ALGEBARSKI IZRAZI (m©h)

kvadrat zbroja	$(a + b)^2 = a^2 + 2ab + b^2$
kvadrat razlike	$(a - b)^2 = a^2 - 2ab + b^2$
kub zbroja	$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
kub razlike	$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
kvadrat trinoma	$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
kub trinoma	$(a + b + c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3ab^2 + 3ac^2 + 3b^2c + 3bc^2 + 6abc$
razlika kvadrata	$a^2 - b^2 = (a - b) \cdot (a + b)$, $a - b = (\sqrt{a} - \sqrt{b}) \cdot (\sqrt{a} + \sqrt{b})$
razlika kubova	$a^3 - b^3 = (a - b) \cdot (a^2 + ab + b^2)$
razlika bikvadrata	$a^4 - b^4 = (a^2 - b^2) \cdot (a^2 + b^2) = (a - b) \cdot (a + b) \cdot (a^2 + b^2)$
zbroj kvadrata	$a^2 + b^2$ = nema rastava nad **R** (skup realnih brojeva)
zbroj kubova	$a^3 + b^3 = (a + b) \cdot (a^2 - ab + b^2)$
zbroj bikvadrata	$a^4 + b^4 = (a^2 - ab\sqrt{2} + b^2) \cdot (a^2 + ab\sqrt{2} + b^2)$
generalizirane relacije	$a^n - b^n = (a - b) \cdot (a^{n-1} + a^{n-2} \cdot b + ... + a \cdot b^{n-2} + b^{n-1})$
	$a^{2n+1} + b^{2n+1} = (a + b) \cdot (a^{2n} - a^{2n-1} \cdot b + ... - a \cdot b^{2n-1} + b^{2n})$
Pozor!	$(-a - b)^2 = (a + b)^2$, $(-a - b)^3 = -(a + b)^3$

POTENCIJE

potencije broja	$a^n = a \cdot a \cdot a \cdot ... \cdot a$ (a se množi n puta sam sa sobom)
potenciranje broja jedinicom	$a^1 = a$
potenciranje broja nulom	$a^0 = 1, a \neq 0$
recipročna vrijednost potencije	$a^{-n} = \frac{1}{a^n}$, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
množenje potencija jednakih baza	$a^n \cdot a^m = a^{n+m}$
dijeljenje potencija jednakih baza	$a^n : a^m = a^{n-m}$, $\frac{a^n}{a^m} = a^{n-m}$
potenciranje umnoška brojeva	$(a \cdot b)^n = a^n \cdot b^n$
množenje potencija jednakih eksponenata	$a^n \cdot b^n = (a \cdot b)^n$
potenciranje količnika brojeva	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $(a : b)^n = a^n : b^n$
dijeljenje potencija jednakih eksponenata	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$, $a^n : b^n = (a : b)^n$
potenciranje potencije	$\left(a^n\right)^m = \left(a^m\right)^n = a^{n \cdot m}$
rastuća funkcija	$a > 1 \ , \ x < y \ \Rightarrow \ a^x < a^y$
Primjer	$2^{3x+4} < 2^{x+10} \ \Rightarrow \ 3x + 4 < x + 10 \ \Rightarrow \ 2x < 6 \ \Rightarrow \ x < 3$
padajuća funkcija	$0 < a < 1 \ , \ x < y \ \Rightarrow \ a^x > a^y$
Primjer	$\left(\frac{1}{2}\right)^{3x+4} < \left(\frac{1}{2}\right)^{x+10} \ \Rightarrow \ 3x + 4 > x + 10 \ \Rightarrow \ 2x > 6 \ \Rightarrow \ x > 3$
jednakost potencija	$a^x = a^y \ \Rightarrow \ x = y$, $a^x = b^x \ \Rightarrow \ a = b$
Primjeri	$0.1^x = 100 \ \Rightarrow \ 10^{-x} = 10^2 \ \Rightarrow \ -x = 2 \ \Rightarrow \ x = -2$
	$8^x = 4^{x+1} \ \Rightarrow \ \left(2^3\right)^x = \left(2^2\right)^{x+1} \ \Rightarrow \ 2^{3x} = 2^{2x+2} \ \Rightarrow \ 3x = 2x + 2 \ \Rightarrow \ x = 2$
nejednakost potencija	$n > 0 \ , \ a > b > 0 \ \Rightarrow \ a^n > b^n$
	$n > 0 \ , \ a > b > 0 \ \Rightarrow \ \frac{1}{a^n} < \frac{1}{b^n}$
potenciranje korijena	$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$, $\left(\sqrt[n]{a^p}\right)^m = \sqrt[n]{a^{p \cdot m}}$

