Zadatak 121 (Alen, gimnazija)

Izračunajte graničnu vrijednost funkcije $f(x)=\frac{4^x-2^x}{x}$ u točki x = 0.

Rješenje 121

Ponovimo!

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a \quad , \quad \frac{a-b}{n}=\frac{a}{n}-\frac{b}{n} \quad , \quad a^0=1 \quad , \quad \ln\frac{a}{b}=\ln a-\ln b.$$

$$\left(a^n\right)^m=\left(a^m\right)^n=a^{n\cdot m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

Ako postoje $\lim_{x\to a} f(x)$ i $\lim_{x\to a} g(x)$, tada vrijedi poučak:

$$\lim_{x\to a}(f(x)\pm g(x))=\lim_{x\to a} f(x)\pm\lim_{x\to a} g(x).$$

$$\lim_{x\to a}\left(f(x)\cdot g(x)\right)=\lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x).$$

1.inačica

$$\lim_{x\to 0}\frac{4^x-2^x}{x}=\lim_{x\to 0}\frac{4^x-1-2^x+1}{x}=\lim_{x\to 0}\frac{\left(4^x-1\right)-\left(2^x-1\right)}{x}=$$

$$=\lim_{x\to 0}\left(\frac{4^x-1}{x}-\frac{2^x-1}{x}\right)=\lim_{x\to 0}\frac{4^x-1}{x}-\lim_{x\to 0}\frac{2^x-1}{x}=\ln 4-\ln 2=\ln\frac{4}{2}=\ln 2.$$

2.inačica

$$\lim_{x\to 0}\frac{4^x-2^x}{x}=\lim_{x\to 0}\frac{\left(2^2\right)^x-2^x}{x}=\lim_{x\to 0}\frac{\left(2^x\right)^2-2^x}{x}=\lim_{x\to 0}\frac{2^x\cdot\left(2^x-1\right)}{x}=$$

$$=\lim_{x\to 0}\left(2^x\cdot\frac{2^x-1}{x}\right)=\lim_{x\to 0}2^x\cdot\lim_{x\to 0}\frac{2^x-1}{x}=2^0\cdot\ln 2=1\cdot\ln 2=\ln 2.$$

Vježba 121

Izračunajte graničnu vrijednost funkcije $f(x)=\frac{6^x-3^x}{x}$ u točki x = 0.

Rezultat: $\ln 2.$

Zadatak 122 (Tomislav, gimnazija)

Neko se tijelo giba po zakonu $s(t)=4\cdot t-t^2$. Koliki put ovo tijelo prijeđe u vremenu od t_1 = 1 s do t_2 = 1.5 s?

Rješenje 122

Ponovimo!

Neka je $x_0\in D_f$ (domena funkcije f) i Δx prirast varijable x. Prirast funkcije y = f(x) u točki x_0

označavamo s Δy ili Δf(x$_0$), a definiramo ovako:

$$\left.\begin{array}{l} x_1 = x_0 \\ x_2 = x_0 + \Delta x \end{array}\right\} \Rightarrow \Delta y = f\left(x_2\right) - f\left(x_1\right) \quad , \quad \Delta y = f\left(x_0 + \Delta x\right) - f\left(x_0\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Računamo prijeđeni put Δs.

$$\Delta s = s\left(t_2\right) - s\left(t_1\right) \Rightarrow \Delta s = 4 \cdot t_2 - t_2^2 - \left(4 \cdot t_1 - t_1^2\right) \Rightarrow \Delta s = 4 \cdot t_2 - t_2^2 - 4 \cdot t_1 + t_1^2 \Rightarrow$$

$$\Rightarrow \Delta s = 4 \cdot \left(t_2 - t_1\right) + t_1^2 - t_2^2 = \left[\begin{array}{l} t_1 = 1 \\ t_2 = 1.5 \end{array}\right] = 4 \cdot (1.5 - 1) + 1^2 - 1.5^2 = 0.75.$$

Prijeđeni put je 0.75 m.

Vježba 122

Neko se tijelo giba po zakonu $s(t) = 4 \cdot t - t^2$. Koliki put ovo tijelo prijeđe u vremenu od t_1 = 1 s do t_2 = 1.3 s?

Rezultat: 0.51 m.

