Zadatak 061 (Mira, gimnazija)

Odredite realan broj x ako je: $\operatorname{Im}\frac{1-x\cdot i}{1+i}=0.$

Rješenje 061

Ponovimo!

$$i^2=-1 \quad , \quad \frac{a+b}{c}=\frac{a}{c}+\frac{b}{c} \quad , \quad \frac{a}{b}=0 \ , \ b\neq 0 \Rightarrow a=0.$$

$$\operatorname{Im}\frac{1-x\cdot i}{1+i}=0 \Rightarrow \operatorname{Im}\left(\frac{1-x\cdot i}{1+i}\cdot\frac{1-i}{1-i}\right)=0 \Rightarrow \operatorname{Im}\left(\frac{1-i-x\cdot i+x\cdot i^2}{1^2+1^2}\right)=0 \Rightarrow \operatorname{Im}\left(\frac{1-i-x\cdot i-x}{2}\right)=0 \Rightarrow$$

$$\Rightarrow \operatorname{Im}\left(\frac{(1-x)+(-1-x)\cdot i}{2}\right)=0 \Rightarrow \operatorname{Im}\left(\frac{1-x}{2}+\frac{-1-x}{2}\cdot i\right)=0 \Rightarrow \frac{-1-x}{2}=0 \Rightarrow -1-x=0 \Rightarrow x=-1.$$

Vježba 061

Odredite realan broj x ako je: $\operatorname{Re}\frac{1-x\cdot i}{1+i}=0.$

Rezultat: x = 1.

Zadatak 062 (Anchy, gimnazija)

Riješite jednadžbu u skupu C: $i-1=z-|z|.$

Rješenje 062

Ponovimo!

$$a+b\cdot i=c+d\cdot i \Rightarrow a=c \ , \ b=d \quad , \quad z=a+b\cdot i \Rightarrow |z|=\sqrt{a^2+b^2} \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$\left.\begin{array}{l} z=x+y\cdot i \\ i-1=z-|z| \end{array}\right\} \Rightarrow i-1=x+y\cdot i-|x+y\cdot i| \Rightarrow -1+i=x+y\cdot i-\sqrt{x^2+y^2} \Rightarrow$$

$$\Rightarrow -1+i=\left(x-\sqrt{x^2+y^2}\right)+y\cdot i \Rightarrow \left.\begin{array}{l} x-\sqrt{x^2+y^2}=-1 \\ y=1 \end{array}\right\} \Rightarrow x-\sqrt{x^2+1^2}=-1 \Rightarrow$$

$$\Rightarrow x-\sqrt{x^2+1}=-1 \Rightarrow -\sqrt{x^2+1}=-x-1 \ /\cdot(-1) \Rightarrow \sqrt{x^2+1}=x+1 \Rightarrow \sqrt{x^2+1}=x+1 \ /^2 \Rightarrow$$

$$\Rightarrow x^2+1=x^2+2\cdot x+1 \Rightarrow 0=2\cdot x \ /:2 \Rightarrow x=0.$$

Rješenje je:

$$(x,\ y)=(0,1) \Rightarrow z=i.$$

Vježba 062

Riješite jednadžbu u skupu C: $|z|-z=1-i.$

Rezultat: z = i.

Zadatak 063 (Otkačena, hotelijerska škola)

Izračunajte: $z=\left(\frac{2}{1+i\cdot\sqrt{3}}\right)^4.$

Rješenje 063

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

1.inačica

$$z=\left(\frac{2}{1+i\cdot\sqrt{3}}\right)^4=\left(\frac{2}{1+i\cdot\sqrt{3}}\cdot\frac{1-i\cdot\sqrt{3}}{1-i\cdot\sqrt{3}}\right)^4=\left(\frac{2\cdot\left(1-i\cdot\sqrt{3}\right)}{1^2+\left(\sqrt{3}\right)^2}\right)^4=\left(\frac{2\cdot\left(1-i\cdot\sqrt{3}\right)}{1+3}\right)^4=\left(\frac{2\cdot\left(1-i\cdot\sqrt{3}\right)}{4}\right)^4=$$

$$=\left(\frac{1-i\cdot\sqrt{3}}{2}\right)^4=\frac{\left(1-i\cdot\sqrt{3}\right)^4}{2^4}=\frac{\left(\left(1-i\cdot\sqrt{3}\right)^2\right)^2}{16}=\frac{\left(1-2\cdot i\cdot\sqrt{3}-3\right)^2}{16}=\frac{\left(-2-2\cdot i\cdot\sqrt{3}\right)^2}{16}=$$

$$=\frac{\left(-2\cdot\left(1+i\cdot\sqrt{3}\right)\right)^2}{16}=\frac{(-2)^2\cdot\left(1+i\cdot\sqrt{3}\right)^2}{16}=\frac{4\cdot\left(1+i\cdot\sqrt{3}\right)^2}{16}=\frac{\left(1+i\cdot\sqrt{3}\right)^2}{4}=\frac{1+2\cdot i\cdot\sqrt{3}-3}{4}=$$

$$=\frac{-2+2\cdot i\cdot\sqrt{3}}{4}=\frac{-2}{4}+\frac{2\cdot i\cdot\sqrt{3}}{4}=-\frac{1}{2}+\frac{\sqrt{3}}{2}\cdot i.$$

2.inačica

$$z=\left(\frac{2}{1+i\cdot\sqrt{3}}\right)^4=\frac{2^4}{\left(1+i\cdot\sqrt{3}\right)^4}=\frac{16}{\left(\left(1+i\cdot\sqrt{3}\right)^2\right)^2}=\frac{16}{\left(1+2\cdot i\cdot\sqrt{3}-3\right)^2}=\frac{16}{\left(-2+2\cdot i\cdot\sqrt{3}\right)^2}=$$

$$=\frac{16}{\left(-2\cdot\left(1-i\cdot\sqrt{3}\right)\right)^2}=\frac{16}{(-2)^2\cdot\left(1-i\cdot\sqrt{3}\right)^2}=\frac{16}{4\cdot\left(1-i\cdot\sqrt{3}\right)^2}=\frac{4}{\left(1-i\cdot\sqrt{3}\right)^2}=\frac{4}{1-2\cdot i\cdot\sqrt{3}-3}=$$

$$=\frac{4}{-2-2\cdot i\cdot\sqrt{3}}=\frac{4}{-2\cdot\left(1+i\cdot\sqrt{3}\right)}=\frac{-2}{1+i\cdot\sqrt{3}}=\frac{-2}{1+i\cdot\sqrt{3}}\cdot\frac{1-i\cdot\sqrt{3}}{1-i\cdot\sqrt{3}}=\frac{-2\cdot\left(1-i\cdot\sqrt{3}\right)}{1^2+\left(\sqrt{3}\right)^2}=\frac{-2\cdot\left(1-i\cdot\sqrt{3}\right)}{1+3}=$$

$$=\frac{-2\cdot\left(1-i\cdot\sqrt{3}\right)}{4}=\frac{-\left(1-i\cdot\sqrt{3}\right)}{2}=-\frac{1}{2}+\frac{\sqrt{3}}{2}\cdot i.$$

Vježba 063

Izračunajte: $z=\left(\frac{2}{1+i}\right)^4.$

Rezultat: $-4.$

Zadatak 064 (Marija, gimnazija)

Izračunajte: $z=\left(\frac{i^5+2}{i^{19}+1}\right)^2.$

Rješenje 064

Ponovimo!