samo potpuno jednake potencije možemo zbrajati i oduzimati

$a^n \pm a^m = ?$, $a^n \pm b^n = ?$, $a^n + a^n = 2a^n$

veza potencije i korijena: $a^{\frac{1}{n}} = \sqrt[n]{a}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

predznaci kod množenja: $+a\cdot(+b) = +ab$, $-a\cdot(-b) = +ab$

$+a\cdot(-b) = -ab$, $-a\cdot(+b) = -ab$

KORIJENI

veza korijena i potencije: $\sqrt[n]{a} = a^{\frac{1}{n}}$, $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

korjenovanje umnoška brojeva: $\sqrt[n]{a\cdot b} = \sqrt[n]{a}\cdot\sqrt[n]{b}$

množenje korijena istih eksponenata: $\sqrt[n]{a}\cdot\sqrt[n]{b} = \sqrt[n]{a\cdot b}$

množenje korijena istih radikanada: $\sqrt[n]{a}\cdot\sqrt[m]{a} = \sqrt[n\cdot m]{a^{n+m}}$

množenje korijena: $\sqrt[n]{a}\cdot\sqrt[m]{b} = \sqrt[n\cdot m]{a^m\cdot b^n}$

korjenovanje količnika brojeva: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, $\sqrt[n]{a:b} = \sqrt[n]{a}:\sqrt[n]{b}$

dijeljenje korijena istih eksponenata: $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, $\sqrt[n]{a}:\sqrt[n]{b} = \sqrt[n]{a:b}$

dijeljenje korijena istih radikanada: $\frac{\sqrt[n]{a}}{\sqrt[m]{a}} = \sqrt[n\cdot m]{a^{m-n}}$, $\sqrt[n]{a}:\sqrt[m]{a} = \sqrt[n\cdot m]{a^{m-n}}$

dijeljenje korijena: $\frac{\sqrt[n]{a}}{\sqrt[m]{b}} = \sqrt[n\cdot m]{\frac{a^m}{b^n}}$, $\sqrt[n]{a}:\sqrt[m]{b} = \sqrt[n\cdot m]{a^m:b^n}$

proširivanje korijena: $\sqrt[n]{a^m} = \sqrt[n\cdot p]{a^{m\cdot p}}$

skraćivanje korijena: $\sqrt[n\cdot p]{a^{m\cdot p}} = \sqrt[n]{a^m}$, $\sqrt[n]{a^n} = 1$, $\sqrt{a^2} = |a|$

korjenovanje korijena: $\sqrt[n]{\sqrt[m]{a}} = \sqrt[m]{\sqrt[n]{a}} = \sqrt[n\cdot m]{a}$, $\sqrt[n]{a\cdot\sqrt[m]{b}} = \sqrt[n\cdot m]{a^m\cdot b}$

djelomično korjenovanje: $\sqrt[n]{a^n\cdot b} = a\cdot\sqrt[n]{b}$, $\sqrt[n]{a^{p\cdot n}\cdot b} = a^p\cdot\sqrt[n]{b}$

unošenje pod korijen: $a\cdot\sqrt[n]{b} = \sqrt[n]{a^n\cdot b}$, $a^p\cdot\sqrt[n]{b} = \sqrt[n]{a^{p\cdot n}\cdot b}$

racionalizacija nazivnika: $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}}\cdot\frac{\sqrt{b}}{\sqrt{b}} = \frac{a\cdot\sqrt{b}}{b}$, $\frac{a}{\sqrt[n]{b}} = \frac{a}{\sqrt[n]{b}}\cdot\frac{\sqrt[n]{b^{n-1}}}{\sqrt[n]{b^{n-1}}} = \frac{a\cdot\sqrt[n]{b^{n-1}}}{b}$

$$\frac{a}{\sqrt{b}\pm\sqrt{c}} = \frac{a}{\sqrt{b}\pm\sqrt{c}}\cdot\frac{\sqrt{b}\mp\sqrt{c}}{\sqrt{b}\mp\sqrt{c}} = \frac{a\cdot\left(\sqrt{b}\mp\sqrt{c}\right)}{b-c}$$