Zadatak 123 (Tomislav, gimnazija)

Neko se tijelo giba po zakonu $s(t) = 4 \cdot t - t^2$. Odredi srednju brzinu gibanja u vremenu od t_1 = 1 s do t_2 = 1.5 s.

Rješenje 123

Ponovimo!

Neka je $x_0 \in D_f$ (domena funkcije f) i Δx prirast varijable x. Prirast funkcije y = f(x) u točki x_0 označavamo s Δy ili Δf(x$_0$), a definiramo ovako:

$$\left.\begin{array}{l} x_1 = x_0 \\ x_2 = x_0 + \Delta x \end{array}\right\} \Rightarrow \Delta y = f\left(x_2\right) - f\left(x_1\right) \quad , \quad \Delta y = f\left(x_0 + \Delta x\right) - f\left(x_0\right).$$

Prosječna brzina u vremenskom intervalu od t_1 = t do t_2 = t + Δt definira se:

$$\bar{v} = \frac{\Delta s}{\Delta t} \quad , \quad \bar{v} = \frac{s\left(t_2\right) - s\left(t_1\right)}{t_2 - t_1} \quad , \quad \bar{v} = \frac{s(t + \Delta t) - s(t)}{\Delta t},$$

gdje je s put, t vrijeme.

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Prosječna brzina u vremenskom intervalu od t_1 do t_2 iznosi:

$$\bar{v} = \frac{\Delta s}{\Delta t} \Rightarrow \bar{v} = \frac{s\left(t_2\right) - s\left(t_1\right)}{t_2 - t_1} \Rightarrow \bar{v} = \frac{4 \cdot t_2 - t_2^2 - \left(4 \cdot t_1 - t_1^2\right)}{t_2 - t_1} \Rightarrow$$

$$\Rightarrow \bar{v} = \frac{4 \cdot t_2 - t_2^2 - 4 \cdot t_1 + t_1^2}{t_2 - t_1} \Rightarrow \bar{v} = \frac{4 \cdot \left(t_2 - t_1\right) + t_1^2 - t_2^2}{t_2 - t_1} =$$

$$= \left[\begin{array}{l} t_1 = 1 \\ t_2 = 1.5 \end{array}\right] = \frac{4 \cdot (1.5 - 1) + 1^2 - 1.5^2}{1.5 - 1} = 1.5.$$

Vježba 123

Neko se tijelo giba po zakonu $s(t) = 4 \cdot t - t^2$. Odredi srednju brzinu gibanja u vremenu od $t_1 = 1$ s do $t_2 = 1.3$ s.

Rezultat: 1.7 m/s.

Zadatak 124 (Sanja, veleučilište)

Odredite graničnu vrijednost (limes) izraza: $\lim\limits_{n \to \infty} \frac{(n+1) \cdot (3 \cdot n + 2)}{n^2}$.

Rješenje 124

Ponovimo!

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \frac{a+b}{n} = \frac{a}{n} + \frac{b}{n} \quad , \quad \frac{a \cdot b}{c \cdot d} = \frac{a}{c} \cdot \frac{b}{d}.$$

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

Ako su nizovi { a_n } i { b_n } konvergentni, tada vrijede sljedeća pravila o graničnoj vrijednosti (limesu):

$$\lim_{n \to \infty} c = c \quad , \quad c - \text{konstanta} \quad , \quad \lim_{n \to \infty} \frac{c}{n} = 0 \quad , \quad c - \text{konstanta}.$$

$$\lim_{n \to \infty} (c \cdot a_n) = c \cdot \lim_{n \to \infty} a_n \quad , \quad c - \text{konstanta} \quad , \quad \lim_{n \to \infty} (a_n \pm b_n) = \lim_{n \to \infty} a_n \pm \lim_{n \to \infty} b_n.$$

