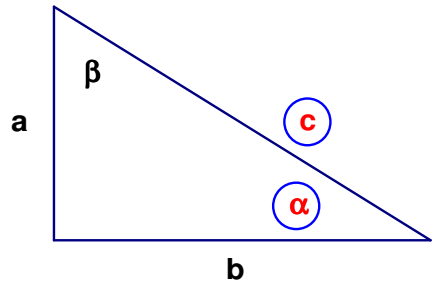
PRAVOKUTAN TROKUT (m©h)

Riješiti pravokutan trokut znači izračunati sve njegove stranice, sve njegove kutove, opseg i površinu. Pravokutan se trokut može zadati osnovnim elementima na jedan od četiri načina:

1) hipotenuzom i jednim šiljastim kutom
2) katetom i jednim šiljastim kutom
3) hipotenuzom i jednom katetom
4) dvjema katetama

- **Zadano je: hipotenuza c i šiljasti kut α**



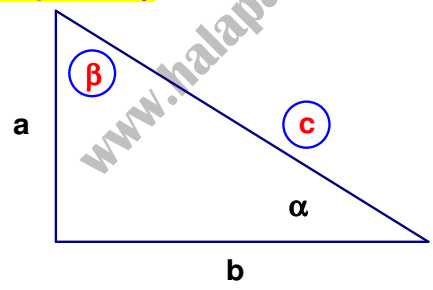
$$\alpha+\beta=90^0 \Rightarrow \beta=90^0-\alpha$$

$$\sin\alpha=\frac{a}{c} \Rightarrow \sin\alpha=\frac{a}{c}\ /\cdot c \Rightarrow a=c\cdot\sin\alpha \quad , \quad \cos\alpha=\frac{b}{c} \Rightarrow \cos\alpha=\frac{b}{c}\ /\cdot c \Rightarrow b=c\cdot\cos\alpha$$

$$O=a+b+c=c\cdot\sin\alpha+c\cdot\cos\alpha+c=c\cdot(\sin\alpha+\cos\alpha+1)$$

$$P=\frac{1}{2}\cdot a\cdot b=\frac{1}{2}\cdot c\cdot\sin\alpha\cdot c\cdot\cos\alpha=\frac{1}{2}\cdot c^2\cdot\sin\alpha\cdot\cos\alpha=\frac{1}{4}\cdot c^2\cdot\sin 2\alpha$$

- **Zadano je: hipotenuza c i šiljasti kut β**



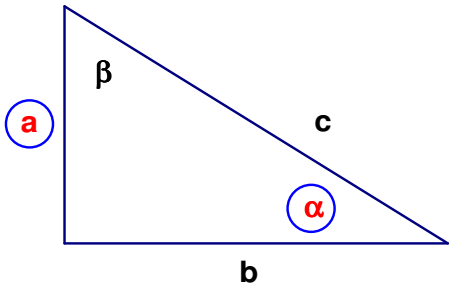
$$\alpha+\beta=90^0 \Rightarrow \alpha=90^0-\beta$$

$$\sin\beta=\frac{b}{c} \Rightarrow \sin\beta=\frac{b}{c}\ /\cdot c \Rightarrow b=c\cdot\sin\beta \quad , \quad \cos\beta=\frac{a}{c} \Rightarrow \cos\beta=\frac{a}{c}\ /\cdot c \Rightarrow a=c\cdot\cos\beta$$

$$O=a+b+c=c\cdot\cos\beta+c\cdot\sin\beta+c=c\cdot(\cos\beta+\sin\beta+1)$$

$$P=\frac{1}{2}\cdot a\cdot b=\frac{1}{2}\cdot c\cdot\cos\beta\cdot c\cdot\sin\beta=\frac{1}{2}\cdot c^2\cdot\cos\beta\cdot\sin\beta$$

- **Zadano je: kateta a i šiljasti kut α**



$$\alpha+\beta=90^0 \Rightarrow \alpha=90^0-\beta$$

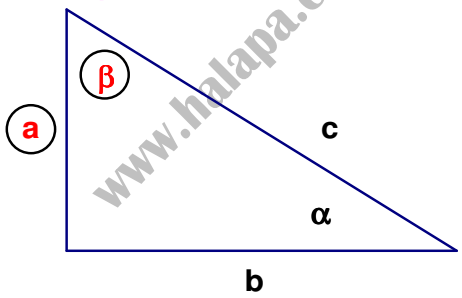
$$\sin\alpha=\frac{a}{c} \Rightarrow \sin\alpha=\frac{a}{c} \ /\cdot c \Rightarrow c\cdot\sin\alpha=a \ /:\sin\alpha \Rightarrow c=\frac{a}{\sin\alpha}$$

