x\to\infty}\left(\frac{a}{b}\right)^x=0 \text{ za } a<b.$$

$$\lim_{x\to-\infty}\frac{3^x-2^x}{5^x-2^x}=\begin{bmatrix}\text{u brojniku izlučimo } 2^x\\ \text{u nazivniku izlučimo } 2^x\end{bmatrix}=\lim_{x\to-\infty}\frac{2^x\cdot\left(\frac{3^x}{2^x}-1\right)}{2^x\cdot\left(\frac{5^x}{2^x}-1\right)}=\lim_{x\to-\infty}\frac{2^x\cdot\left(\frac{3^x}{2^x}-1\right)}{2^x\cdot\left(\frac{5^x}{2^x}-1\right)}=$$

$$=\lim_{x\to-\infty}\frac{\frac{3^x}{2^x}-1}{\frac{5^x}{2^x}-1}=\lim_{x\to-\infty}\frac{\left(\frac{3}{2}\right)^x-1}{\left(\frac{5}{2}\right)^x-1}=\begin{bmatrix}\text{supstitucija}\\ -x=y\ \Rightarrow\ x=-y\\ x\to-\infty\ \Rightarrow\ y\to\infty\end{bmatrix}=\lim_{y\to\infty}\frac{\left(\frac{3}{2}\right)^{-y}-1}{\left(\frac{5}{2}\right)^{-y}-1}=$$

$$=\lim_{y\to\infty}\frac{\left(\frac{2}{3}\right)^y-1}{\left(\frac{2}{5}\right)^y-1}=\frac{0-1}{0-1}=\frac{-1}{-1}=1.$$

Vježba 072

Odredite graničnu vrijednost (limes): $\lim\limits_{x \to -\infty} \frac{2^x - 3^x}{2^x - 5^x}$.

Rezultat: 1.

Zadatak 073 (Vedrana, studentica)

Odredite graničnu vrijednost (limes): $\lim\limits_{x \to 0} \frac{3^x - 2^x}{5^x - 2^x}$.

Rješenje 073

Ponovimo!

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad , \quad \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} \; , \; \lim_{x \to a} g(x) \neq 0 \quad , \quad \lim_{x \to 0} \frac{a^x - 1}{x} = \ln a.$$

$$\lim_{x \to 0} \frac{3^x - 2^x}{5^x - 2^x} = \left[\begin{array}{l} \text{u brojniku izlučimo } 2^x \\ \text{u nazivniku izlučimo } 2^x \end{array}\right] = \lim_{x \to 0} \frac{2^x \cdot \left(\frac{3^x}{2^x} - 1\right)}{2^x \cdot \left(\frac{5^x}{2^x} - 1\right)} = \lim_{x \to 0} \frac{2^x \cdot \left(\frac{3^x}{2^x} - 1\right)}{2^x \cdot \left(\frac{5^x}{2^x} - 1\right)} =$$

$$= \lim_{x \to 0} \frac{\frac{3^x}{2^x} - 1}{\frac{5^x}{2^x} - 1} = \lim_{x \to 0} \frac{\left(\frac{3}{2}\right)^x - 1}{\left(\frac{5}{2}\right)^x - 1} = \left[\begin{array}{l} \text{brojnik i nazivnik} \\ \text{dijelimo sa } x \end{array}\right] = \lim_{x \to 0} \frac{\frac{\left(\frac{3}{2}\right)^x - 1}{x}}{\frac{\left(\frac{5}{2}\right)^x - 1}{x}} =$$

$$= \frac{\lim\limits_{x \to 0} \frac{\left(\frac{3}{2}\right)^x - 1}{x}}{\lim\limits_{x \to 0} \frac{\left(\frac{5}{2}\right)^x - 1}{x}} = \frac{\ln \frac{3}{2}}{\ln \frac{5}{2}} = \frac{\ln 3 - \ln 2}{\ln 5 - \ln 2}.$$

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Vježba 073

Odredite graničnu vrijednost (limes): $\lim\limits_{x \to 0} \frac{2^x - 3^x}{2^x - 5^x}$.

Rezultat: $\frac{\ln 3 - \ln 2}{\ln 5 - \ln 2}$.