1.inačica

$$\lim_{n \to \infty} \frac{(n+1) \cdot (3 \cdot n + 2)}{n^2} = \lim_{n \to \infty} \frac{3 \cdot n^2 + 2 \cdot n + 3 \cdot n + 2}{n^2} = \lim_{n \to \infty} \frac{3 \cdot n^2 + 5 \cdot n + 2}{n^2} =$$

$$= \lim_{n \to \infty} \left(\frac{3 \cdot n^2}{n^2} + \frac{5 \cdot n}{n^2} + \frac{2}{n^2} \right) = \lim_{n \to \infty} \left(\frac{3 \cdot n^2}{n^2} + \frac{5 \cdot n}{n^2} + \frac{2}{n^2} \right) = \lim_{n \to \infty} \left(3 + \frac{5}{n} + \frac{2}{n^2} \right) =$$

$$= \lim_{n \to \infty} 3 + \lim_{n \to \infty} \frac{5}{n} + \lim_{n \to \infty} \frac{2}{n^2} = 3 + 5 \cdot \lim_{n \to \infty} \frac{1}{n} + 2 \cdot \lim_{n \to \infty} \frac{1}{n^2} = 3 + 5 \cdot 0 + 2 \cdot 0 = 3 + 0 + 0 = 3.$$

2.inačica

$$\lim_{n \to \infty} \frac{(n+1) \cdot (3 \cdot n + 2)}{n^2} = \lim_{n \to \infty} \left(\frac{n+1}{n} \cdot \frac{3 \cdot n + 2}{n} \right) = \lim_{n \to \infty} \frac{n+1}{n} \cdot \lim_{n \to \infty} \frac{3 \cdot n + 2}{n} =$$

$$= \lim_{n \to \infty} \left(\frac{n}{n} + \frac{1}{n} \right) \cdot \lim_{n \to \infty} \left(\frac{3 \cdot n}{n} + \frac{2}{n} \right) = \lim_{n \to \infty} \left(\frac{n}{n} + \frac{1}{n} \right) \cdot \lim_{n \to \infty} \left(\frac{3 \cdot n}{n} + \frac{2}{n} \right) =$$

$$= \lim_{n \to \infty} \left(1 + \frac{1}{n} \right) \cdot \lim_{n \to \infty} \left(3 + \frac{2}{n} \right) = (1+0) \cdot (3+0) = 1 \cdot 3 = 3.$$

Vježba 124

Odredite graničnu vrijednost (limes) izraza: $\lim\limits_{n \to \infty} \frac{(n+4) \cdot (3 \cdot n + 1)}{n^2}$.

Rezultat: 3.

Zadatak 125 (Petar, gimnazija)

Odredite graničnu vrijednost (limes) izraza: $\lim\limits_{n\to\infty}\dfrac{2^n+3^{n+1}}{2^{n+1}+3^{n+1}}$.

Rješenje 125

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

Ako su nizovi { a_n } i { b_n } konvergentni, tada vrijede sljedeća pravila o graničnoj vrijednosti (limesu):

$$\lim_{n\to\infty} c=c \;,\; c-\text{konstanta} \quad , \quad \lim_{n\to\infty}\frac{a_n}{b_n}=\frac{\lim\limits_{n\to\infty} a_n}{\lim\limits_{n\to\infty} b_n} \;,\; \lim_{n\to\infty} b_n\neq 0.$$

$$\lim_{n\to\infty}(c\cdot a_n)=c\cdot\lim_{n\to\infty} a_n \;,\; c-\text{konstanta} \quad , \quad \lim_{n\to\infty}(a_n\pm b_n)=\lim_{n\to\infty} a_n\pm\lim_{n\to\infty} b_n.$$

$$\lim_{n\to\infty} a^n=0,\ \text{za}\ |a|<1.$$

1.inačica

$$\lim_{n\to\infty}\frac{2^n+3^{n+1}}{2^{n+1}+3^{n+1}}=\lim_{n\to\infty}\frac{2^n+3^n\cdot 3^1}{2^n\cdot 2^1+3^n\cdot 3^1}=\lim_{n\to\infty}\frac{2^n+3^n\cdot 3}{2^n\cdot 2+3^n\cdot 3}=$$

$$=\lim_{n\to\infty}\frac{3^n\cdot\left(\frac{2^n}{3^n}+3\right)}{3^n\cdot\left(\frac{2^n}{3^n}\cdot 2+3\right)}=\lim_{n\to\infty}\frac{3^n\cdot\left(\frac{2^n}{3^n}+3\right)}{3^n\cdot\left(\frac{2^n}{3^n}\cdot 2+3\right)}=\lim_{n\to\infty}\frac{\frac{2^n}{3^n}+3}{2\cdot\frac{2^n}{3^n}+3}=\lim_{n\to\infty}\frac{\left(\frac{2}{3}\right)^n+3}{2\cdot\left(\frac{2}{3}\right)^n+3}=$$