$$tg\,\alpha=\frac{a}{b} \Rightarrow tg\,\alpha=\frac{a}{b} \ /\cdot b \Rightarrow b\cdot tg\,\alpha=a \ /:tg\,\alpha \Rightarrow b=\frac{a}{tg\,\alpha}$$

$$O=a+b+c=a+\frac{a}{tg\,\alpha}+\frac{a}{\sin\alpha}=a\cdot\left(1+\frac{1}{tg\,\alpha}+\frac{1}{\sin\alpha}\right)$$

$$P=\frac{1}{2}\cdot a\cdot b=\frac{1}{2}\cdot a\cdot\frac{a}{tg\,\alpha}=\frac{1}{2}\cdot a^2\cdot\frac{1}{tg\,\alpha}$$

- **Zadano je: kateta a i šiljasti kut β**



$$\alpha+\beta=90^0 \Rightarrow \alpha=90^0-\beta$$

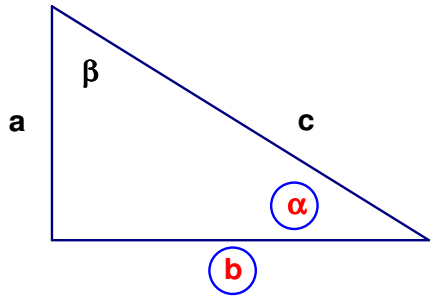
$$\cos\beta=\frac{a}{c} \Rightarrow \cos\beta=\frac{a}{c} \ /\cdot c \Rightarrow c\cdot\cos\beta=a \ /:\cos\beta \Rightarrow c=\frac{a}{\cos\beta}$$

$$tg\,\beta=\frac{b}{a} \Rightarrow tg\,\beta=\frac{b}{a} \ /\cdot a \Rightarrow b=a\cdot tg\,\beta$$

$$O=a+b+c=a+a\cdot tg\,\beta+\frac{a}{\cos\beta}=a\cdot\left(1+tg\,\beta+\frac{1}{\cos\beta}\right)$$

$$P=\frac{1}{2}\cdot a\cdot b=\frac{1}{2}\cdot a\cdot a\cdot tg\,\beta=\frac{1}{2}\cdot a^2\cdot tg\,\beta$$

- **Zadano je: kateta b i šiljasti kut α**



$$\alpha+\beta=90^0 \Rightarrow \beta=90^0-\alpha$$

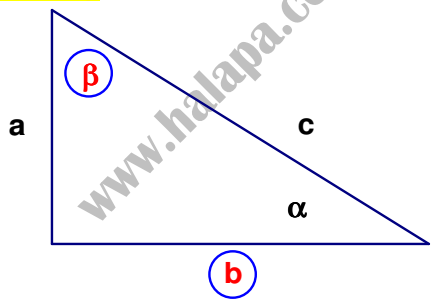
$$tg\,\alpha=\frac{a}{b} \Rightarrow tg\,\alpha=\frac{a}{b}\ /\cdot b \Rightarrow a=b\cdot tg\,\alpha$$

$$\cos\alpha=\frac{b}{c} \Rightarrow \cos\alpha=\frac{b}{c}\ /\cdot c \Rightarrow c\cdot\cos\alpha=b\ /:\cos\alpha \Rightarrow c=\frac{b}{\cos\alpha}$$

$$O=a+b+c=b\cdot tg\,\alpha+b+\frac{b}{\cos\alpha}=b\cdot\left(tg\,\alpha+1+\frac{1}{\cos\alpha}\right)$$

$$P=\frac{1}{2}\cdot a\cdot b=\frac{1}{2}\cdot b\cdot tg\,\alpha\cdot b=\frac{1}{2}\cdot b^2\cdot tg\,\alpha$$

- **Zadano je: kateta b i šiljasti kut β**



$$\alpha+\beta=90^0 \Rightarrow \alpha=90^0-\beta$$

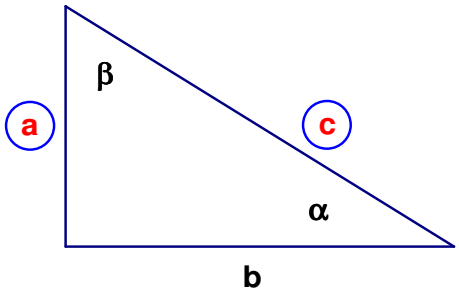
$$\sin\beta=\frac{b}{c} \Rightarrow \sin\beta=\frac{b}{c}\ /\cdot c \Rightarrow c\cdot\sin\beta=b\ /:\sin\beta \Rightarrow c=\frac{b}{\sin\beta}$$