Zadatak 074 (Tea, studentica)

Odredite graničnu vrijednost (limes): $\lim\limits_{x \to 0} \frac{2 \cdot x - \sin 5x}{3 \cdot x - \sin 4x}$.

Rješenje 074

Ponovimo!

$$\frac{a - b}{n} = \frac{a}{n} - \frac{b}{n} \quad , \quad \lim_{x \to 0} \frac{\sin x}{x} = 1 \quad , \quad \lim_{x \to 0} \frac{\sin k \cdot x}{k \cdot x} = 1 \text{ za svako } k \in R \setminus \{0\}.$$

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}\quad,\quad \lim_{x\to a} g(x)\neq 0.$$

$$\lim_{x\to 0}\frac{2\cdot x-\sin 5x}{3\cdot x-\sin 4x}=\left[\begin{array}{l}\text{brojnik i nazivnik}\\ \text{dijelimo sa } x\end{array}\right]=\lim_{x\to 0}\frac{\frac{2\cdot x-\sin 5x}{x}}{\frac{3\cdot x-\sin 4x}{x}}=\lim_{x\to 0}\frac{\frac{2\cdot x}{x}-\frac{\sin 5x}{x}}{\frac{3\cdot x}{x}-\frac{\sin 4x}{x}}=$$

$$=\lim_{x\to 0}\frac{\frac{2\cdot x}{x}-\frac{\sin 5x}{x}}{\frac{3\cdot x}{x}-\frac{\sin 4x}{x}}=\lim_{x\to 0}\frac{2-\frac{\sin 5x}{x}}{3-\frac{\sin 4x}{x}}=\lim_{x\to 0}\frac{2-5\cdot\frac{\sin 5x}{5\cdot x}}{3-4\cdot\frac{\sin 4x}{4\cdot x}}=\frac{2-5\cdot\lim_{x\to 0}\frac{\sin 5x}{5\cdot x}}{3-4\cdot\lim_{x\to 0}\frac{\sin 4x}{4\cdot x}}=$$

$$=\frac{2-5\cdot 1}{3-4\cdot 1}=\frac{2-5}{3-4}=\frac{-3}{-1}=3.$$

Vježba 074

Odredite graničnu vrijednost (limes): $\lim_{x\to 0}\frac{3\cdot x-\sin 4x}{2\cdot x-\sin 5x}$.

Rezultat: $\frac{1}{3}$.

Zadatak 075 (Andro, student)

Odredite graničnu vrijednost (limes): $\lim_{x\to+\infty}\left[\ln(4\cdot x+7)-\ln(2\cdot x+13)+\log 1+\sin\pi\right]$.

Rješenje 075

Ponovimo!

$$\log 1=0\quad,\quad\sin\pi=0\quad,\quad\lim_{n\to\infty}\frac{a_n}{b_n}=\frac{\lim_{n\to\infty}a_n}{\lim_{n\to\infty}b_n}\quad,\quad\lim_{n\to\infty}b_n\neq 0\quad,\quad\lim_{n\to\infty}\frac{1}{n}=0.$$

Ako je funkcija y = f(x) neprekidna u točki a, onda je limes funkcije u točki a jednak funkciji limesa nezavisne varijable x u točki a:

$$\lim_{x\to a} f(x)=f\left(\lim_{x\to a} x\right).$$

Korisno je znati ukoliko egzistira i pozitivno je $\lim_{x\to a} f(x)$, da vrijedi:

$$\lim_{x\to a}\left(\ln f(x)\right)=\ln\left(\lim_{x\to a} f(x)\right).$$

$$\lim_{x\to+\infty}\left[\ln(4\cdot x+7)-\ln(2\cdot x+13)+\log 1+\sin\pi\right]=\lim_{x\to+\infty}\left[\ln(4\cdot x+7)-\ln(2\cdot x+13)+0+0\right]=$$

$$=\lim_{x\to+\infty}\left[\ln(4\cdot x+7)-\ln(2\cdot x+13)\right]=\lim_{x\to+\infty}\ln\frac{4\cdot x+7}{2\cdot x+13}=$$