$$=\frac{\lim\limits_{n\to\infty}\left(\frac{2}{3}\right)^n+3}{2\cdot\lim\limits_{n\to\infty}\left(\frac{2}{3}\right)^n+3}=\frac{0+3}{2\cdot 0+3}=\frac{3}{3}=\frac{3}{3}=1.$$

2.inačica

$$\lim_{n\to\infty}\frac{2^n+3^{n+1}}{2^{n+1}+3^{n+1}}=\lim_{n\to\infty}\frac{2^{n+1}\cdot 2^{-1}+3^{n+1}}{2^{n+1}+3^{n+1}}=\lim_{n\to\infty}\frac{3^{n+1}\cdot\left(\frac{2^{n+1}}{3^{n+1}}\cdot 2^{-1}+1\right)}{3^{n+1}\cdot\left(\frac{2^{n+1}}{3^{n+1}}+1\right)}=$$

$$=\lim_{n\to\infty}\frac{3^{n+1}\cdot\left(\frac{2^{n+1}}{3^{n+1}}\cdot 2^{-1}+1\right)}{3^{n+1}\cdot\left(\frac{2^{n+1}}{3^{n+1}}+1\right)}=\lim_{n\to\infty}\frac{\frac{2^{n+1}}{3^{n+1}}\cdot 2^{-1}+1}{\frac{2^{n+1}}{3^{n+1}}+1}=\lim_{n\to\infty}\frac{\left(\frac{2}{3}\right)^{n+1}\cdot 2^{-1}+1}{\left(\frac{2}{3}\right)^{n+1}+1}=$$

$$=\frac{2^{-1}\cdot\lim\limits_{n\to\infty}\left(\frac{2}{3}\right)^{n+1}+1}{\lim\limits_{n\to\infty}\left(\frac{2}{3}\right)^{n+1}+1}=\frac{2^{-1}\cdot 0+1}{0+1}=\frac{1}{1}=1.$$

Vježba 125

Odredite graničnu vrijednost (limes) izraza: $\lim\limits_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}$.

Rezultat: 3.

Zadatak 126 (Lana, ekonomska škola)

Odredite graničnu vrijednost (limes) izraza: $\lim\limits_{x\to\infty}\frac{1+2\cdot x+x^2}{3\cdot x^2+2}$.

Rješenje 126

Ponovimo!

$$\frac{a}{b}=\frac{\frac{a}{n}}{\frac{b}{n}}\quad,\quad\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}\quad,\quad a^1=a\quad,\quad\frac{a^n}{a^m}=a^{n-m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Ako su nizovi { a_n } i { b_n } konvergentni, tada vrijede sljedeća pravila o graničnoj vrijednosti (limesu):

$$\lim_{n\to\infty}c=c\ ,\ c-\text{konstanta}\quad,\quad\lim_{n\to\infty}\frac{a_n}{b_n}=\frac{\lim\limits_{n\to\infty}a_n}{\lim\limits_{n\to\infty}b_n}\ ,\ \lim_{n\to\infty}b_n\neq 0.$$

$$\lim_{n\to\infty}(a_n\pm b_n)=\lim_{n\to\infty}a_n\pm\lim_{n\to\infty}b_n\quad,\quad\lim_{n\to\infty}(c\cdot a_n)=c\cdot\lim_{n\to\infty}a_n\ ,\ c-\text{konstanta}.$$

$$\lim_{x\to\infty}\frac{1}{x}=0\quad,\quad\lim_{x\to\infty}\frac{1}{x^p}=0\ ,\ p>0.$$

Pri traženju limesa količnika dvaju cijelih polinoma u x kada x → ∞ treba oba člana količnika (brojnik i nazivnik) prethodno podijeliti sa x^n, gdje je n najveća potencija tih polinoma.