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad,\quad(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

1.inačica

$$z=\left(\frac{i^5+2}{i^{19}+1}\right)^2=\left[\begin{matrix}5:4=1 & , & 19:4=4\\ 1 & & 3\end{matrix}\right]=\left(\frac{i^1+2}{i^3+1}\right)^2=\left(\frac{i+2}{-i+1}\right)^2=\left(\frac{2+i}{1-i}\right)^2=\left(\frac{2+i}{1-i}\cdot\frac{1+i}{1+i}\right)^2=$$

$$=\left(\frac{2+2\cdot i+i-1}{1^2+1^2}\right)^2=\left(\frac{1+3\cdot i}{2}\right)^2=\frac{(1+3\cdot i)^2}{2^2}=\frac{1+6\cdot i-9}{4}=\frac{-8+6\cdot i}{4}=-\frac{8}{4}+\frac{6}{4}\cdot i=-2+\frac{3}{2}\cdot i.$$

2.inačica

$$z=\left(\frac{i^5+2}{i^{19}+1}\right)^2=\left[\begin{matrix}5:4=1 & , & 19:4=4\\ 1 & & 3\end{matrix}\right]=\left(\frac{i^1+2}{i^3+1}\right)^2=\left(\frac{i+2}{-i+1}\right)^2=\left(\frac{2+i}{1-i}\right)^2=\frac{(2+i)^2}{(1-i)^2}=$$

$$=\frac{4+4\cdot i-1}{1-2\cdot i-1}=\frac{3+4\cdot i}{-2\cdot i}=\frac{3+4\cdot i}{-2\cdot i}\cdot\frac{2\cdot i}{2\cdot i}=\frac{6\cdot i-8}{-4\cdot i^2}=\frac{-8+6\cdot i}{4}=-\frac{8}{4}+\frac{6}{4}\cdot i=-2+\frac{3}{2}\cdot i.$$

Vježba 064

Izračunajte: $z=\left(\frac{i^5+1}{i^{19}+1}\right)^2.$

Rezultat: – 1.

Zadatak 065 (Martin, gimnazija)

Izračunajte: $z=\left(i^{-23}+i^{96}\right)^{-2}.$

Rješenje 065

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad i^{-1}=-i\quad,\quad i^0=1\quad,\quad i^2=-1\quad,\quad i^3=-i\quad,\quad a^{-n}=\frac{1}{a^n}$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2$$

1.inačica

$$z=\left(i^{-23}+i^{96}\right)^{-2}\Rightarrow z=\left(\left(i^{-1}\right)^{23}+i^{96}\right)^{-2}\Rightarrow z=\left((-i)^{23}+i^{96}\right)^{-2}\Rightarrow z=\left(-i^{23}+i^{96}\right)^{-2}\Rightarrow$$

$$\Rightarrow\left[\begin{matrix}23:4=5 & , & 96:4=24\\ 3 & & 0\end{matrix}\right]\Rightarrow z=\left(-i^3+i^0\right)^{-2}\Rightarrow z=(i+1)^{-2}\Rightarrow z=(1+i)^{-2}\Rightarrow z=\frac{1}{(1+i)^2}\Rightarrow$$

$$\Rightarrow z=\frac{1}{1+2\cdot i-1}\Rightarrow z=\frac{1}{2\cdot i}\Rightarrow z=\frac{1}{2\cdot i}\cdot\frac{-i}{-i}\Rightarrow z=\frac{-i}{-2\cdot i^2}\Rightarrow z=\frac{-i}{2}\Rightarrow z=-\frac{1}{2}\cdot i.$$

2.inačica

$$z=\left(i^{-23}+i^{96}\right)^{-2}\Rightarrow z=\left(\frac{1}{i^{23}}+i^{96}\right)^{-2}\Rightarrow\left[\begin{matrix}23:4=5 & , & 96:4=24\\ 3 & & 0\end{matrix}\right]\Rightarrow z=\left(\frac{1}{i^3}+i^0\right)^{-2}\Rightarrow$$

$$\Rightarrow z=\left(\frac{1}{-i}+1\right)^{-2}\Rightarrow z=\left(\frac{1}{-i}\cdot\frac{i}{i}+1\right)^{-2}\Rightarrow z=\left(\frac{i}{-i^2}+1\right)^{-2}\Rightarrow z=\left(\frac{i}{1}+1\right)^{-2}\Rightarrow z=(i+1)^{-2}\Rightarrow$$

$$\Rightarrow z=\frac{1}{(1+i)^2}\Rightarrow z=\frac{1}{1+2\cdot i-1}\Rightarrow z=\frac{1}{2\cdot i}\Rightarrow z=\frac{1}{2\cdot i}\cdot\frac{-i}{-i}\Rightarrow z=\frac{-i}{-2\cdot i^2}\Rightarrow z=\frac{-i}{2}\Rightarrow z=-\frac{1}{2}\cdot i.$$

Vježba 065

Izračunajte: $z=\left(i^{-20}+i^{96}\right)^2.$

Rezultat: 4.

Zadatak 066 (Kiki, Izzy, ekonomska škola)

Izračunajte |z| ako je zadano: $z = \frac{\left(\sqrt{3}+i\right)^7}{\left(1-i\cdot\sqrt{3}\right)^6}$.

Rješenje 066

Ponovimo!

$$z = x + y\cdot i \Rightarrow |z| = \sqrt{x^2+y^2} \quad , \quad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \quad , \quad |z^n| = |z|^n \quad , \quad \frac{a^m}{a^n} = a^{m-n}.$$

$$z = \frac{\left(\sqrt{3}+i\right)^7}{\left(1-i\cdot\sqrt{3}\right)^6} \Rightarrow |z| = \left|\frac{\left(\sqrt{3}+i\right)^7}{\left(1-i\cdot\sqrt{3}\right)^6}\right| = \frac{\left|\left(\sqrt{3}+i\right)^7\right|}{\left|\left(1-i\cdot\sqrt{3}\right)^6\right|} = \frac{\left|\sqrt{3}+i\right|^7}{\left|1-i\cdot\sqrt{3}\right|^6} = \frac{\left(\sqrt{\left(\sqrt{3}\right)^2+1^2}\right)^7}{\left(\sqrt{1^2+\left(-\sqrt{3}\right)^2}\right)^6} =$$

$$= \frac{\left(\sqrt{3+1}\right)^7}{\left(\sqrt{1+3}\right)^6} = \frac{\left(\sqrt{4}\right)^7}{\left(\sqrt{4}\right)^6} = \frac{2^7}{2^6} = 2.$$

Vježba 066

Izračunajte |z| ako je zadano: $z = \frac{\left(\sqrt{3}+i\right)^7}{\left(1-i\cdot\sqrt{3}\right)^4}$.