$$=\left[\lim_{x\to a}\left(\ln f(x)\right)=\ln\left(\lim_{x\to a} f(x)\right)\right]=\ln\left(\lim_{x\to+\infty}\frac{4\cdot x+7}{2\cdot x+13}\right)=\left[\begin{array}{l}\text{brojnik i nazivnik}\\ \text{dijelimo sa } x\end{array}\right]=$$

$$=\ln\left(\lim_{x\to+\infty}\frac{\frac{4\cdot x}{x}+\frac{7}{x}}{\frac{2\cdot x}{x}+\frac{13}{x}}\right)=\ln\left(\lim_{x\to+\infty}\frac{\frac{4\cdot x}{x}+\frac{7}{x}}{\frac{2\cdot x}{x}+\frac{13}{x}}\right)=\ln\left(\lim_{x\to+\infty}\frac{4+\frac{7}{x}}{2+\frac{13}{x}}\right)=$$

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$$= \ln\left(\frac{4+\lim\limits_{x\to+\infty}\frac{7}{x}}{2+\lim\limits_{x\to+\infty}\frac{13}{x}}\right) = \ln\left(\frac{4+7\cdot\lim\limits_{x\to+\infty}\frac{1}{x}}{2+13\cdot\lim\limits_{x\to+\infty}\frac{1}{x}}\right) = \ln\left(\frac{4+7\cdot 0}{2+13\cdot 0}\right) = \ln\frac{4}{2} = \ln 2.$$

Vježba 075

Odredite graničnu vrijednost (limes): $\lim\limits_{x\to+\infty}\left[\ln(6\cdot x+5)-\ln(6\cdot x+7)+\log 1+\sin\pi\right].$

Rezultat: ln 2.

Zadatak 076 (Zvjezdana, studentica)

Odredite graničnu vrijednost (limes): $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+a\cdot x+c}+\sqrt{x^2+b\cdot x+d}-2\cdot x\right).$

Rješenje 076

Ponovimo!

$$\lim_{x\to a}\left(f(x)+g(x)\right)=\lim_{x\to a}f(x)+\lim_{x\to a}g(x) \quad , \quad \lim_{n\to\infty}\frac{c}{n}=0 \quad , \quad \lim_{n\to\infty}\frac{c}{n^2}=0.$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n} \quad , \quad \frac{\sqrt{a}}{b}=\sqrt{\frac{a}{b^2}}.$$

$$\lim_{n\to\infty}\frac{a_n}{b_n}=\frac{\lim\limits_{n\to\infty}a_n}{\lim\limits_{n\to\infty}b_n} \quad , \quad \lim_{n\to\infty}b_n\neq 0.$$

Ako je funkcija y = f(x) neprekidna u točki a, onda je limes funkcije u točki a jednak funkciji limesa nezavisne varijable x u točki a:

$$\lim_{x\to a}f(x)=f\left(\lim_{x\to a}x\right).$$

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Računamo limes:

$$\lim_{x\to+\infty}\left(\sqrt{x^2+a\cdot x+c}+\sqrt{x^2+b\cdot x+d}-2\cdot x\right)=$$

$$=\lim_{x\to+\infty}\left(\sqrt{x^2+a\cdot x+c}-x+\sqrt{x^2+b\cdot x+d}-x\right)=$$

$$=\lim_{x\to+\infty}\left(\left(\sqrt{x^2+a\cdot x+c}-x\right)+\left(\sqrt{x^2+b\cdot x+d}-x\right)\right)=$$

$$=\lim_{x\to+\infty}\left(\sqrt{x^2+a\cdot x+c}-x\right)+\lim_{x\to+\infty}\left(\sqrt{x^2+b\cdot x+d}-x\right)=\left[\begin{matrix}\text{racionalizacija}\\ \text{brojnika svakog limesa}\end{matrix}\right]=$$

$$=\lim_{x\to+\infty}\left(\sqrt{x^2+a\cdot x+c}-x\right)\cdot\frac{\sqrt{x^2+a\cdot x+c}+x}{\sqrt{x^2+a\cdot x+c}+x}+\lim_{x\to+\infty}\left(\sqrt{x^2+b\cdot x+d}-x\right)\cdot\frac{\sqrt{x^2+b\cdot x+d}+x}{\sqrt{x^2+b\cdot x+d}+x}=$$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+a\cdot x+c}\right)^2-x^2}{\sqrt{x^2+a\cdot x+c}+x}+\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+b\cdot x+d}\right)^2-x^2}{\sqrt{x^2+b\cdot x+d}+x}=$$