$$\frac{a}{\sqrt[3]{b}\pm\sqrt[3]{c}} = \frac{a}{\sqrt[3]{b}\pm\sqrt[3]{c}}\cdot\frac{\sqrt[3]{b^2}\mp\sqrt[3]{b\cdot c}+\sqrt[3]{c^2}}{\sqrt[3]{b^2}\mp\sqrt[3]{b\cdot c}+\sqrt[3]{c^2}} = \frac{a\cdot\left(\sqrt[3]{b^2}\mp\sqrt[3]{b\cdot c}+\sqrt[3]{c^2}\right)}{b\pm c}$$

$$\frac{m}{a\sqrt{b}\pm c\sqrt{d}} = \frac{m}{a\sqrt{b}\pm c\sqrt{d}}\cdot\frac{a\sqrt{b}\mp c\sqrt{d}}{a\sqrt{b}\mp c\sqrt{d}} = \frac{m\cdot\left(a\sqrt{b}\mp c\sqrt{d}\right)}{a^2b-c^2d}$$

potencije korijena

$$\left(\sqrt{a}\right)^2 = a,\ a \ge 0 \quad , \quad \left(\sqrt[2n]{a}\right)^{2n} = a,\ a \ge 0 \quad , \quad \left(\sqrt[3]{a}\right)^3 = a \quad , \quad \left(\sqrt[2n-1]{a}\right)^{2n-1} = a \quad , \quad \sqrt{a^2} = |a|$$

važne formule

$$\sqrt{a+\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}} \quad , \quad \sqrt{a-\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} - \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

nejednakost korijena $a > b > 0 \Rightarrow \sqrt[n]{a} > \sqrt[n]{b}$

samo potpuno jednake korijene možemo zbrajati i oduzimati

$$\sqrt[n]{a} \pm \sqrt[m]{a} = ? \ , \ \sqrt[n]{a} \pm \sqrt[n]{b} = ? \ , \ \sqrt[n]{a} + \sqrt[n]{a} = 2 \cdot \sqrt[n]{a}.$$

LOGARITMI

definicija logaritma $\log_b a = c \Leftrightarrow b^c = a \ , \ b > 0 \ , \ b \ne 1 \ , \ a > 0$

definicija logaritma $\log_b a$ je broj kojim treba potencirati b da se dobije a

$b^c = a \Leftrightarrow c = \log_b a = c$ ili $b^{\log_b a} = a$

svojstva $\log_b b = 1 \ , \ \log_b 1 = 0$

$\log_b 0 = -\infty \ , b > 1 \ , \ \log_b 0 = +\infty \ , 0 < b < 1$

Primjeri $5^{\log_5 2} = 2 \quad , \quad \log_3 3 = 1 \quad , \quad 25^{\log_5 2} = \left(5^2\right)^{\log_5 2} = \left(5^{\log_5 2}\right)^2 = 2^2 = 4$

dekadski logaritam (Briggsov) $\log_{10} a = \lg a = \log a$

prirodni logaritam (Neperov) $\log_e a = \ln a \ , \ e = 2.718281828...$

logaritam umnoška $\log_b (a \cdot c) = \log_b a + \log_b c$

Primjeri $\log_2 6 = \log_2 (2 \cdot 3) = \log_2 2 + \log_2 3 = 1 + \log_2 3$

$$\log_2 (8 \cdot x) = \log_2 8 + \log_2 x = \log_2 2^3 + \log_2 x = 3 \cdot \log_2 2 + \log_2 x = 3 + \log_2 x$$

logaritam kvocijenta $\log_b \frac{a}{c} = \log_b a - \log_b c$

Primjeri $\log \frac{100}{x} = \log 100 - \log x = 2 - \log x$

$\log_3 \left(\frac{2}{3}\right) = \log_3 2 - \log_3 3 = \log_3 2 - 1$

logaritam potencije $\log_b a^n = n \cdot \log_b a \ , \ \log_{b^n} a = \frac{1}{n} \cdot \log_b a \ , \ \log_{b^n} a^n = \log_b a$

$\log_b b^n = n \ , \ \log_{b^{\frac{m}{n}}} a = \frac{n}{m} \cdot \log_b a \ , \ \log_{\frac{1}{b}} \frac{1}{a} = \log_b a$

Primjer $\log_2 32 = \log_2 2^5 = 5 \cdot \log_2 2 = 5 \cdot 1 = 5$

logaritam korijena $\log_b \sqrt[n]{a} = \frac{1}{n} \cdot \log_b a \ , \ \log_b \sqrt[n]{a^m} = \frac{m}{n} \cdot \log_b a$