$$tg\,\beta=\frac{b}{a} \Rightarrow tg\,\beta=\frac{b}{a}\ /\cdot a \Rightarrow a\cdot tg\,\beta=b\ /:tg\,\beta \Rightarrow a=\frac{b}{tg\,\beta}$$

$$O=a+b+c=\frac{b}{tg\,\beta}+b+\frac{b}{\sin\beta}=b\cdot\left(\frac{1}{tg\,\beta}+1+\frac{1}{\sin\beta}\right)$$

$$P=\frac{1}{2}\cdot a\cdot b=\frac{1}{2}\cdot\frac{b}{tg\,\beta}\cdot b=\frac{1}{2}\cdot b^2\cdot\frac{1}{tg\,\beta}$$

- **Zadano je: hipotenuza c i kateta a**



$$b^2 = c^2 - a^2 \Rightarrow b^2 = c^2 - a^2 \ / \sqrt{\ } \Rightarrow b = \sqrt{c^2 - a^2}$$

$$\sin\alpha = \frac{a}{c} \Rightarrow \alpha = \sin^{-1}\left(\frac{a}{c}\right) \quad , \quad \alpha + \beta = 90^0 \Rightarrow \beta = 90^0 - \alpha$$

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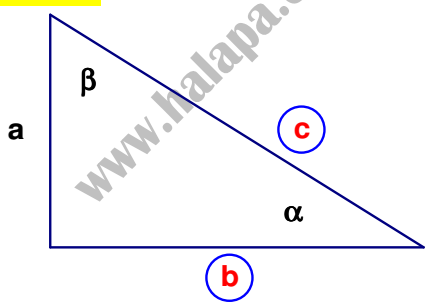
$$\cos\beta = \frac{a}{c} \Rightarrow \beta = \cos^{-1}\left(\frac{a}{c}\right) \quad , \quad \alpha + \beta = 90^0 \Rightarrow \alpha = 90^0 - \beta$$

$$O = a + b + c = a + \sqrt{c^2 - a^2} + c$$

$$P = \frac{1}{2} \cdot a \cdot b = \frac{1}{2} \cdot a \cdot \sqrt{c^2 - a^2}$$

- **Zadano je: hipotenuza c i kateta b**



$$a^2 = c^2 - b^2 \Rightarrow a^2 = c^2 - b^2 \ / \sqrt{\ } \Rightarrow a = \sqrt{c^2 - b^2}$$

$$\cos\alpha = \frac{b}{c} \Rightarrow \alpha = \cos^{-1}\left(\frac{b}{c}\right) \quad , \quad \alpha + \beta = 90^0 \Rightarrow \beta = 90^0 - \alpha$$

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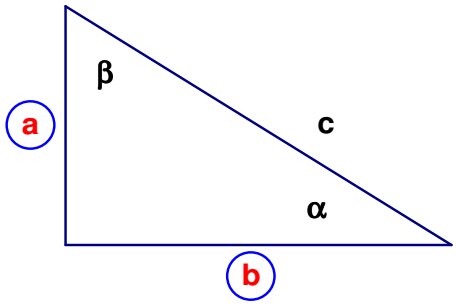
$$\sin\beta = \frac{b}{c} \Rightarrow \beta = \sin^{-1}\left(\frac{b}{c}\right) \quad , \quad \alpha + \beta = 90^0 \Rightarrow \alpha = 90^0 - \beta$$

$$O = a + b + c = \sqrt{c^2 - b^2} + b + c$$

$$P = \frac{1}{2} \cdot a \cdot b = \frac{1}{2} \cdot b \cdot \sqrt{c^2 - b^2}$$

- **Zadano je: kateta a i kateta b**



$$c^2 = a^2 + b^2 \Rightarrow c^2 = a^2 + b^2 \ / \sqrt{\ } \Rightarrow c = \sqrt{a^2 + b^2}$$

$$tg\,\alpha = \frac{a}{b} \Rightarrow \alpha = tg^{-1}\left(\frac{a}{b}\right) \quad , \quad \alpha + \beta = 90^0 \Rightarrow \beta = 90^0 - \alpha$$

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$$tg\,\beta = \frac{b}{a} \Rightarrow \beta = tg^{-1}\left(\frac{b}{a}\right) \quad , \quad \alpha + \beta = 90^0 \Rightarrow \alpha = 90^0 - \beta$$

$$O = a + b + c = a + b + \sqrt{a^2 + b^2}$$

$$P = \frac{1}{2} \cdot a \cdot b$$