$$=\lim_{x\to+\infty}\frac{x^2+a\cdot x+c-x^2}{\sqrt{x^2+a\cdot x+c}+x}+\lim_{x\to+\infty}\frac{x^2+b\cdot x+d-x^2}{\sqrt{x^2+b\cdot x+d}+x}=$$

$$= \lim_{x\to+\infty} \frac{x^2+a\cdot x+c-x^2}{\sqrt{x^2+a\cdot x+c}+x} + \lim_{x\to+\infty}\frac{x^2+b\cdot x+d-x^2}{\sqrt{x^2+b\cdot x+d}+x} =$$

$$= \lim_{x\to+\infty} \frac{a\cdot x+c}{\sqrt{x^2+a\cdot x+c}+x} + \lim_{x\to+\infty}\frac{b\cdot x+d}{\sqrt{x^2+b\cdot x+d}+x} = \left[\text{brojnik i nazivnik svakog razlomka podijelimo s } x\right] =$$

$$= \lim_{x\to+\infty} \frac{\frac{a\cdot x+c}{x}}{\frac{\sqrt{x^2+a\cdot x+c}+x}{x}} + \lim_{x\to+\infty}\frac{\frac{b\cdot x+d}{x}}{\frac{\sqrt{x^2+b\cdot x+d}+x}{x}} =$$

$$= \lim_{x\to+\infty} \frac{\frac{a\cdot x}{x}+\frac{c}{x}}{\frac{\sqrt{x^2+a\cdot x+c}}{x}+\frac{x}{x}} + \lim_{x\to+\infty}\frac{\frac{b\cdot x}{x}+\frac{d}{x}}{\frac{\sqrt{x^2+b\cdot x+d}}{x}+\frac{x}{x}} =$$

$$= \lim_{x\to+\infty} \frac{\frac{a\cdot x}{x}+\frac{c}{x}}{\frac{\sqrt{x^2+a\cdot x+c}}{x}+\frac{x}{x}} + \lim_{x\to+\infty}\frac{\frac{b\cdot x}{x}+\frac{d}{x}}{\frac{\sqrt{x^2+b\cdot x+d}}{x}+\frac{x}{x}} =$$

$$= \lim_{x\to+\infty} \frac{a+\frac{c}{x}}{\frac{\sqrt{x^2+a\cdot x+c}}{x}+1} + \lim_{x\to+\infty}\frac{b+\frac{d}{x}}{\frac{\sqrt{x^2+b\cdot x+d}}{x}+1} =$$

$$= \lim_{x\to+\infty} \frac{a+\frac{c}{x}}{\sqrt{\frac{x^2+a\cdot x+c}{x^2}}+1} + \lim_{x\to+\infty}\frac{b+\frac{d}{x}}{\sqrt{\frac{x^2+b\cdot x+d}{x^2}}+1} =$$

$$= \lim_{x\to+\infty} \frac{a+\frac{c}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{a\cdot x}{x^2}+\frac{c}{x^2}}+1} + \lim_{x\to+\infty}\frac{b+\frac{d}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{b\cdot x}{x^2}+\frac{d}{x^2}}+1} =$$

$$= \lim_{x\to+\infty} \frac{a+\frac{c}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{a\cdot x}{x^2}+\frac{c}{x^2}}+1} + \lim_{x\to+\infty}\frac{b+\frac{d}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{b\cdot x}{x^2}+\frac{d}{x^2}}+1} =$$

$$= \lim_{x\to+\infty} \frac{a+\frac{c}{x}}{\sqrt{1+\frac{a}{x}+\frac{c}{x^2}}+1} + \lim_{x\to+\infty}\frac{b+\frac{d}{x}}{\sqrt{1+\frac{b}{x}+\frac{d}{x^2}}+1} =$$