Rezultat: 8.

Zadatak 067 (Izzy, Kiki, ekonomska škola)

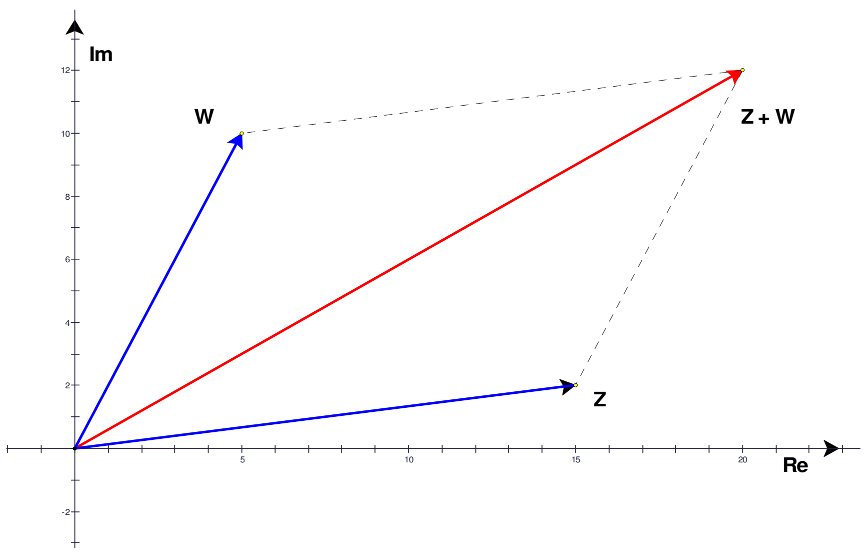
Računski i grafički izvedite računsku operaciju: (15 + 2 · i) + (5 + 10 · i).

Rješenje 067

Računski:

$$\left.\begin{aligned} z &= 15 + 2\cdot i \\ w &= 5 + 10\cdot i \end{aligned}\right\} \Rightarrow z + w = 15 + 2\cdot i + 5 + 10\cdot i = 20 + 12\cdot i.$$

Grafički:



Vježba 067

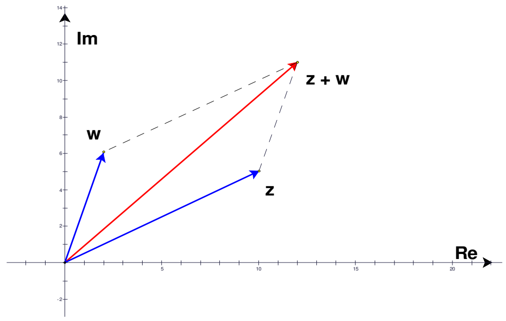
Računski i grafički izvedite računsku operaciju: (10 + 5 · i) + (2 + 6 · i).

Rezultat:

Računski:

$$\left.\begin{array}{l} z=10+5\cdot i \\ w=2+6\cdot i \end{array}\right\} \Rightarrow z+w=10+5\cdot i+2+6\cdot i=12+11\cdot i.$$

Grafički:



Zadatak 068 (2A, hotelijerska škola)

Dokažite da je: $\sqrt{1+i\cdot\sqrt{3}}+\sqrt{1-i\cdot\sqrt{3}}=\sqrt{6}.$

Rješenje 068

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad (a+b\cdot i)\cdot(a-b\cdot i)=a^2+b^2.$$

1.inačica

$$\sqrt{1+i\cdot\sqrt{3}}+\sqrt{1-i\cdot\sqrt{3}}=\sqrt{6} \Rightarrow \sqrt{1+i\cdot\sqrt{3}}+\sqrt{1-i\cdot\sqrt{3}}=\sqrt{6} \; /^2 \Rightarrow$$

$$\Rightarrow 1+i\cdot\sqrt{3}+2\cdot\sqrt{1+i\cdot\sqrt{3}}\cdot\sqrt{1-i\cdot\sqrt{3}}+1-i\cdot\sqrt{3}=6 \Rightarrow 2+2\cdot\sqrt{\left(1+i\cdot\sqrt{3}\right)\cdot\left(1-i\cdot\sqrt{3}\right)}=6 \Rightarrow$$

$$\Rightarrow 2+2\cdot\sqrt{1^2+\left(\sqrt{3}\right)^2}=6 \Rightarrow 2+2\cdot\sqrt{1+3}=6 \Rightarrow 2+2\cdot\sqrt{4}=6 \Rightarrow 2+2\cdot 2=6 \Rightarrow 6=6.$$

2.inačica

$$x=\sqrt{1+i\cdot\sqrt{3}}+\sqrt{1-i\cdot\sqrt{3}} \Rightarrow x=\sqrt{1+i\cdot\sqrt{3}}+\sqrt{1-i\cdot\sqrt{3}} \; /^2 \Rightarrow$$

$$\Rightarrow x^2=1+i\cdot\sqrt{3}+2\cdot\sqrt{1+i\cdot\sqrt{3}}\cdot\sqrt{1-i\cdot\sqrt{3}}+1-i\cdot\sqrt{3} \Rightarrow x^2=2+2\cdot\sqrt{\left(1+i\cdot\sqrt{3}\right)\cdot\left(1-i\cdot\sqrt{3}\right)} \Rightarrow$$

$$\Rightarrow x^2=2+2\cdot\sqrt{1^2+\left(\sqrt{3}\right)^2} \Rightarrow x^2=2+2\cdot\sqrt{1+3} \Rightarrow x^2=2+2\cdot\sqrt{4} \Rightarrow x^2=2+2\cdot 2 \Rightarrow$$

$$\Rightarrow x^2=6 \; /\sqrt{\ } \Rightarrow x=\sqrt{6}.$$

Vježba 068

Dokažite da je: $\sqrt{1+i\cdot\sqrt{15}}+\sqrt{1-i\cdot\sqrt{15}}=\sqrt{10}.$

Rezultat: Dokaz analogan.