Primjer $\log_2 \sqrt[3]{2} = \frac{1}{3} \cdot \log_2 2 = \frac{1}{3} \cdot 1 = \frac{1}{3}$

formule za promjenu baze $\log_b a = \frac{\log_c a}{\log_c b} = \log_b c \cdot \log_c a \ , \ \frac{\log_b c}{\log_{ab} c} = 1 + \log_b a$

Primjer $\log_{100} 2 = \dfrac{\log 2}{\log 100} = \dfrac{\log 2}{2}$

recipročnost logaritama $\log_b a = \dfrac{1}{\log_a b}$

veza s dekadskim logaritmima $\log_b a = \dfrac{\log a}{\log b}$

Primjer $\log_{100} 1000 = \dfrac{\log 1000}{\log 100} = \dfrac{3}{2} = 1.5$

rastuća funkcija $b > 1 \ , \ x < y \ \Rightarrow \ \log_b x < \log_b y$

Primjer
$$\left.\begin{array}{l}\log_2(2x+4) < \log_2(x+6) \Rightarrow 2x+4 < x+6 \Rightarrow x < 2 \\ 2x+4 > 0 \ \text{ i } \ x+6 > 0 \Rightarrow x > -2\end{array}\right\} \Rightarrow x \in \langle -2, 2 \rangle$$

padajuća funkcija $0 < b < 1 \ , \ x < y \ \Rightarrow \ \log_b x > \log_b y$

Primjeri
$$\left.\begin{array}{l}\log_{\frac{1}{2}}(2x+6) < \log_{\frac{1}{2}}(x+8) \Rightarrow 2x+6 > x+8 \Rightarrow x > 2 \\ 2x+6 > 0 \ \text{ i } \ x+8 > 0 \Rightarrow x > -3\end{array}\right\} \Rightarrow x > 2$$

$$\left.\begin{array}{l}\log_{\frac{1}{2}}(2x+5) > \log_{\frac{1}{2}}(x+8) \Rightarrow 2x+5 < x+8 \Rightarrow x < 3 \\ 2x+5 > 0 \ \text{ i } \ x+8 > 0 \Rightarrow x > -\frac{5}{2}\end{array}\right\} \Rightarrow x \in \left\langle -\frac{5}{2}, 3 \right\rangle$$

injektivnost $\log_b x = \log_b y \ \Rightarrow \ x = y$

Primjeri
$$\left.\begin{array}{l}\log(3x+2) = \log(2x+7) \Rightarrow 3x+2 = 2x+7 \Rightarrow x = 5 \\ 3x+2 > 0 \ \text{ i } \ 2x+7 > 0 \Rightarrow x > -\frac{2}{3}\end{array}\right\} \Rightarrow x = 5$$

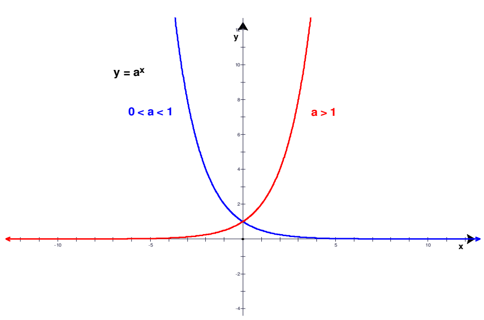
$$\left.\begin{array}{l}\log 2 + \log(x-3) = 1 \Rightarrow \log 2 \cdot (x-3) = \log 10 \Rightarrow \\ \Rightarrow 2 \cdot (x-3) = 10 \Rightarrow 2x - 6 = 10 \Rightarrow 2x = 16 \Rightarrow x = 8 \\ x - 3 > 0 \Rightarrow x > 3\end{array}\right\} \Rightarrow x = 8$$

množenje istih logaritama $\log_b a \cdot \log_b a = (\log_b a)^2 = \log^2{}_b\, a$

$$x^{\log_a y} = y^{\log_a x}$$

EKSPONENCIJALNA FUNKCIJA

$f : R \to R^+ \ , \ f(x) = a^x \ , \ a > 0 \ , \ a \neq 1$



LOGARITAMSKA FUNKCIJA

$f : R^+ \to R \ , \ f(x) = a^x \ , \ a > 0 \ , \ a \neq 1$