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$$=\frac{a+\lim\limits_{x\to+\infty}\frac{c}{x}}{\sqrt{1+\lim\limits_{x\to+\infty}\frac{a}{x}+\lim\limits_{x\to+\infty}\frac{c}{x^2}}+1}+\frac{b+\lim\limits_{x\to+\infty}\frac{d}{x}}{\sqrt{1+\lim\limits_{x\to+\infty}\frac{b}{x}+\lim\limits_{x\to+\infty}\frac{d}{x^2}}+1}=$$

$$=\frac{a+0}{\sqrt{1+0+0}+1}+\frac{b+0}{\sqrt{1+0+0}+1}=\frac{a}{\sqrt{1}+1}+\frac{b}{\sqrt{1}+1}=\frac{a}{1+1}+\frac{b}{1+1}=\frac{a}{2}+\frac{b}{2}=\frac{a+b}{2}.$$

Vježba 076

Odredite graničnu vrijednost (limes): $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x+c}+\sqrt{x^2+x+d}-2\cdot x\right).$

Rezultat: 1.

Zadatak 077 (Tihomir, student)

Odredite graničnu vrijednost (limes): $\lim\limits_{n\to\infty}\frac{1}{n^2}\cdot\left(1+2+3+4+\ \dots\ +n\right).$

Rješenje 077

Ponovimo!

$$1+2+3+4+\ \dots\ +n=\frac{n\cdot(n+1)}{2}\quad,\quad\lim_{n\to\infty}\frac{1}{n}=0\quad,\quad\lim_{n\to\infty}\frac{a_n}{b_n}=\frac{\lim\limits_{n\to\infty}a_n}{\lim\limits_{n\to\infty}b_n}\ ,\ \lim_{n\to\infty}b_n\neq0.$$

$$\lim_{n\to\infty}(a_n+b_n)=\lim_{n\to\infty}a_n+\lim_{n\to\infty}b_n\quad,\quad\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}\quad,\quad\lim_{n\to\infty}a=a.$$

$$\lim_{n\to\infty}\frac{1}{n^2}\cdot\left(1+2+3+4+\ \dots\ +n\right)=\lim_{n\to\infty}\frac{1}{n^2}\cdot\frac{n\cdot(n+1)}{2}=\lim_{n\to\infty}\frac{1}{n^2}\cdot\frac{n\cdot(n+1)}{2}=$$

$$=\lim_{n\to\infty}\frac{n+1}{2\cdot n}=\left[\begin{matrix}\text{brojnik i nazivnik}\\ \text{podijelimo sa } n\end{matrix}\right]=\lim_{n\to\infty}\frac{\frac{n+1}{n}}{\frac{2\cdot n}{n}}=\lim_{n\to\infty}\frac{\frac{n}{n}+\frac{1}{n}}{\frac{2\cdot n}{n}}=\lim_{n\to\infty}\frac{\frac{n}{n}+\frac{1}{n}}{\frac{2\cdot n}{n}}=$$

$$=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{2}=\frac{\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)}{\lim\limits_{n\to\infty}2}=\frac{\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\frac{1}{n}}{2}=\frac{1+0}{2}=\frac{1}{2}.$$

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Vježba 077

Odredite graničnu vrijednost (limes): $\lim\limits_{n\to\infty}\frac{n^2}{1+2+3+4+\ \dots\ +n}.$

Rezultat: 2.

Zadatak 078 (Tihomir, student)

Odredite graničnu vrijednost (limes): $\lim\limits_{n\to\infty}\left(\frac{1+2+3+4+\ \dots.\ +n}{n+2}-\frac{n}{2}\right).$

Rješenje 078

Ponovimo!