Zadatak 069 (2A, hotelijerska škola)

Dokažite da vrijedi: $\left|\frac{1}{z}\right|=\frac{1}{|z|}.$

Rješenje 069

Ponovimo!

$$z = x + y \cdot i \Rightarrow |z| = \sqrt{x^2 + y^2} \quad , \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}.$$

Napišimo standardni ili algebarski oblik kompleksnog broja: z = x + y · i. Tada vrijedi:

$$\left|\frac{1}{z}\right| = \left|\frac{1}{x + y \cdot i}\right| = \left|\frac{1}{x + y \cdot i} \cdot \frac{x - y \cdot i}{x - y \cdot i}\right| = \left|\frac{x - y \cdot i}{x^2 + y^2}\right| = \left|\frac{x}{x^2 + y^2} - \frac{y}{x^2 + y^2} \cdot i\right| =$$

$$= \sqrt{\left(\frac{x}{x^2 + y^2}\right)^2 + \left(\frac{-y}{x^2 + y^2}\right)^2} = \sqrt{\frac{x^2}{\left(x^2 + y^2\right)^2} + \frac{y^2}{\left(x^2 + y^2\right)^2}} = \sqrt{\frac{x^2 + y^2}{\left(x^2 + y^2\right)^2}} = \sqrt{\frac{1}{x^2 + y^2}} =$$

$$= \frac{1}{\sqrt{x^2 + y^2}} = \frac{1}{|z|}.$$

Vježba 069

Dokažite da vrijedi: $\left|\frac{2}{z}\right| = \frac{2}{|z|}.$

Rezultat: Dokaz analogan.

Zadatak 070 (2A, hotelijerska škola)

Dokažite da vrijedi: $|z_1 \cdot z_2| = |z_1| \cdot |z_2|.$

Rješenje 070

Ponovimo!

$$z = x + y \cdot i \Rightarrow |z| = \sqrt{x^2 + y^2} \quad , \quad (x - y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad (x + y)^2 = x^2 + 2 \cdot x \cdot y + y^2$$

$$\sqrt{x \cdot y} = \sqrt{x} \cdot \sqrt{y}.$$

1.inačica

$$\left.\begin{array}{l} z_1 = a + b \cdot i \\ z_2 = c + d \cdot i \end{array}\right\} \Rightarrow \left.\begin{array}{l} |z_1| = \sqrt{a^2 + b^2} \\ |z_2| = \sqrt{c^2 + d^2} \end{array}\right\}.$$

Slijedi dokaz:

$$|z_1 \cdot z_2| = |(a + b \cdot i) \cdot (c + d \cdot i)| = \left|a \cdot c + a \cdot d \cdot i + b \cdot c \cdot i + b \cdot d \cdot i^2\right| = |a \cdot c + a \cdot d \cdot i + b \cdot c \cdot i - b \cdot d| =$$

$$= |(a \cdot c - b \cdot d) + (a \cdot d + b \cdot c) \cdot i| = \sqrt{(a \cdot c - b \cdot d)^2 + (a \cdot d + b \cdot c)^2} =$$

$$= \sqrt{a^2 \cdot c^2 - 2 \cdot a \cdot c \cdot b \cdot d + b^2 \cdot d^2 + a^2 \cdot d^2 + 2 \cdot a \cdot d \cdot b \cdot c + b^2 \cdot c^2} =$$

$$= \sqrt{a^2 \cdot c^2 + b^2 \cdot d^2 + a^2 \cdot d^2 + b^2 \cdot c^2} = \sqrt{\left(a^2 \cdot c^2 + a^2 \cdot d^2\right) + \left(b^2 \cdot c^2 + b^2 \cdot d^2\right)} =$$

$$= \sqrt{a^2 \cdot \left(c^2 + d^2\right) + b^2 \cdot \left(c^2 + d^2\right)} = \sqrt{\left(c^2 + d^2\right) \cdot \left(a^2 + b^2\right)} = \sqrt{\left(a^2 + b^2\right) \cdot \left(c^2 + d^2\right)} =$$

$$= \sqrt{a^2 + b^2} \cdot \sqrt{c^2 + d^2} = |z_1| \cdot |z_2|.$$

2.inačica

$$\left.\begin{array}{l} z_1 = a + b \cdot i \\ z_2 = c + d \cdot i \end{array}\right\} \Rightarrow \left.\begin{array}{l} |z_1| = \sqrt{a^2 + b^2} \\ |z_2| = \sqrt{c^2 + d^2} \end{array}\right\}.$$

Da bismo dokazali jednakost

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|,$$

izračunamo posebno njezinu lijevu stranu, a posebno desnu. Dobivene rezultate usporedimo. Ako su jednaki, vrijedi jednakost.

Lijeva strana:

$$|z_1 \cdot z_2| = |(a + b \cdot i) \cdot (c + d \cdot i)| = |a \cdot c + a \cdot d \cdot i + b \cdot c \cdot i + b \cdot d \cdot i^2| = |a \cdot c + a \cdot d \cdot i + b \cdot c \cdot i - b \cdot d| =$$

$$= |(a \cdot c - b \cdot d) + (a \cdot d + b \cdot c) \cdot i| = \sqrt{(a \cdot c - b \cdot d)^2 + (a \cdot d + b \cdot c)^2} =$$

$$= \sqrt{a^2 \cdot c^2 - 2 \cdot a \cdot c \cdot b \cdot d + b^2 \cdot d^2 + a^2 \cdot d^2 + 2 \cdot a \cdot d \cdot b \cdot c + b^2 \cdot c^2} =$$

$$= \sqrt{a^2 \cdot c^2 + b^2 \cdot d^2 + a^2 \cdot d^2 + b^2 \cdot c^2}. \qquad (1)$$

Desna strana:

$$|z_1| \cdot |z_2| = |a + b \cdot i| \cdot |c + d \cdot i| = \sqrt{a^2 + b^2} \cdot \sqrt{c^2 + d^2} = \sqrt{(a^2 + b^2) \cdot (c^2 + d^2)} =$$

$$= \sqrt{a^2 \cdot c^2 + a^2 \cdot d^2 + b^2 \cdot c^2 + b^2 \cdot d^2} = \sqrt{a^2 \cdot c^2 + b^2 \cdot d^2 + a^2 \cdot d^2 + b^2 \cdot c^2}. \qquad (2)$$

Iz (1) i (2) slijedi jednakost:

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|.$$

Vježba 070

Provjerite jednakost: $|(1 + 2 \cdot i) \cdot (3 + 4 \cdot i)| = |1 + 2 \cdot i| \cdot |3 + 4 \cdot i|.$

Rezultat: Točno je.