Budući da su nizovi u brojniku i nazivniku neograničeni kažemo da je ovaj limes neodređeni oblik tipa $\frac{\infty}{\infty}$. Zato moramo najprije brojnik i nazivnik podijeliti sa x^2, a onda koristiti poznate limese

$$\lim_{x\to\infty}\frac{1}{x}=0\quad,\quad\lim_{x\to\infty}\frac{1}{x^2}=0.$$

$$\lim_{x\to\infty}\frac{1+2\cdot x+x^2}{3\cdot x^2+2}=\left[\begin{array}{c}\text{brojnik i nazivnik}\\ \text{podijelimo sa } x^2\end{array}\right]=\lim_{x\to\infty}\frac{\frac{1+2\cdot x+x^2}{x^2}}{\frac{3\cdot x^2+2}{x^2}}=$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x^2}+\frac{2\cdot x}{x^2}+\frac{x^2}{x^2}}{\frac{3\cdot x^2}{x^2}+\frac{2}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x^2}+\frac{2\cdot x}{x^2}+\frac{x^2}{x^2}}{\frac{3\cdot x^2}{x^2}+\frac{2}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x^2}+\frac{2}{x}+1}{3+\frac{2}{x^2}} =$$

$$= \frac{\lim\limits_{x\to\infty}\frac{1}{x^2}+\lim\limits_{x\to\infty}\frac{2}{x}+1}{3+\lim\limits_{x\to\infty}\frac{2}{x^2}} = \frac{\lim\limits_{x\to\infty}\frac{1}{x^2}+2\cdot\lim\limits_{x\to\infty}\frac{1}{x}+1}{3+2\cdot\lim\limits_{x\to\infty}\frac{1}{x^2}} = \frac{0+2\cdot 0+1}{3+2\cdot 0} = \frac{1}{3}.$$

Vježba 126

Odredite graničnu vrijednost (limes) izraza: $\lim\limits_{x\to\infty} \frac{5+3\cdot x+x^2}{3\cdot x^2+7}$.

Rezultat: $\frac{1}{3}$.

Zadatak 127 (Lana, ekonomska škola)

Odredite graničnu vrijednost (limes) izraza: $\lim\limits_{n\to\infty} \left(\frac{n+1}{n-1}\right)^{n-1}$.

Rješenje 127

Ponovimo!

$$\frac{a+b}{n} = \frac{a}{n}+\frac{b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \quad , \quad n \ne 0 \quad , \quad n \ne 1.$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \quad , \quad \lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n = e^2 \quad , \quad \lim_{n\to\infty}\left(1+\frac{\alpha}{n}\right)^n = e^{\alpha}.$$

$$\lim_{n\to\infty}\left(\frac{n+1}{n-1}\right)^{n-1} = \lim_{n\to\infty}\left(\frac{n-1+2}{n-1}\right)^{n-1} = \lim_{n\to\infty}\left(\frac{(n-1)+2}{n-1}\right)^{n-1} =$$

$$= \lim_{n\to\infty}\left(\frac{n-1}{n-1}+\frac{2}{n-1}\right)^{n-1} = \lim_{n\to\infty}\left(\frac{n-1}{n-1}+\frac{2}{n-1}\right)^{n-1} = \lim_{n\to\infty}\left(1+\frac{2}{n-1}\right)^{n-1} =$$

$$= \begin{bmatrix} \text{zamjena} \\ x = n-1 \\ n\to\infty \ \Rightarrow \ x\to\infty \end{bmatrix} = \lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x = e^2.$$

Vježba 127

Odredite graničnu vrijednost (limes) izraza: $\lim\limits_{n\to\infty} \left(\frac{n+2}{n+1}\right)^{n+1}$.

Rezultat: e^2.

Zadatak 128 (Anamarija, srednja škola)

Odredite graničnu vrijednost (limes) funkcije $f(x)=\sqrt{x^{2}+a\cdot x+c}+\sqrt{x^{2}+b\cdot x+d}-2\cdot x$, kad x → ∞ (a, b, c i d su realni brojevi).