$$1+2+3+4+\ \dots\ +n=\frac{n\cdot(n+1)}{2}\quad,\quad\lim_{n\to\infty}\frac{1}{n}=0\quad,\quad\lim_{n\to\infty}\frac{a_n}{b_n}=\frac{\lim\limits_{n\to\infty}a_n}{\lim\limits_{n\to\infty}b_n}\ ,\ \lim_{n\to\infty}b_n\neq0.$$

$$\lim_{n\to\infty}(a_n+b_n)=\lim_{n\to\infty}a_n+\lim_{n\to\infty}b_n\quad,\quad\lim_{n\to\infty}c\cdot a_n=c\cdot\lim_{n\to\infty}a_n\ ,\ c\ \text{konstanta}.$$

$$\lim_{n\to\infty} a = a.$$

$$\lim_{n\to\infty}\left(\frac{1+2+3+4+\ ....\ +n}{n+2}-\frac{n}{2}\right)=\lim_{n\to\infty}\left(\frac{\frac{n\cdot(n+1)}{2}}{n+2}-\frac{n}{2}\right)=\lim_{n\to\infty}\left(\frac{n\cdot(n+1)}{2\cdot(n+2)}-\frac{n}{2}\right)=$$

$$=\lim_{n\to\infty}\frac{n\cdot(n+1)-n\cdot(n+2)}{2\cdot(n+2)}=\lim_{n\to\infty}\frac{n^2+n-n^2-2\cdot n}{2\cdot(n+2)}=\lim_{n\to\infty}\frac{n^2+n-n^2-2\cdot n}{2\cdot(n+2)}=$$

$$=\lim_{n\to\infty}\frac{-n}{2\cdot(n+2)}=-\frac{1}{2}\cdot\lim_{n\to\infty}\frac{n}{n+2}=\left[\begin{array}{l}\text{brojnik i nazivnik}\\ \text{podijelimo sa } n\end{array}\right]=-\frac{1}{2}\cdot\lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{n+2}{n}}=$$

$$=-\frac{1}{2}\cdot\lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{n}{n}+\frac{2}{n}}=-\frac{1}{2}\cdot\lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{n}{n}+\frac{2}{n}}=-\frac{1}{2}\cdot\lim_{n\to\infty}\frac{1}{1+\frac{2}{n}}=-\frac{1}{2}\cdot\frac{\lim_{n\to\infty}1}{\lim_{n\to\infty}\left(1+\frac{2}{n}\right)}=$$

$$=-\frac{1}{2}\cdot\frac{1}{\lim_{n\to\infty}1+\lim_{n\to\infty}\frac{2}{n}}=-\frac{1}{2}\cdot\frac{1}{1+2\cdot\lim_{n\to\infty}\frac{1}{n}}=-\frac{1}{2}\cdot\frac{1}{1+2\cdot 0}=-\frac{1}{2}\cdot\frac{1}{1+0}=-\frac{1}{2}\cdot\frac{1}{1}=-\frac{1}{2}.$$

Vježba 078

Odredite graničnu vrijednost (limes): $\lim_{n\to\infty}\left(\frac{n}{2}-\frac{1+2+3+4+\ ....\ +n}{n+2}\right).$

Rezultat: $\frac{1}{2}.$

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Zadatak 079 (Mila, studentica)

Odredite graničnu vrijednost (limes): $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\frac{4}{n^2}+\ ...\ +\frac{n}{n^2}\right).$

Rješenje 079

Ponovimo!

$$1+2+3+4+\ ...\ +n=\frac{n\cdot(n+1)}{2}\quad,\quad\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}\quad,\quad\lim_{n\to\infty}\frac{1}{n}=0.$$

$$\lim_{n\to\infty}(a_n+b_n)=\lim_{n\to\infty}a_n+\lim_{n\to\infty}b_n\quad,\quad\lim_{n\to\infty}c\cdot a_n=c\cdot\lim_{n\to\infty}a_n\ ,\ c\text{ konstanta}.$$

$$\lim_{n\to\infty} a = a.$$

$$\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\frac{4}{n^2}+\ ...\ +\frac{n}{n^2}\right)=\lim_{n\to\infty}\frac{1+2+3+4+\ ...\ +n}{n^2}=$$

$$=\lim_{n\to\infty}\frac{\frac{n\cdot(n+1)}{2}}{n^2}=\lim_{n\to\infty}\frac{n\cdot(n+1)}{2\cdot n^2}=\lim_{n\to\infty}\frac{n\cdot(n+1)}{2\cdot n^2}=\lim_{n\to\infty}\frac{n+1}{2\cdot n}=\lim_{n\to\infty}\left(\frac{n}{2\cdot n}+\frac{1}{2\cdot n}\right)=$$