Zadatak 071 (2A, hotelijerska škola)

Dokažite da vrijedi: $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}.$

Rješenje 071

1.inačica

$$\left.\begin{array}{l} z_1 = a + b \cdot i \\ z_2 = c + d \cdot i \end{array}\right\} \Rightarrow \left.\begin{array}{l} \overline{z_1} = a - b \cdot i \\ \overline{z_2} = c - d \cdot i \end{array}\right\}.$$

Slijedi dokaz:

$$\overline{z_1 - z_2} = \overline{(a + b \cdot i) - (c + d \cdot i)} = \overline{a + b \cdot i - c - d \cdot i} = \overline{(a - c) + (b - d) \cdot i} = (a - c) - (b - d) \cdot i =$$

$$= a - c - b \cdot i + d \cdot i = a - b \cdot i - c + d \cdot i = (a - b \cdot i) - (c - d \cdot i) = \overline{(a + b \cdot i)} - \overline{(c + d \cdot i)} = \overline{z_1} - \overline{z_2}.$$

2.inačica

$$\left.\begin{array}{l} z_1 = a + b \cdot i \\ z_2 = c + d \cdot i \end{array}\right\} \Rightarrow \left.\begin{array}{l} \overline{z_1} = a - b \cdot i \\ \overline{z_2} = c - d \cdot i \end{array}\right\}.$$

Da bismo dokazali jednakost

$$\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}.$$

izračunamo posebno njezinu lijevu stranu, a posebno desnu. Dobivene rezultate usporedimo. Ako su jednaki, vrijedi jednakost.

Lijeva strana:

$$\overline{z_1 - z_2} = \overline{(a + b \cdot i) - (c + d \cdot i)} = \overline{a + b \cdot i - c - d \cdot i} = \overline{(a - c) + (b - d) \cdot i} = (a - c) - (b - d) \cdot i. \qquad (1)$$

Desna strana:

$$\overline{z_1} - \overline{z_2} = \overline{(a + b \cdot i)} - \overline{(c + d \cdot i)} = a - b \cdot i - (c - d \cdot i) = a - b \cdot i - c + d \cdot i = (a - c) - (b - d) \cdot i. \qquad (2)$$

Iz (1) i (2) slijedi jednakost:

$$\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}.$$

Vježba 071

Dokažite da vrijedi: $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}.$

Rezultat: Dokaz analogan.

Zadatak 072 (2A, hotelijerska škola)

Ako su x, y realni brojevi, i imaginarna jedinica te ako vrijedi (1 + i) · x + (2 + 3 · i) · y = 1 – i, koliko iznosi x + y?

Rješenje 072

Ponovimo definiciju jednakosti kompleksnih brojeva. Kada su dva kompleksna broja jednaka?

Dva su kompleksna broja jednaka ako i samo ako su im međusobno jednaki realni dijelovi i međusobno jednaki imaginarni dijelovi, tj.

$$a + bi = c + di \quad \Leftrightarrow \quad a = c\,,\; b = d.$$

$$(1 + i) \cdot x + (2 + 3 \cdot i) \cdot y = 1 - i \Rightarrow x + x \cdot i + 2 \cdot y + 3 \cdot y \cdot i = 1 - i \Rightarrow (x + 2 \cdot y) + (x + 3 \cdot y) \cdot i = 1 - i \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x + 2 \cdot y = 1 \\ x + 3 \cdot y = -1 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{bmatrix} \Rightarrow \left.\begin{matrix} x + 2 \cdot y = 1 \;/ \cdot (-1) \\ x + 3 \cdot y = -1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} -x - 2 \cdot y = -1 \\ x + 3 \cdot y = -1 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow y = -2 \Rightarrow \left.\begin{matrix} y = -2 \\ x + 2 \cdot y = 1 \end{matrix}\right\} \Rightarrow x + 2 \cdot (-2) = 1 \Rightarrow x - 4 = 1 \Rightarrow x = 5.$$

Zbroj x i y iznosi:

$$x + y = 5 + (-2) = 3.$$

Vježba 072

Ako su x, y realni brojevi, i imaginarna jedinica te ako vrijedi (1 + i) · x + (2 + 3 · i) · y = 1 – i, koliko iznosi x – y?

Rezultat: 7.

Zadatak 073 (2A, hotelijerska škola)

Svi kompleksni brojevi koji imaju modul kao broj $z = 1 + i \cdot \sqrt{3}$ u koordinatnom sustavu nalaze se:

A. u I. kvadrantu B. na imaginarnoj osi C. na realnoj osi D. na kružnici

Rješenje 073

Ponovimo!

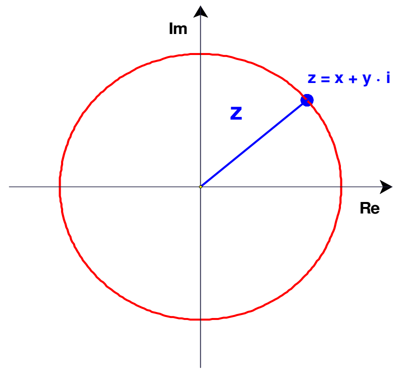
$$z = x + y \cdot i \Rightarrow |z| = \sqrt{x^2 + y^2}.$$

Modul (apsolutna vrijednost) zadanog kompleksnog broja iznosi:

$$\left.\begin{matrix} z = 1 + \sqrt{3} \cdot i \\ z = x + y \cdot i \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |z| = \sqrt{1^2 + \left(\sqrt{3}\right)^2} \\ |z| = \sqrt{x^2 + y^2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |z| = \sqrt{4} \\ |z| = \sqrt{x^2 + y^2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |z| = 2 \\ |z| = \sqrt{x^2 + y^2} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \sqrt{x^2 + y^2} = 2 \ /^2 \ \Rightarrow \ x^2 + y^2 = 2^2.$$

Budući da središnja jednadžba kružnice glasi $x^2 + y^2 = r^2$, svi kompleksni brojevi koji imaju modul kao broj $z = 1 + i \cdot \sqrt{3}$ u koordinatnom sustavu nalaze se na kružnici polumjera 2. Odgovor je pod D.



Vježba 073

Svi kompleksni brojevi koji imaju modul kao broj z = 3 + 4 · i u koordinatnom sustavu nalaze se:

A. u I. kvadrantu B. na imaginarnoj osi C. na realnoj osi D. na kružnici

Rezultat: Odgovor je pod D.

Zadatak 074 (Rony, gimnazija)

Nađite modul kompleksnog broja z ≠ 0 koji zadovoljava uvjete | z + 1 | = | z + i | = 1.