Rješenje 128

Ponovimo!

$$a\cdot\frac{b}{c}=\frac{a\cdot b}{c}\quad,\quad(a-b)\cdot(a+b)=a^{2}-b^{2}\quad,\quad\left(\sqrt{a}\right)^{2}=a.$$

$$a^{1}=a\quad,\quad\frac{a^{n}}{a^{m}}=a^{n-m}\quad,\quad\frac{\sqrt{a}}{b}=\sqrt{\frac{a}{b^{2}}}\quad,\quad\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq0\ ,\ n\neq1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq0\ ,\ n\neq1.$$

Ako postoje $\lim\limits_{x\to a}f(x)$ i $\lim\limits_{x\to a}g(x)$, tada vrijedi poučak:

$$\lim_{x\to a}(f(x)\pm g(x))=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x).$$

$$\lim_{x\to a}c\cdot f(x)=c\cdot\lim_{x\to a}f(x)\ ,\ c\text{ konstanta}\quad,\quad\lim_{x\to\infty}\frac{1}{x}=0.$$

Ako je funkcija y = f(x) neprekidna u točki a, onda je limes funkcije u točki a jednak funkciji limesa nezavisne varijable x u točki a:

$$\lim_{x\to a}f(x)=f\left(\lim_{x\to a}x\right).$$

$$\lim_{x\to\infty}f(x)=\lim_{x\to\infty}\left(\sqrt{x^{2}+a\cdot x+c}+\sqrt{x^{2}+b\cdot x+d}-2\cdot x\right)=$$

$$=\lim_{x\to\infty}\left(\sqrt{x^{2}+a\cdot x+c}-x+\sqrt{x^{2}+b\cdot x+d}-x\right)=$$

$$=\lim_{x\to\infty}\left(\left(\sqrt{x^{2}+a\cdot x+c}-x\right)+\left(\sqrt{x^{2}+b\cdot x+d}-x\right)\right)=$$

$$=\lim_{x\to\infty}\left(\sqrt{x^{2}+a\cdot x+c}-x\right)+\lim_{x\to\infty}\left(\sqrt{x^{2}+b\cdot x+d}-x\right)=$$

$$=\lim_{x\to\infty}\left(\sqrt{x^{2}+a\cdot x+c}-x\right)\cdot\frac{\sqrt{x^{2}+a\cdot x+c}+x}{\sqrt{x^{2}+a\cdot x+c}+x}+\lim_{x\to\infty}\left(\sqrt{x^{2}+b\cdot x+d}-x\right)\cdot\frac{\sqrt{x^{2}+b\cdot x+d}+x}{\sqrt{x^{2}+b\cdot x+d}+x}=$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^{2}+a\cdot x+c}\right)^{2}-x^{2}}{\sqrt{x^{2}+a\cdot x+c}+x}+\lim_{x\to\infty}\frac{\left(\sqrt{x^{2}+b\cdot x+d}\right)^{2}-x^{2}}{\sqrt{x^{2}+b\cdot x+d}+x}=$$

$$= \lim_{x\to\infty} \frac{x^2 + a\cdot x + c - x^2}{\sqrt{x^2 + a\cdot x + c} + x} + \lim_{x\to\infty} \frac{x^2 + b\cdot x + d - x^2}{\sqrt{x^2 + b\cdot x + d} + x} =$$

$$= \lim_{x\to\infty} \frac{x^2 + a\cdot x + c - x^2}{\sqrt{x^2 + a\cdot x + c} + x} + \lim_{x\to\infty} \frac{x^2 + b\cdot x + d - x^2}{\sqrt{x^2 + b\cdot x + d} + x} =$$

$$= \lim_{x\to\infty} \frac{a\cdot x + c}{\sqrt{x^2 + a\cdot x + c} + x} + \lim_{x\to\infty} \frac{b\cdot x + d}{\sqrt{x^2 + b\cdot x + d} + x} =$$

$$= \lim_{x\to\infty} \frac{\frac{a\cdot x + c}{x}}{\frac{\sqrt{x^2 + a\cdot x + c} + x}{x}} + \lim_{x\to\infty} \frac{\frac{b\cdot x + d}{x}}{\frac{\sqrt{x^2 + b\cdot x + d} + x}{x}} =$$

$$= \lim_{x\to\infty} \frac{\frac{a\cdot x}{x} + \frac{c}{x}}{\frac{\sqrt{x^2 + a\cdot x + c}}{x} + \frac{x}{x}} + \lim_{x\to\infty} \frac{\frac{b\cdot x}{x} + \frac{d}{x}}{\frac{\sqrt{x^2 + b\cdot x + d}}{x} + \frac{x}{x}} =$$