$$=\lim_{n\to\infty}\left(\frac{n}{2\cdot n}+\frac{1}{2\cdot n}\right)=\lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2\cdot n}\right)=\lim_{n\to\infty}\frac{1}{2}+\lim_{n\to\infty}\frac{1}{2\cdot n}=\frac{1}{2}+\frac{1}{2}\cdot\lim_{n\to\infty}\frac{1}{n}=$$

$$=\frac{1}{2}+\frac{1}{2}\cdot 0=\frac{1}{2}+0=\frac{1}{2}.$$

Vježba 079

Odredite graničnu vrijednost (limes): $\lim\limits_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\frac{4}{n^2}+\ ...\ +\frac{n-1}{n^2}\right).$

Rezultat: $\frac{1}{2}.$

Zadatak 080 (Dragan, student)

Odredite graničnu vrijednost (limes): $\lim\limits_{n\to\infty} n\cdot\left(\sqrt{n^2+1}-n\right).$

Rješenje 080

Ponovimo!

$$(a-b)\cdot(a+b)=a^2-b^2\quad,\quad\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}\quad,\quad\lim_{n\to\infty}\frac{1}{n}=0\quad,\quad\lim_{n\to\infty}\frac{1}{n^2}=0\quad,\quad\frac{\sqrt{a}}{b}=\sqrt{\frac{a}{b^2}}.$$

Ako je funkcija y = f(x) neprekidna u točki a, onda je limes funkcije u točki a jednak funkciji limesa nezavisne varijable x u točki a:

$$\lim_{x\to a} f(x)=f\left(\lim_{x\to a} x\right).$$

$$\lim_{n\to\infty} n\cdot\left(\sqrt{n^2+1}-n\right)=\left[\begin{array}{l}\text{racionalizacija}\\ \text{brojnika}\end{array}\right]=\lim_{n\to\infty} n\cdot\left(\sqrt{n^2+1}-n\right)\cdot\frac{\sqrt{n^2+1}+n}{\sqrt{n^2+1}+n}=$$

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$$=\lim_{n\to\infty} n\cdot\frac{\left(\sqrt{n^2+1}-n\right)\cdot\left(\sqrt{n^2+1}+n\right)}{\sqrt{n^2+1}+n}=\lim_{n\to\infty} n\cdot\frac{\left(\sqrt{n^2+1}\right)^2-n^2}{\sqrt{n^2+1}+n}=$$

$$=\lim_{n\to\infty} n\cdot\frac{n^2+1-n^2}{\sqrt{n^2+1}+n}=\lim_{n\to\infty} n\cdot\frac{n^2+1-n^2}{\sqrt{n^2+1}+n}=\lim_{n\to\infty} n\cdot\frac{1}{\sqrt{n^2+1}+n}=\lim_{n\to\infty}\frac{n}{\sqrt{n^2+1}+n}=$$

$$=\left[\begin{array}{l}\text{podijelimo brojnik}\\ \text{i nazivnik sa } n\end{array}\right]=\lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{\sqrt{n^2+1}+n}{n}}=\lim_{n\to\infty}\frac{\frac{n}{n}}{\frac{\sqrt{n^2+1}}{n}+\frac{n}{n}}=\lim_{n\to\infty}\frac{1}{\frac{\sqrt{n^2+1}}{n}+\frac{n}{n}}=$$

$$=\lim_{n\to\infty}\frac{1}{\frac{\sqrt{n^2+1}}{n}+1}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n^2+1}{n^2}}+1}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}+1}=\lim_{n\to\infty}\frac{1}{\sqrt{\frac{n^2}{n^2}+\frac{1}{n^2}}+1}=$$

$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{n^2}}+1}=\frac{1}{\sqrt{1+\lim\limits_{n\to\infty}\frac{1}{n^2}}+1}=\frac{1}{\sqrt{1+0}+1}=\frac{1}{\sqrt{1}+1}=\frac{1}{1+1}=\frac{1}{2}.$$

Vježba 080

Odredite graničnu vrijednost (limes): $\lim\limits_{n\to\infty} n\cdot\left(\sqrt{n^2+1}-\sqrt{n^2-1}\right).$

Rezultat: 0.