Rješenje 074

Ponovimo!

$$z = x + y \cdot i \Rightarrow |z| = \sqrt{x^2 + y^2}.$$

1.inačica

U zadani uvjet uvrstimo kompleksan broj z = x + y · i:

$$\left.\begin{array}{l} |z+1| = |z+i| \\ z = x + y \cdot i \end{array}\right\} \Rightarrow |x + y \cdot i + 1| = |x + y \cdot i + i| \Rightarrow |(x+1) + y \cdot i| = |x + (y+1) \cdot i| \Rightarrow$$

$$\Rightarrow \sqrt{(x+1)^2 + y^2} = \sqrt{x^2 + (y+1)^2} \ /^2 \ \Rightarrow (x+1)^2 + y^2 = x^2 + (y+1)^2 \Rightarrow$$

$$\Rightarrow x^2 + 2 \cdot x + 1 + y^2 = x^2 + y^2 + 2 \cdot y + 1 \Rightarrow x^2 + 2 \cdot x + 1 + y^2 = x^2 + y^2 + 2 \cdot y + 1 \Rightarrow$$

$$\Rightarrow 2 \cdot x = 2 \cdot y \ /:2 \ \Rightarrow x = y.$$

U bilo koji zadani izraz uvrstimo x = y:

$$\left.\begin{array}{l} |z+1| = 1 \\ x = y \end{array}\right\} \Rightarrow |x + x \cdot i + 1| = 1 \Rightarrow |(x+1) + y \cdot i| = 1 \Rightarrow \sqrt{(x+1)^2 + x^2} = 1 \ /^2 \ \Rightarrow$$

$$\Rightarrow (x+1)^2 + x^2 = 1 \Rightarrow x^2 + 2 \cdot x + 1 + x^2 = 1 \Rightarrow x^2 + 2 \cdot x + 1 + x^2 = 1 \Rightarrow 2 \cdot x^2 + 2 \cdot x = 0 \Rightarrow$$

$$\Rightarrow 2 \cdot x^2 + 2 \cdot x = 0 \ /:2 \ \Rightarrow x^2 + x = 0 \Rightarrow x \cdot (x+1) = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x = 0 \ \text{nema smisla zbog uvjeta zadatka } z \neq 0 \\ x + 1 = 0 \end{array}\right\} \Rightarrow x = -1.$$

Iz jednakosti x = y slijedi y = – 1. Kompleksan broj glasi: z = – 1 – i, a njegov modul je:

$$|z| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}.$$

2.inačica

U zadane uvjete uvrstimo kompleksan broj z = x + y · i:

$$\left.\begin{matrix} |z+1|=1 \\ |z+i|=1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |x+y\cdot i+1|=1 \\ |x+y\cdot i+i|=1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} |(x+1)+y\cdot i|=1 \\ |x+(y+1)\cdot i|=1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \sqrt{(x+1)^2+y^2}=1\ /^2 \\ \sqrt{x^2+(y+1)^2}=1\ /^2 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} (x+1)^2+y^2=1 \\ x^2+(y+1)^2=1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2+2\cdot x+1+y^2=1 \\ x^2+y^2+2\cdot y+1=1 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{oduzmemo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow$$

$$\Rightarrow x^2+2\cdot x+1+y^2-x^2-y^2-2\cdot y-1=1-1 \Rightarrow 2\cdot x-2\cdot y=0\ /:2 \Rightarrow x-y=0 \Rightarrow x=y.$$

U bilo koji zadani izraz uvrstimo x = y:

$$\left.\begin{matrix} |z+1|=1 \\ x=y \end{matrix}\right\} \Rightarrow |x+x\cdot i+1|=1 \Rightarrow |(x+1)+y\cdot i|=1 \Rightarrow \sqrt{(x+1)^2+x^2}=1\ /^2 \Rightarrow$$

$$\Rightarrow (x+1)^2+x^2=1 \Rightarrow x^2+2\cdot x+1+x^2=1 \Rightarrow x^2+2\cdot x+1+x^2=1 \Rightarrow 2\cdot x^2+2\cdot x=0 \Rightarrow$$

$$\Rightarrow 2\cdot x^2+2\cdot x=0\ /:2 \Rightarrow x^2+x=0 \Rightarrow x\cdot(x+1)=0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x=0 \ \text{nema smisla zbog uvjeta zadatka } z\neq 0 \\ x+1=0 \end{matrix}\right\} \Rightarrow x=-1.$$

Iz jednakosti x = y slijedi y = – 1. Kompleksan broj glasi: z = – 1 – i, a njegov modul je:

$$|z|=\sqrt{(-1)^2+(-1)^2}=\sqrt{2}.$$

Vježba 074

Nađite kompleksan broj z ≠ 0 koji zadovoljava uvjete | z + 1 | = | z + i | = 1.

Rezultat: z = – 1 – i .

Zadatak 075 (Anamarija, gimnazija)

Izračunajte modul kompleksnog broja:

$$z=4\cdot\left(\cos\frac{5\pi}{3}-i\cdot\sin\frac{7\pi}{6}\right)^2.$$

Rješenje 075

Ponovimo!

Svođenje na prvi kvadrant: $\cos(2\pi-\alpha)=\cos\alpha \quad , \quad \sin(\pi+\alpha)=-\sin\alpha.$

Modul (apsolutna vrijednost) kompleksnog broja: $z=x+y\cdot i \Rightarrow |z|=\sqrt{x^2+y^2}.$

Svojstva modula: $|z\cdot w|=|z|\cdot|w| \quad , \quad |z^n|=|z|^n.$

$$z=4\cdot\left(\cos\frac{5\pi}{3}-i\cdot\sin\frac{7\pi}{6}\right)^2 \Rightarrow \left.\begin{matrix} \cos\frac{5\pi}{3}=\cos\left(2\pi-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2} \\ \sin\frac{7\pi}{6}=\sin\left(\pi+\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2} \end{matrix}\right\} \Rightarrow z=4\cdot\left(\frac{1}{2}+\frac{1}{2}\cdot i\right)^2 \Rightarrow$$

$$\Rightarrow |z|=\left|4\cdot\left(\frac{1}{2}+\frac{1}{2}\cdot i\right)^2\right| \Rightarrow |z|=|4|\cdot\left|\left(\frac{1}{2}+\frac{1}{2}\cdot i\right)^2\right| \Rightarrow |z|=4\cdot\left|\left(\frac{1}{2}+\frac{1}{2}\cdot i\right)\right|^2 \Rightarrow$$

$$\Rightarrow |z| = 4 \cdot \left(\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \right)^2 \Rightarrow |z| = 4 \cdot \left(\frac{1}{4} + \frac{1}{4} \right) \Rightarrow |z| = 4 \cdot \frac{2}{4} \Rightarrow |z| = 2.$$

Vježba 075

Izračunajte modul kompleksnog broja:

$$z = 2 \cdot \left(\cos \frac{5\pi}{3} - i \cdot \sin \frac{7\pi}{6} \right)^2.$$

Rezultat: 1.

Zadatak 076 (Nata, hotelijerska škola)

Nađite $|z|$ ako je $z = \frac{-1-i}{(1+i)^3}$.

Rješenje 076

Ponovimo!