$$= \lim_{x\to\infty} \frac{\frac{a\cdot x}{x} + \frac{c}{x}}{\frac{\sqrt{x^2 + a\cdot x + c}}{x} + \frac{x}{x}} + \lim_{x\to\infty} \frac{\frac{b\cdot x}{x} + \frac{d}{x}}{\frac{\sqrt{x^2 + b\cdot x + d}}{x} + \frac{x}{x}} =$$

$$= \lim_{x\to\infty} \frac{a + \frac{c}{x}}{\frac{\sqrt{x^2 + a\cdot x + c}}{x} + 1} + \lim_{x\to\infty} \frac{b + \frac{d}{x}}{\frac{\sqrt{x^2 + b\cdot x + d}}{x} + 1} =$$

$$= \lim_{x\to\infty} \frac{a + \frac{c}{x}}{\sqrt{\frac{x^2 + a\cdot x + c}{x^2}} + 1} + \lim_{x\to\infty} \frac{b + \frac{d}{x}}{\sqrt{\frac{x^2 + b\cdot x + d}{x^2}} + 1} =$$

$$= \lim_{x\to\infty} \frac{a + \frac{c}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{a\cdot x}{x^2} + \frac{c}{x^2}} + 1} + \lim_{x\to\infty} \frac{b + \frac{d}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{b\cdot x}{x^2} + \frac{d}{x^2}} + 1} =$$

$$= \lim_{x\to\infty} \frac{a + \frac{c}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{a\cdot x}{x^2} + \frac{c}{x^2}} + 1} + \lim_{x\to\infty} \frac{b + \frac{d}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{b\cdot x}{x^2} + \frac{d}{x^2}} + 1} =$$

$$= \lim_{x\to\infty} \frac{a + \frac{c}{x}}{\sqrt{1 + \frac{a}{x} + \frac{c}{x^2}} + 1} + \lim_{x\to\infty} \frac{b + \frac{d}{x}}{\sqrt{1 + \frac{b}{x} + \frac{d}{x^2}} + 1} =$$

$$=\frac{a+\lim\limits_{x\to\infty}\frac{c}{x}}{\sqrt{1+\lim\limits_{x\to\infty}\frac{a}{x}+\lim\limits_{x\to\infty}\frac{c}{x^{2}}}+1}+\frac{b+\lim\limits_{x\to\infty}\frac{d}{x}}{\sqrt{1+\lim\limits_{x\to\infty}\frac{b}{x}+\lim\limits_{x\to\infty}\frac{d}{x^{2}}}+1}=$$

$$=\frac{a+c\cdot\lim\limits_{x\to\infty}\frac{1}{x}}{\sqrt{1+a\cdot\lim\limits_{x\to\infty}\frac{1}{x}+c\cdot\lim\limits_{x\to\infty}\frac{1}{x^{2}}}+1}+\frac{b+d\cdot\lim\limits_{x\to\infty}\frac{1}{x}}{\sqrt{1+b\cdot\lim\limits_{x\to\infty}\frac{1}{x}+d\cdot\lim\limits_{x\to\infty}\frac{1}{x^{2}}}+1}=$$

$$=\frac{a+c\cdot 0}{\sqrt{1+a\cdot 0+c\cdot 0}+1}+\frac{b+d\cdot 0}{\sqrt{1+b\cdot 0+d\cdot 0}+1}=\frac{a+0}{\sqrt{1+0+0}+1}+\frac{b+0}{\sqrt{1+0+0}+1}=$$

$$=\frac{a}{\sqrt{1}+1}+\frac{b}{\sqrt{1}+1}=\frac{a}{1+1}+\frac{b}{1+1}=\frac{a}{2}+\frac{b}{2}=\frac{a+b}{2}.$$

Vježba 128

Odredite graničnu vrijednost (limes) funkcije $f(x)=\sqrt{x^{2}+a\cdot x+1}+\sqrt{x^{2}+b\cdot x+1}-2\cdot x$, kad x → ∞ (a, b, c i d su realni brojevi).

Rezultat: $\frac{a+b}{2}.$