Modul (apsolutna vrijednost) kompleksnog broja: $z = x + y \cdot i \Rightarrow |z| = \sqrt{x^2 + y^2}$.

Svojstva modula: $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$, $|z^n| = |z|^n$. Kub zbroja: $(a+b)^3 = a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3$.

1.inačica

$$z = \frac{-1-i}{(1+i)^3} \Rightarrow |z| = \left| \frac{-1-i}{(1+i)^3} \right| \Rightarrow |z| = \frac{|-1-i|}{\left|(1+i)^3\right|} \Rightarrow |z| = \frac{|-1-i|}{|1+i|^3} \Rightarrow |z| = \frac{\sqrt{(-1)^2 + (-1)^2}}{\left(\sqrt{1^2 + 1^2}\right)^3} \Rightarrow$$

$$\Rightarrow |z| = \frac{\sqrt{1+1}}{\left(\sqrt{1+1}\right)^3} \Rightarrow |z| = \frac{\sqrt{2}}{\left(\sqrt{2}\right)^3} \Rightarrow |z| = \frac{1}{\left(\sqrt{2}\right)^2} \Rightarrow |z| = \frac{1}{2}.$$

2.inačica

Najprije nađemo standardni oblik kompleksnog broja, a zatim njegovu apsolutnu vrijednost:

$$z = \frac{-1-i}{(1+i)^3} \Rightarrow z = \frac{-1-i}{1^3 + 3 \cdot 1^2 \cdot i + 3 \cdot 1 \cdot i^2 + i^3} \Rightarrow z = \frac{-1-i}{1 + 3 \cdot i - 3 - i} \Rightarrow z = \frac{-1-i}{-2 + 2 \cdot i} \Rightarrow$$

$$\Rightarrow z = \frac{-1-i}{-2+2 \cdot i} \cdot \frac{-2-2 \cdot i}{-2-2 \cdot i} \Rightarrow z = \frac{2 + 2 \cdot i + 2 \cdot i - 2}{(-2)^2 + 2^2} \Rightarrow z = \frac{4 \cdot i}{4+4} \Rightarrow z = \frac{4 \cdot i}{8} \Rightarrow z = \frac{1}{2} \cdot i \Rightarrow z = 0 + \frac{1}{2} \cdot i.$$

$$|z| = \sqrt{0^2 + \left(\frac{1}{2}\right)^2} \Rightarrow |z| = \sqrt{\left(\frac{1}{2}\right)^2} \Rightarrow |z| = \frac{1}{2}.$$

Vježba 076

Nađite $|z|$ ako je $z = \frac{1+i}{(1-i)^2}$.

Rezultat: $\frac{\sqrt{2}}{2}$.

Zadatak 077 (Maturant, gimnazija)

Nađite apsolutnu vrijednost kompleksnog broja $\frac{1-2 \cdot i}{3+4 \cdot i} + \frac{i-4}{6 \cdot i - 8}$.

Rješenje 077

Ponovimo!

$$z = a + b \cdot i \Rightarrow |z| = \sqrt{a^2 + b^2} \quad , \quad \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}.$$

1.inačica

$$\left| \frac{1-2\cdot i}{3+4\cdot i} + \frac{i-4}{6\cdot i - 8} \right| = \left| \frac{1-2\cdot i}{3+4\cdot i} + \frac{-4+i}{-8+6\cdot i} \right| = \left| \frac{1-2\cdot i}{3+4\cdot i} \cdot \frac{3-4\cdot i}{3-4\cdot i} + \frac{-4+i}{-8+6\cdot i} \cdot \frac{-8-6\cdot i}{-8-6\cdot i} \right| =$$

$$= \left| \frac{(1-2\cdot i)\cdot(3-4\cdot i)}{(3+4\cdot i)\cdot(3-4\cdot i)} + \frac{(-4+i)\cdot(-8-6\cdot i)}{(-8+6\cdot i)\cdot(-8-6\cdot i)} \right| = \left| \frac{3-4\cdot i-6\cdot i-8}{9+16} + \frac{32+24\cdot i-8\cdot i+6}{64+36} \right| =$$

$$= \left| \frac{-5-10\cdot i}{25} + \frac{38+16\cdot i}{100} \right| = \left| \frac{4\cdot(-5-10\cdot i)+38+16\cdot i}{100} \right| = \left| \frac{-20-40\cdot i+38+16\cdot i}{100} \right| = \left| \frac{18-24\cdot i}{100} \right| =$$

$$= \left| \frac{2\cdot(9-12\cdot i)}{100} \right| = \left| \frac{9-12\cdot i}{50} \right| = \frac{|9-12\cdot i|}{50} = \frac{\sqrt{9^2+(-12)^2}}{50} = \frac{\sqrt{81+144}}{50} = \frac{\sqrt{225}}{50} = \frac{15}{50} = \frac{3}{10}.$$

2.inačica

$$\left| \frac{1-2\cdot i}{3+4\cdot i} + \frac{i-4}{6\cdot i - 8} \right| = \left| \frac{1-2\cdot i}{3+4\cdot i} + \frac{-4+i}{-8+6\cdot i} \right| = \left| \frac{(1-2\cdot i)\cdot(-8+6\cdot i)+(-4+i)\cdot(3+4\cdot i)}{(3+4\cdot i)\cdot(-8+6\cdot i)} \right| =$$

$$= \left| \frac{-8+6\cdot i+16\cdot i+12-12-16\cdot i+3\cdot i-4}{-24+18\cdot i-32\cdot i-24} \right| = \left| \frac{-12+9\cdot i}{-48-14\cdot i} \right| = \frac{|-12+9\cdot i|}{|-48-14\cdot i|} = \frac{\sqrt{(-12)^2+9^2}}{\sqrt{(-48)^2+(-14)^2}} =$$

$$= \frac{\sqrt{144+81}}{\sqrt{2304+196}} = \frac{\sqrt{225}}{\sqrt{2500}} = \frac{15}{50} = \frac{3}{10}.$$

Vježba 077

Nađite apsolutnu vrijednost kompleksnog broja $\frac{1-2\cdot i}{1+2\cdot i}$.

Rezultat: 1.

Zadatak 078 (Luka, gimnazija)

Neka je $f(z) = \frac{1}{z+3}$. Izračunajte koliko je $f\left(\sqrt{2}-3-i\right)$.

Rješenje 078

Ponovimo!

$$(x - y\cdot i)\cdot(x + y\cdot i) = x^2 + y^2.$$

$$f\left(\sqrt{2}-3-i\right) = \frac{1}{\sqrt{2}-3-i+3} = \frac{1}{\sqrt{2}-i} \cdot \frac{\sqrt{2}+i}{\sqrt{2}+i} = \frac{\sqrt{2}+i}{\left(\sqrt{2}\right)^2+1^2} = \frac{\sqrt{2}+i}{2+1} = \frac{\sqrt{2}+i}{3} = \frac{\sqrt{2}}{3} + \frac{1}{3}\cdot i.$$

Vježba 078

Neka je $f(z) = \frac{1}{z+2}$. Izračunajte koliko je $f\left(\sqrt{2}-2-i\right)$.

Rezultat: $\frac{\sqrt{2}}{3} + \frac{1}{3}\cdot i.$

Zadatak 079 (Mario, Ana, Mirjana, Mateo, srednja škola)

Kompleksan broj z = 2 + 2 · i napišite u trigonometrijskom obliku.

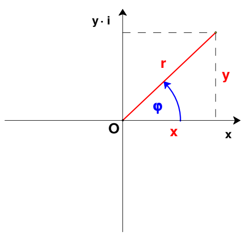
Rješenje 079

Ponovimo!

Kompleksan broj z = x + y · i napisan u trigonometrijskom obliku glasi

$$z = r \cdot (\cos\varphi + i \cdot \sin\varphi),$$

gdje je r apsolutna vrijednost ili modul kompleksnog broja (udaljenost kompleksnog broja od ishodišta kompleksne ravnine), φ argument kompleksnog broja.



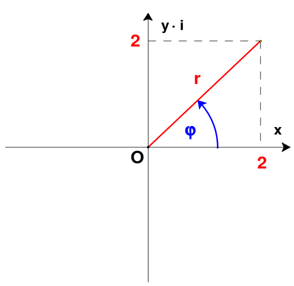
$$r = \sqrt{x^2 + y^2},$$

$$tg\,\varphi = \frac{y}{x} \Rightarrow \varphi = arctg\,\frac{y}{x}$$

ili

$$\text{tg}\,\varphi = \frac{y}{x} \Rightarrow \varphi = tg^{-1}\left(\frac{y}{x}\right).$$

Ovaj kompleksan broj nalazi se u I. kvadrantu kompleksne ravnine, Gaussove ravnine, (x > 0, y > 0). Da bismo odredili njegov trigonometrijski oblik treba naći apsolutnu vrijednost r i argument φ:



$$\left.\begin{array}{l} z = 2 + 2 \cdot i \ , \ x = 2 \ , \ y = 2 \\ r = \sqrt{x^2 + y^2} \\ tg\,\varphi = \dfrac{y}{x} \end{array}\right\} \Rightarrow \left.\begin{array}{l} r = \sqrt{2^2 + 2^2} \\ tg\,\varphi = \dfrac{2}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} r = \sqrt{4+4} \\ tg\,\varphi = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} r = \sqrt{4 \cdot 2} \\ \varphi = arctg\,1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} r = 2 \cdot \sqrt{2} \\ \varphi = \dfrac{\pi}{4} \end{array}\right\}.$$

Trigonometrijski oblik kompleksnog broja glasi:

$$z = 2 \cdot \sqrt{2} \cdot \left(\cos\frac{\pi}{4} + i \cdot \sin\frac{\pi}{4}\right).$$

Vježba 079

Kompleksan broj z = 3 + 3 · i napišite u trigonometrijskom obliku.

Rezultat: $z = 3 \cdot \sqrt{2} \cdot \left(\cos\frac{\pi}{4} + i \cdot \sin\frac{\pi}{4}\right).$

Zadatak 080 (Mario, Ana, Mirjana, Mateo, srednja škola)

Kompleksan broj z = – 2 + 2 · i napišite u trigonometrijskom obliku.

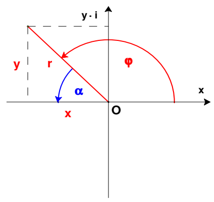
Rješenje 080

Ponovimo!

Kompleksan broj z = x + y · i napisan u trigonometrijskom obliku glasi

$$z = r \cdot (\cos\varphi + i \cdot \sin\varphi),$$

gdje je r apsolutna vrijednost ili modul kompleksnog broja (udaljenost kompleksnog broja od ishodišta kompleksne ravnine), φ argument kompleksnog broja.



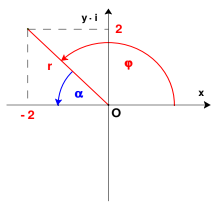
$$r = \sqrt{x^2 + y^2},$$

$$tg\,\alpha = \frac{y}{x} \Rightarrow \alpha = arctg\,\frac{y}{x} \text{ ili } \mathrm{tg}\,\alpha = \frac{y}{x} \Rightarrow \alpha = tg^{-1}\left(\frac{y}{x}\right).$$

Argument φ iznosi:

$$\varphi = \pi - \alpha.$$

Ovaj kompleksan broj nalazi se u II. kvadrantu kompleksne ravnine, Gaussove ravnine, ($x < 0$, $y > 0$). Da bismo odredili njegov trigonometrijski oblik treba naći apsolutnu vrijednost r i argument φ:



$$\left.\begin{array}{l} z = -2 + 2 \cdot i \;,\; x = -2 \;,\; y = 2 \\ r = \sqrt{x^2 + y^2} \\ tg\,\alpha = \dfrac{y}{x} \\ \varphi = \pi - \alpha \end{array}\right\} \Rightarrow \left.\begin{array}{l} r = \sqrt{(-2)^2 + 2^2} \\ tg\,\alpha = \dfrac{2}{2} \\ \varphi = \pi - \alpha \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} r = \sqrt{4+4} \\ tg\,\alpha = 1 \\ \varphi = \pi - \alpha \end{array}\right\} \Rightarrow \left.\begin{array}{l} r = \sqrt{4 \cdot 2} \\ \alpha = arctg\,1 \\ \varphi = \pi - \alpha \end{array}\right\} \Rightarrow \left.\begin{array}{l} r = 2 \cdot \sqrt{2} \\ \alpha = \dfrac{\pi}{4} \\ \varphi = \pi - \alpha \end{array}\right\} \Rightarrow \left.\begin{array}{l} r = 2 \cdot \sqrt{2} \\ \varphi = \pi - \dfrac{\pi}{4} \end{array}\right\} \Rightarrow \left.\begin{array}{l} r = 2 \cdot \sqrt{2} \\ \varphi = \dfrac{3 \cdot \pi}{4} \end{array}\right\}.$$

Trigonometrijski oblik kompleksnog broja glasi:

$$z = 2 \cdot \sqrt{2} \cdot \left(\cos\frac{3 \cdot \pi}{4} + i \cdot \sin\frac{3 \cdot \pi}{4}\right).$$

Vježba 080

Kompleksan broj $z = -3 + 3 \cdot i$ napišite u trigonometrijskom obliku.

Rezultat: $z = 3 \cdot \sqrt{2} \cdot \left(\cos\frac{3 \cdot \pi}{4} + i \cdot \sin\frac{3 \cdot \pi}{4}\right).$